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PROCEEDINGS
OF THE
ROYAL SOCIETY OF EDINBURGH.



PROCEEDINGS
OF
THE ROYAL SOCIETY
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PROCEEDINGS
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VOL. XXXIII.

1912-13.

I.—On a Method of determining Vapour Densities at High Temperatures, and on a new form of Quartz Manometer. By G. E. Gibson, B.Sc., Ph.D. (Carnegie Fellow, 1912). *Communicated by Professor JOHN GIBSON.*

(MS. received November 6, 1912. Read January 6, 1913.)

ALL methods of vapour density determination hitherto in use, in which quartz or glass manometers are employed, suffer from the disadvantage that the highest temperature attainable is about 750° C.

It is, however, possible to overcome this difficulty whenever the substance under investigation has a considerable vapour pressure below 750° C.*

If we heat the bulb (volume V) which contains the main bulk of the vapour in one furnace whose temperature T may be varied at will, and the manometer (volume v) in a separate furnace at a constant temperature t (less than 750° C.), we can calculate the vapour density Δ , relative to hydrogen at the temperature T , by the formula :

$$\Delta = \frac{m}{\mu} \frac{RT}{pv} - \frac{vT}{Vt} \delta \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here R is the gas constant, m is the total weight of substance in the apparatus, μ is the molecular weight of hydrogen (2.016), p is the measured pressure, and δ is the vapour density of the substance at the temperature t .

* Starck and Bodenstein (*Zeit.f. Elektrochem.*, vol. xvi. p. 161, 1910) describe a method similar to mine. I was unaware of the existence of this paper until mine was ready for publication.

It is essential for the success of the method to employ a suitable form of manometer. The author has devised two forms of manometer, either of which may be employed with advantage, although the second is generally to be preferred.

The first form, a sketch of which is shown in fig. 1, has been employed in a slightly modified form by C. G. Jackson * in the investigation of the cuprous-cupric bromide equilibrium. The manometer and the method of using it are fully described in Mr Jackson's paper.

Fig. 2 is a drawing of the second form of manometer.† A bulb *a*

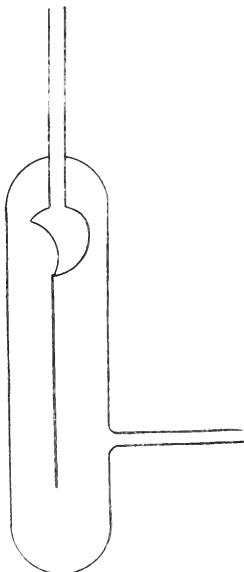


FIG. 1.

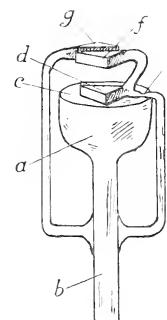


FIG. 2.

about 1 c.c. in capacity is blown on the end of a quartz tube *b*, 3 mm. in diameter, and flattened at one end so as to form a flexible quartz membrane *c* about $\frac{1}{10}$ mm. thick. This membrane is deformed by differences in pressure in the same manner as is the flexible metallic membrane of an ordinary aneroid barometer.

The quartz plate *d* is in intimate contact with the membrane, and is so adjusted that a deformation of the membrane, caused by a difference in pressure between the outside and the inside of the bulb, causes *d* to

* "Owing to an unfortunate misunderstanding, this method was not clearly referred to in my paper (*Trans. Chem. Soc.*, 1911, vol. xcix. p. 1066) as having been devised by Dr G. E. Gibson.—C. G. JACKSON."

† This is an improved form of a manometer which was described in the thesis presented by the author for the degree of Ph.D. at Breslau in July 1911.

move outwards or inwards. The quartz rod which connects the plate *d* with the tube *b* is constricted at *e*, so that it is flexible at this point. The constriction is made as close to the plate *d* as possible, so that the angular displacement of the plate for a given displacement of the membrane may be as great as possible. Both surfaces of *d* are dull so that they do not reflect. A small quartz plate *f* is rigidly connected with *d*. The inner surface of *f* (nearest the membrane) is dull, while the outer surface is polished so as to act as a mirror. A third quartz plate *g*, exactly similar to *f*, is rigidly connected with the tube *b*, and adjusted so that its polished surface is as nearly as possible in the same plane with the polished surface of *f*.

Fig. 3 shows the method of using the manometer for vapour density determinations. The main bulk of the vapour is contained in the bulb *A*. *E* and *F* are sealed off at *D* before the determination. *A* is connected with

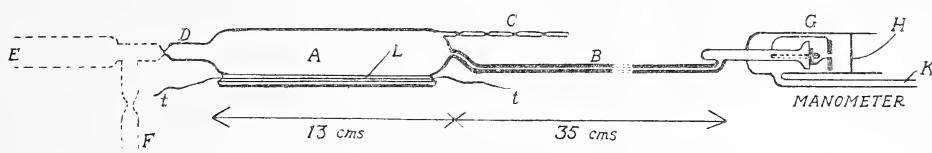


FIG. 3.

the manometer by means of a long capillary tube *B*. A series of small tubes *C* permit of the removal of small portions of the substance under investigation, without the necessity of opening the apparatus. A portion of the substance is distilled or sublimed into the outermost end of the last tube, which is then sealed off at the constriction. This is a great saving of trouble when it is desired to make a series of determinations at varying concentrations, as for example in the investigation of a dissociating gas.

The manometer is enclosed in a quartz jacket *G* which is closed at one end by a plane disk *H* of polished quartz. The tube *K* leads to the mercury manometer and to the regulating apparatus, a sketch of which is shown in fig. 4.

In order to determine the pressure of the gas in *A*, the pressure of the air inside the quartz jacket *G* is regulated until the movable mirror *f* (fig. 2) is in the zero position, *i.e.* in the position which it occupies when the pressure in *A* is equal to the pressure in *G*. The pressure is then read off on the mercury manometer *g* (fig. 4).

The deflection of the mirror is magnified and observed by the method which is employed in the Thomson mirror galvanometer.

An illuminated slit, or Nernst filament, and a convex lens of about

60 cm. focal length are placed so that a sharp image of the slit or filament would be produced at a distance of about 6 m. from the lens.

The manometer is placed in the beam of light coming from the lens, at a distance of about 3 m. from it, in such a manner that the mirrors *f* and *g* (fig. 2) are both illuminated. In this way two reflected images of the slit or filament are produced and are caught on a white cardboard screen at a distance of 3 m. from the manometer. The image from the

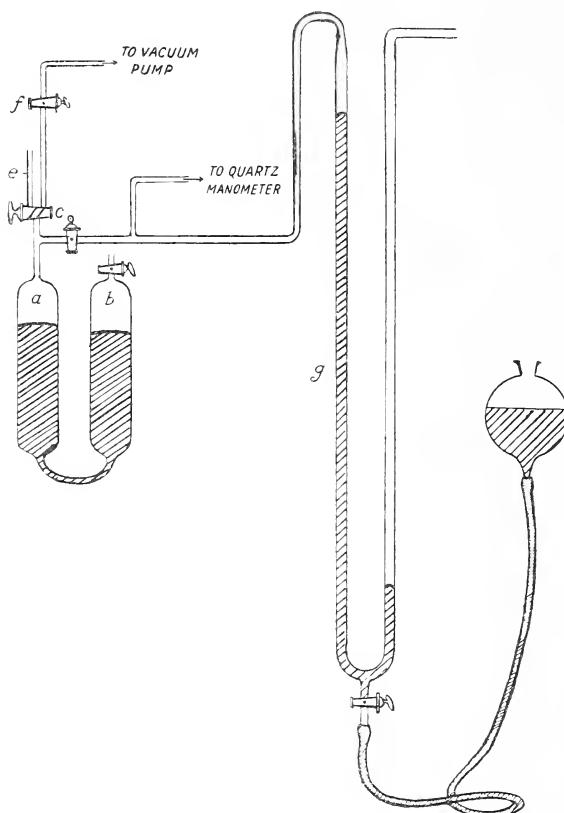


FIG. 4.

fixed mirror *g* serves to determine the zero position of the manometer. In the construction of the manometer the mirrors *f* and *g* are adjusted so that the two reflected images are as close together as possible. They need not coincide exactly, as the zero position can be marked on the screen by drawing two fine lines in ink to coincide with corresponding boundaries of the fixed and movable images.

An alteration of the zero distance between the images may be avoided by eliminating the following sources of error:—

- (1) An alteration of the relative position of mirrors, lens, slit, and image.

To avoid this source of error it is sufficient to mark the position of manometer, slit, and screen on the table. A displacement of any one of these by one or two centimetres has no appreciable effect. It is convenient to have the slit and lens securely fixed, as an alteration in the comparatively small distance between them produces the greatest effect.

(2) If the screen is rotated about a vertical axis out of its position perpendicular to the ray of light, the relative distance of the images is increased in inverse ratio to the cosine of the angular displacement. This source of error is, however, very small. It is sufficient to place the screen by eye as nearly as possible perpendicular to the ray of light.

(3) A rotation of the manometer about an axis perpendicular to the plane of the mirrors produces a displacement which is greater, the greater the initial distance between the images.

When a point of light is substituted for the slit, the two point images should not be more than 1 or 2 mm. apart. It is quite easy to adjust the mirrors prior to sealing the membrane into the outer quartz cover G, so that this may be the case. A rotation of the manometer during a determination by more than an exceedingly small amount can be avoided by securing the quartz tube K (fig. 3) to a wooden support which is kept in position on the table by a heavy weight.

When the above-mentioned sources of error are avoided, the zero point—*i.e.* the distance between the two images—is perfectly constant. As quartz is the only material used in the construction of the manometer, change of temperature does not cause a relative displacement of the mirrors. A good manometer should give a deflection of $\frac{1}{2}$ mm. on the screen, with a pressure difference of 1 mm. of mercury, and should stand a pressure difference of $1\frac{1}{2}$ atmospheres without breaking. With this degree of sensibility the pressure readings are reliable to less than 0.5 mm. of mercury.

The apparatus for adjusting the pressure in the outer jacket G (fig. 3) is shown in fig. 4.

When the three-way tap *c* is in the position shown in the drawing, the vacuum pump is in communication with the vessel *a*. By turning the tap *c* through 180° *a* is brought into communication with the atmosphere. The space above the mercury in *b* is nearly completely evacuated, but a little air is left in it to act as a cushion and prevent the top of the vessel from being broken by the impact of the mercury, should air be let too suddenly into *a*.

A slight increase in pressure can be produced in *a* by closing the top of the tube *e* with the finger and allowing *e* to communicate with *a* through the tap *c*. A slight decrease in pressure can be produced in *a* by closing the tap *f* and allowing the evacuated space between *c* and *f* to communicate with *a*.

In this way any pressure up to one atmosphere can be obtained. By blowing air through *e* into the apparatus, readings may be made up to nearly 2 atmospheres.

To test the efficiency of the method, the vapour density of mercury was determined. The accuracy of the determination depends necessarily on the accuracy with which the temperature of the vapour in the bulb A (fig. 3) can be measured.

The bulb A was fitted with about 1 mm. play into the heating-tube of a Heraeus electric furnace 60 cm. in length. A Le Chatelier thermocouple *tt* was made to pass through the quartz tube L (fig. 3). L was sealed into the bulb A and passed through the interior of A, so that it was completely surrounded by mercury vapour. In this way the thermocouple was prevented from coming into direct contact with the walls of the furnace, whose temperature often varies from point to point in an irregular manner. Those parts of the furnace wall which are covered with the heating spiral of platinum foil are generally at a considerably higher temperature than the immediately adjacent parts which are not covered by the heating foil.

The thermocouple could be moved freely in L so that the temperature at different parts of L could be determined. The bulb A was placed at a portion of the furnace which by previous trial had been found to have a nearly uniform temperature.

The temperature at the ends of A was two degrees lower at 400° C., and five degrees lower at 900° C., than at the middle.

For the calculation of the vapour density the temperature of the vapour in A was taken to be the mean of the temperature at the middle and of the temperature at either end.

The thermocouple used had been calibrated at the Physik. Techn. Reichsanstalt in Berlin, up to 1400° C. The E.M.F. was measured with a Siemens and Halske Millivoltmeter which was tested by direct comparison with a standard instrument.

The cold joints were immersed in water of approximately 20° C. contained in vacuum vessels. Their temperature was measured at each determination.

The arrangement was also tested by direct comparison with a Zincke

thermometer which had been calibrated at the Reichsanstalt. Immersed in a bath of sodium and potassium nitrates, the thermocouple and the thermometer gave readings which agreed to within one degree at 400° C.

The manometer was kept at a constant temperature of $580^{\circ} \pm 5^{\circ}$ C. in another furnace, which was heated by a spiral of platinum wire.

The ends of the furnace which contained the bulb A were closed by means of tight-fitting asbestos plugs. This was found necessary to prevent the disturbing effect of draughts of air. The end of the manometer furnace, through which the ray of light from the illuminated slit had to pass, was closed by means of a polished plate of transparent quartz.

Without this plate, irregularly heated vortices of air were set up at the mouth of the furnace, which, on account of their varying refractive index, caused the images of the slit to flicker, and made the reading of the zero point very difficult.

With the plate in position the images were practically steady, and the zero point could be read with ease and certainty.

A space of 6 cm. separated the ends of the two furnaces, and had to be traversed by part of the capillary B (fig. 3). This portion of B was wrapped in asbestos fibre and heated by means of a bunsen flame. A correction was made for the vapour in this part of B. Its temperature was estimated to be 800° C. An error of several hundred degrees in the estimation of this temperature would have no appreciable effect on the accuracy of the result.

0·1030 gram of Kahlbaum's purest mercury was weighed out in a small quartz tube and introduced into the bulb A (fig. 3). The whole apparatus was then evacuated by means of an automatic Töpler mercury pump. When a vacuum of less than 0·01 mm. of mercury had been attained, the bulb A was sealed off at D with an oxygen-gas blowpipe.

The Heraeus electric furnace was then moved into the position which by previous trial had been found to give the most uniform temperature in A. The furnace was mounted on iron rails to facilitate this operation.

The determination was then proceeded with as already described. At the conclusion of the determination the apparatus was taken to pieces and the volume of the various parts was determined by weighing out with mercury.

The volume of the bulb A, including the tube C and the portion of B contained in the large Heraeus furnace, was found to be 63·73 c.c.

The volume of the connecting portion of the capillary tube B which was heated by means of the bunsen flame was 0·06 c.c. The volume of the

manometer, including the portion of B contained in the smaller furnace was found to be 1.30 c.c.

The vapour density is therefore given by the equation :

$$\Delta = \left\{ \frac{0.1030 R}{2.016 \times 63.73} \cdot \frac{1}{p} - \frac{99.21}{63.73} \left(\frac{0.06}{1073} + \frac{1.30}{853} \right) \right\} T \quad . \quad . \quad (2)$$

The barometer readings and the readings of the mercury manometer were reduced to 0° C.

At the conclusion of the experiment the outside and inside of the bulb A (fig. 2), were both opened to the atmosphere. The zero point had not appreciably altered.

The measured pressures and temperatures and the values of Δ calculated according to equation (2) are given in the table :

Pressure in mm.	Temperature in ° C.	Vapour Density, Δ .
346.5	437	100.8
383.5	513	100.6
419.0	593	101.2
466.0	677	99.5
536.5	823	99.6
556.5	870	99.9
576.0	912	100.0
		Mean = 100.2

The mean value of Δ is 1 per cent. higher than the theoretical value, 99.21, calculated on the assumption that mercury vapour is monatomic. There is, however, a slight tendency towards a decrease at the higher temperatures, so that the discrepancy may be due to slight association.

The apparatus was subsequently used by the author in conjunction with Professor A. Stock and Dr E. Stamm in an investigation on the dissociation of phosphorus vapour, the results of which are at present in course of publication. Highly accurate measurements were made up to 1250° C.

In conclusion, the author desires to thank Professor Stock for his kind permission to carry out this research in the Chemical Laboratory of the Technische Hochschule in Breslau, and to acknowledge with thanks the receipt of a grant for apparatus from the Carnegie Trust.

II.—The Hydroid Zoophytes collected by the British Antarctic Expedition of Sir Ernest Shackleton, 1908. By James Ritchie, M.A., D.Sc., Royal Scottish Museum, Edinburgh.* *Communicated by W. EAGLE CLARKE, F.L.S.*

(MS. received September 11, 1912. Read December 2, 1912.)

HYDROID Zoophytes were collected by the *Nimrod* at eleven different stations within the Antarctic Circle. These stations centre round three definite areas of Victoria Land—the open waters of M'Murdo Sound to the west and north-west of Cape Royds (“M'Murdo Sound” in the following pages); the open waters of the Sound from Cape Royds southwards (“Cape Royds” in the following pages); and the Bay, a small indentation in the land to the east of Cape Royds.

There are differences in the conditions of these areas, for the two former are in the open Sound, exposed to strong currents. The bottom, as one would expect in such a case, is destitute of mud, and is composed of stones and pebbles down to a depth of 25 fathoms. Beyond this its character is unknown: nothing but living things came up in the dredge, and this led to the supposition that here the floor of the sea is entirely carpeted with organic growths, of which sponges form the main constituent. The Bay, on the other hand, is sheltered and shallow. The bottom is composed of a very fine, tenacious black mud, the result of the decomposition of lava; and the animals are attached to small pebbles, and to shells and other organisms. All the stations, however, lie within half a mile of each other, on the western shore of Ross Island, roughly in latitude 77° 5' S., longitude 166° 1' E., far within the Antarctic Circle.

Since this position is only some twenty miles north of that investigated by the *Discovery*, it was natural to expect that the Hydroid collection should resemble that described by Professor Hickson and Mr Gravely. And this is indeed so, for of the sixteen species found (two of which remain undetermined) eight appear in the *Discovery* report. To these should be added *Campanulina belgicæ*, which seems to be identical with *Campanulina A* of that report.

Leaving the undetermined *Tubularia* out of count, the remaining six are recorded for the first time from the neighbourhood of Victoria Land—*Clavatella* (?) juv., *Eudendrium ramosum* (perhaps the *E. insigne* of Hickson and Gravely ranges with this species, as Vanhöffen suggests),

* The investigation of this collection was aided by a grant from the Carnegie Trustees.

Campanularia volubilis var. *antarctica*, *Filellum* (?) *expansum*, *Sertularia cumberlandica*, and *Sertularia glacialis*. Four of these have not hitherto been found in Antarctic regions: the doubtful *Clavatella* juv., representing a genus unknown from the Antarctic; the new variety of *Campanularia volubilis* and *Filellum expansum*, both species hitherto known only from the northern hemisphere; and *Sertularia cumberlandica*, which so far has been found only in a sub-antarctic locality, and that on the opposite side of the Antarctic continent.

The dearth of representatives of the family Plumularidæ in the Antarctic fauna of this area, as illustrated in this collection and in that of the *Discovery*, is remarkable, especially when one takes into consideration the abundance of such forms in the neighbouring Australian fauna.

Amongst Antarctic faunas outside the Victoria Land region, this small sample appears most to resemble that investigated by the German Antarctic Expedition, to the south of Kerguelen.

LIST OF SPECIES.

GYMNOBLASTEA.

Family BOUGAINVILLIDÆ.

Atractylis antarcticus (Hickson and Gravely), p. 11.

Family EUDENDRIDÆ.

Eudendrium ramosum (Linn.), p. 12.

Family CORYNIDÆ.

Clavatella (?) juv., p. 13.

Family TUBULARIDÆ.

Tubularia indet., p. 13.

Lampra parvula, Hickson and Gravely, p. 14.

CALYPTOBLASTEA.

Family HALECIDÆ.

Haleciunum tenellum, Hincks, p. 14.

Ophiodes arboreus (Allman), p. 15.

Family CAMPANULARIDÆ.

Campanularia laevis, Hartlaub, p. 19.

„ *lobata*, Vanhöffen, p. 21.

„ *volubilis*, var. *antarctica*, nov., p. 22.

Family CAMPANULINIDÆ.

Campanulina belgicae, Hartlaub, p. 24.

Lafœina longitheca, Jäderholm, p. 25.

Family LAFOËIDÆ.

Filellum (?) *expansum*, Levinsen, p. 26.

Family SERTULARIDÆ.

Sertularia cumberlandica, Jäderholm, p. 27.

„ *glacialis*, Jäderholm, p. 29.

„ *plectilis*, Hickson and Gravely, p. 30.

GYMNOBLASTEA.

Family BOUGAINVILLIDÆ.

Atractylis antarcticus (Hickson and Gravely).

Perigonimus sp., Hartlaub, 1904, p. 8, Taf. i. fig. 2.

Perigonimus antarcticus, Hickson and Gravely, 1907, p. 4, pl. i. figs. 1, 2, 3, and pl. iv. fig. 32.

Atractylis antarcticus, Vanhöffen, 1910, p. 283, fig. 8.

For the most part very poor specimens, without gonosome and with the polyps obscured by adhering diatoms and other débris, indicate the presence of this species on the stems and branches of various larger hydroids. The occurrence of a few distinct polyps and of exceedingly scarce sporosacs, however, makes identification sure.

The polyps observed are much contracted, and the hypostome is produced to a point as sharp as that of a tentacle. The gonophores, all of which bear female products, differ from Hickson and Gravely's figure in possessing a considerably longer stalk, narrow at the base and widening gradually upwards until it merges in the base of the sporosac proper. The lower portion of the stalk is marked by indefinite annular wrinkles. In one case the upper portion of the stalk appears to form a cup, from the centre of which the sporosac arises; but this is probably due to the destruction of an earlier sporosac, and the regeneration of a new one within the ruins of the old.

The dimensions of our specimens are smaller than those of the specimens collected by the *Discovery*.

I have followed Vanhöffen in attributing this species, on account of its sporosac reproduction, to Wright's genus *Atractylis*.

Dimensions.—

Hydrocaulus, height	up to 5 mm.
diameter	0·08-0·12 mm.
Hydranth, length	0·23-0·28 mm.
diameter	0·18 mm.
Sporosac (female), length (excluding stalk)	0·80-0·85 mm.
diameter	0·50-0·62 mm.
length of stalk	0·40 mm.

Localities.—M'Murdo Sound, 25-50 fathoms, 6th July 1908, growing on *Ophiodes arboreus*. Cape Royds, 30-60 fathoms, 15th August 1908, on *Ophiodes arboreus*; 30-80 fathoms, August 1908, with sporosacs, on same host.

Distribution.—This species is known only from Antarctic seas, but there it is widely distributed; for it has already been recorded from three sides of the Antarctic continent— $70^{\circ} 15'$ S. lat., $87^{\circ} 39'$ W. long., 100 metres (Hartlaub); M'Murdo "Bay," 0–130 fathoms (Hickson and Gravely); Gauss-Berg, 46 metres, and Gauss Station, 385 metres (Vanhöffen); and the present records.

Family EUDENDRIDÆ.

Eudendrium ramosum (Linnæus).

Tubularia ramosa, Linn., 1758, p. 804.

Eudendrium ramosum, Ehrenberg, 1834, p. 296; *idem*, Hincks, 1868, p. 82, pl. xiii.; *idem*, Allman, 1872, p. 332, pl. xiii.; and others.

Many colonies from several localities are referred to this species. Unfortunately, the cœnosarc has wholly disappeared, so that skeletal characters alone have to be relied upon. These, however, agree well with those of the above species, for the colonies, which reach a height of 60 mm., are dark brown in colour, exhibit a slender habit, have long, almost simple stems with only a trace of fasciculation near the base, and long simple branches. The main stem bears groups of a few rings at odd intervals, and each branch and branchlet has seven or more rings at its base.

The chitinous envelopes of female gonophores occur rarely, in groups of four or five, perched near the summit of ramuli.

Localities.—Bay (east of Cape Royds), 7–8 and 7–20 fathoms, June 1908; 10–20 fathoms, May 1908. M'Murdo Sound, 25–50 fathoms. Cape Royds, 20–30 fathoms, July 1908; 30–60 fathoms, 15th August 1908; 30–80 fathoms, August 1908.

Distribution.—This species is very generally distributed, and appears, according to Broch (1909, p. 201), to have its headquarters in the warmer parts of the Atlantic and Pacific Oceans, although it occurs generally in sub-arctic and sporadically in arctic waters. It has been recorded from the Antarctic, from localities between 70° and $71^{\circ} 15'$ S. lat., and $80^{\circ} 48'$ and $89^{\circ} 15'$ W. long., doubtfully by Hartlaub (1904); and from Gauss Station by Vanhöffen (1909), in addition to the present records. But Vanhöffen (1909, pp. 298, 299) regards the species as that generally occurring in the Antarctic Ocean, and is inclined to refer to it the forms described under other specific names from widely separated parts of this area, by Jäderholm, from South Georgia (as *E. rameum* ?, 1905); by Billard, from Booth-Wandel Island (as *E. capillare* ?, 1906); and by Hickson and Gravely, from M'Murdo "Bay" (as *E. insigne*, 1907).

Family CORYNIDÆ.

Clavatella (?) juv.

A young specimen, the characters of which are so ill-defined that its genus is recorded with hesitation. It consists of a solitary polyp with well-developed, club-shaped head, about the middle of which occur six short capitate tentacles set in two whorls, and charged with large nematocysts. There is a short stalk, terminating in a still shorter hydrorhiza, which creeps upon the perisarc of *Sertularella cumberlandica*. An exceedingly fine

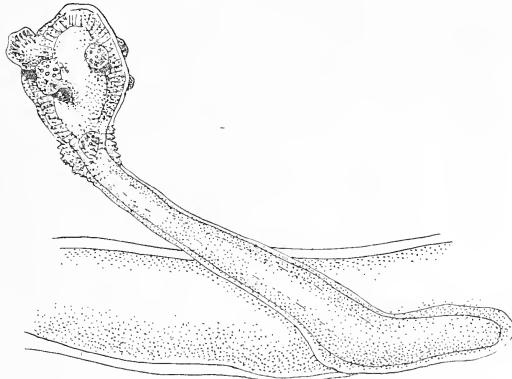


FIG. 1.—*Clavatella* (?) juv. on branch of *Sertularella cumberlandica*. $\times 87$.

perisarc clothes the hydrorhiza and hydrocaulus, and is crumpled at the junction of the latter with the hydranth, where it ceases.

There is no trace of gonosome.

Dimensions.—

Hydrocaulus, length	0·45 mm.
diameter at summit	0·055 mm.

Hydranth, length	0·19 mm.
diameter	0·14 mm.

Locality.—On *Sertularella cumberlandica* from Bay (east of Cape Royds), 7-20 fathoms, June 1908.

Family TUBULARIDÆ.

Tubularia indet.

The stems of a species of Tubularia occur in several collections, but there is present no key to specific identity. The perisarc bears closely set, regular, but very ill-defined annulations.

Localities.—Cape Royds, 10-25 fathoms, and 20-30 fathoms, July 1908;

30–60 fathoms, August 1908. M'Murdo Sound, 25–50 fathoms, 6th July 1908. Specimens labelled only “4 July 1908” (? M'Murdo Sound).

Lampra parvula, Hickson and Gravely.

Lampra parvula, Hickson and Gravely, 1907, p. 17, pl. iii. figs. 15, 16; pl. iv. fig. 35.

Examples of this species, the striking brick-red colour of which is easily recognisable in the collections, occur at several stations. They are poorly preserved, however, but the majority bear at least indications of well-developed gonophores. The largest specimen reaches a length of 28 mm., and the largest head is about 3·5 mm. in diameter.

Localities.—M'Murdo Sound, 25 fathoms and 25–50 fathoms, 6th July 1908. Cape Royds, 10–25 fathoms, July 1908; 30–60 fathoms, 15th August 1908; 50–80 fathoms, 20th August 1908. Specimens labelled only “4 July 1908” (? M'Murdo Sound).

This species has so far been found only in the neighbourhood of Ross Island, Victoria Land.

CALYPTOBLASTEA.

Family HALECIDÆ.

Haleci um tenellum, Hincks.

Haleci um tenellum, Hincks, 1861, p. 252, pl. vi. figs. 1–4; and 1868, p. 226, pl. xlv. fig. 1. *Idem*, Jäderholm, 1909, p. 55, pl. iv. fig. 12; and others.

Characteristic examples of this cosmopolitan species occur on several hosts from several areas. They differ from the examples commonly met with only in their lack of all wrinkling in the stems, and in the regularity with which a branch arises from a definite position beneath a hydrotheca. Such a branch is frequently balanced by another branch placed a very little lower on the stem and on the opposite side—an appearance of false dichotomy being approximated, as in the specimen figured by Jäderholm, from Bohuslän (1909, pl. iv. fig. 12). It is also worthy of note that the hydrothecæ themselves are borne on short terminal internodes, separated by a distinct constriction from the stem or branch proper.

A few broadly ovate gonangia are present.

Dimensions.—

Stolon, diameter	0·12 mm.
Stem, diameter	0·08–0·10 mm.
Hydrotheca, depth	0·045 mm.
diameter at mouth . . .	0·14–0·15 mm.
Gonangium, length including stalk .	1·01–1·08 mm.
greatest diameter . . .	0·52–0·60 mm.

Localities.—Bay (east of Cape Royds), 7-8 fathoms, on *Campanularia lobata*, and 7-20 fathoms, June 1908, on *Polyzoon*; 10-20 fathoms, May 1908, on *Sertularella cumberlandica*. Cape Royds, 10-25 fathoms, July 1908; 30-60 fathoms, 15th August 1908; 30-80 fathoms, August 1908, on *Sertularella cumberlandica*.

Distribution.—*Halecium tenellum* has been recognised in all the great oceans, although it is more abundant and more widely distributed in the Atlantic and Arctic areas. In the Antarctic region fresh records are gradually exhibiting a general distribution: 70° 48' S. lat., 91° 54' W. long.; 70° 00' S. lat., 80° 48' W. long. (west of Alexander Land), Hartlaub, 1904; Booth-Wandel Island, Billard, 1906; M'Murdo "Bay" and neighbourhood, Hickson and Gravely, 1907; Gauss Station and Gauss-Berg, Vanhöffen, 1909; and the present records.

Ophiodes arboreus (Allman).

Halecium robustum, Allman, 1888, pl. iv. figs. 1-3; but not *Halecium robustum*, Allm., Ritchie, 1907, p. 524; nor Vanhöffen, 1910, p. 319, fig. 35.

Halecium arboreum, Allman, 1888, p. 10. *Idem*, Hickson and Gravely, 1907, p. 27, pl. iv. figs. 17-29; but probably not *H. arboreum*, Allm., Jäderholm, 1905, p. 11, Taf. v. fig. 4.

Ophiodes arboreus, Billard, 1910, p. 4.

Many fragments which agree with Hickson and Gravely's description of this robust species occur in the collections from various localities. Unfortunately, the fragments of the trophosome are in so bare a condition that only with the greatest difficulty could any trace of hydrothecæ be discovered. These, however, enable me to represent them somewhat less diagrammatically than the authors of the *Discovery* report have done. This is the more important since the hydrothecæ in the Antarctic specimens differ in detail from those of Allman's type, as both Vanhöffen and Billard have remarked. The difference, however, as regards the length of the hydrothecæ and their supports, is usually less marked than the figures in the *Discovery* report would indicate; although, on the other hand, I cannot agree altogether with the remark (p. 28), contradicted by the figures, that the hydrotheca is "adnate throughout its whole length." As a matter of fact, there is some variation in the condition exhibited by the hydrothecæ. While in the majority of cases a hydrotheca is closely apposed to the inter-node, at least the margin is free, although it does not, or scarcely does, reach beyond the node. Sometimes, however, a hydrotheca occurs, generally towards the base of a hydroclade, which is not only quite free, but which, by the prolongation of the tubular portion which it caps, has been carried some distance beyond the node. These remarks refer to primary hydro-

theæ; where secondary and subsequent individuals are present they are free, and may reach far beyond their own internode.

The dimensions of the hydrothecæ are greater than those of the type specimen, judging from Billard's measurement.

Several of the extraordinary aggregates of gonangia-bearing branches occur. They resemble the "coppinia" masses which occur in the Lafoëidæ only in a very general way; for here is none of the aggregating of gonangia

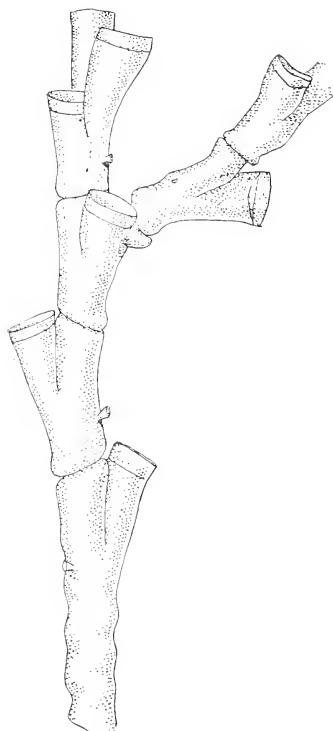


FIG. 2.—Part of branch of *Ophiodes arboreus* from Cape Royds. $\times 25$.



FIG. 3.—Part of branch of *Ophiodes arboreus* from Cape Royds, showing three hydrothecæ. $\times 25$.

into closely compressed, compact, unified groups radiating from the supporting stem. The aggregate is of a more casual type, the essence of which is that the stolon-like branches, upon which the gonangia are grouped in pairs, arise from a limited portion of the stem, and ramify to such an extent that an inextricable tangle arises. The hook-like tube at the distal end of a gonangium is usually even longer and more curved than Hickson and Gravely represent in their figure. All the gonangia observed contained female reproductive products.

There has just come to my notice the description of a species from North-

East Greenland, *Halecium grænlandicum*, Kramp,* in which the author has found aggregates of reproductive bodies remarkably like those of *Ophiodes arboreus*, a species to which Kramp makes no reference. The trophosome of the Arctic species differs, however, from that of the Antarctic, and in the former nematophores have not been observed.

Dimensions.—

Internode, length	0·55-1·08 mm.
diameter	0·22-0·35 mm.
Hydrotheca, diameter	0·26-0·30 mm.
depth	0·06-0·085 mm.
Gonangium, length	1·02-1·09 mm.
Greatest diameter	0·49-0·52 mm.

Localities.—M'Murdo Sound, 25-50 fathoms, 6th July 1908 (gonangial clusters). Cape Royds, 20-30 fathoms, July 1908; 30-60 fathoms, 15th August 1908 (stem fragments); 30-80 fathoms, August 1908 (stem fragments); 50-80 fathoms, 20th August 1908, and 60-80 fathoms, 20th August 1908 (gonangial clusters).

Distribution.—The only secure records of this species are: off Kerguelen Island, 105 fathoms (Allman), and the Antarctic Ross Island records of Hickson and Gravely, together with those given above.

Note on Synonymy.—As regards the two specific names which Allman used in his original description, preference has generally been given to *arboreum*, but Vanhöffen rejects this in favour of *robustum*, alleging that the former is only a mistaken subscription to the figure. This view cannot be maintained. Allman originally named the species *Halecium robustum*, but this name had been used for a different species of *Halecium* by Verrill in 1873, and again by Pieper in 1884, and, probably having discovered this, Allman was compelled to select a new specific name. This he publishes in the text which accompanies the plate representing his species, as *Halecium arboreum*, and adds that he has found it "necessary to change the name since the plate was printed off"; the name on the plate is "*robustum*." This last name, therefore, cannot stand.

As to the transference of the species to the genus *Ophiodes*, Hickson and Gravely have by comparison identified their specimens with the type specimens. On both, nematophores occurred on the internodes, a fact which Billard has confirmed for Allman's type, and which my present specimens also confirm. This is the distinctive and definite character of the group of species referred to *Ophiodes*, and it appears to me that more evidence is required before the character can be regarded as of so little importance

* Kramp, 1911, p. 367, pl. xxii.

that species possessing or lacking it are regarded in the same light; and that much more evidence is required before it can be held that the character is evanescent within the limits of a single species—in spite of the identifications of Jäderholm (1905) and myself (1907), misled by Allman's incomplete description, and of Vanhöffen (1910), who suggests that the nematophores first appear in older branchlets. But even the shortest branchlets in the

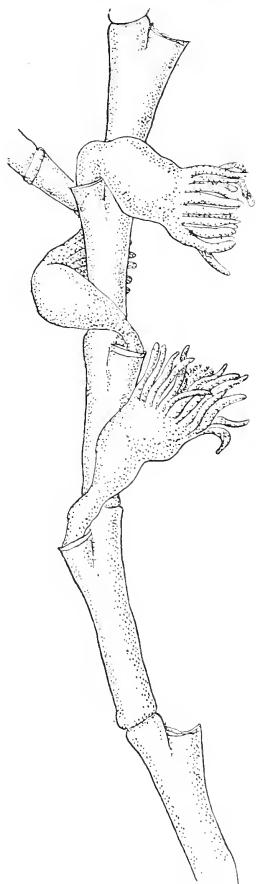


FIG. 4.—Part of branch of *Halecium macrocephalum*, from off Coats Land, previously recorded as *Halecium robustum*. $\times 25$.

Antarctic specimens in the present collection bear nematophores. On account of this character, which appears to me to be definite and decisive, I follow Billard in transferring the species to genus *Ophiodes*.

The species, lacking nematophores, which was collected by the Scottish Antarctic Expedition off Coats Land and at St Helena, and was referred by me (1907) to *Halecium robustum* (= *arboreum*), Allman, must now be separated from this nematophore-bearing species. I therefore rank it with *Halecium macrocephalum*, Allman, 1877, a species with trophosome similar

to that of *Ophiodes arboreum*, but without nematophores, and of less robust growth. With *Halecium macrocephalum* also I reckon Jäderholm's *Halecium arboreum* from the Falkland Islands and their neighbourhood, and from South Georgia (Jäderholm, 1905); but Vanhöffen's *Halecium robustum* from the Antarctic Gauss Station (Vanhöffen, 1910) bears more resemblance to the finer *Halecium sessile*, Norman.

Family CAMPANULARIDÆ.

Campanularia lœvis, Hartlaub.

Campanularia lœvis, Hartlaub, 1905, p. 565, fig. p'.

Idem, Hickson and Gravely, 1907, p. 25, pl. iv. fig. 26.

Idem, Vanhöffen, 1909, p. 298, fig. 18.

The material of this species, although it has been obtained from two localities, is scanty, and the trophosomes are in poor condition, the majority of the hydrothecæ being detached from their stems. The trophosomes, however, agree with the figures of Hartlaub and of Vanhöffen, less well with that of Hickson and Gravely. There are differences in size to be noted, for while our specimens are rather larger than those originally described by Hartlaub (judging from his magnified figure), and agree fairly closely with the measurements given by Hickson and Gravely, yet they correspond only to the minimum given by Vanhöffen. Both Hartlaub and Vanhöffen emphasise the absence of annulations at the base of the pedicel, but, although some of our specimens are smooth, a few bear very faintly marked corrugations.

Gonosome.—Hickson and Gravely have already drawn attention to the dissimilarity which exists between the gonangia described and figured by Hartlaub and those found by themselves. It is extraordinary that our examples are different from either. They are twice as tall, and are set on moderately long stalks ornamented with about six to eight strong and definite twists. In shape they are flask-like, and exceedingly graceful, widening from the stalk into a long body, one of the profiles of which is usually more convex than the other. This tapers gradually upwards into a distinct neck, which widens slightly again before the truncated termination is reached. The aperture is circular and lies at right angles to the long axis of the gonangium. Our specimens thus resemble Hartlaub's more closely than they do Hickson and Gravely's. They bear considerable likeness to the gonangia of *Campanularia volubilis*, as figured by Hincks (1868, pl. xxiv. fig. 2).

Some of the gonangia contain many much-decomposed ova. The difference between these gonangia and those formerly described may be due in

great part to sexual dimorphism, but unfortunately the sex of the earlier examples is unknown.

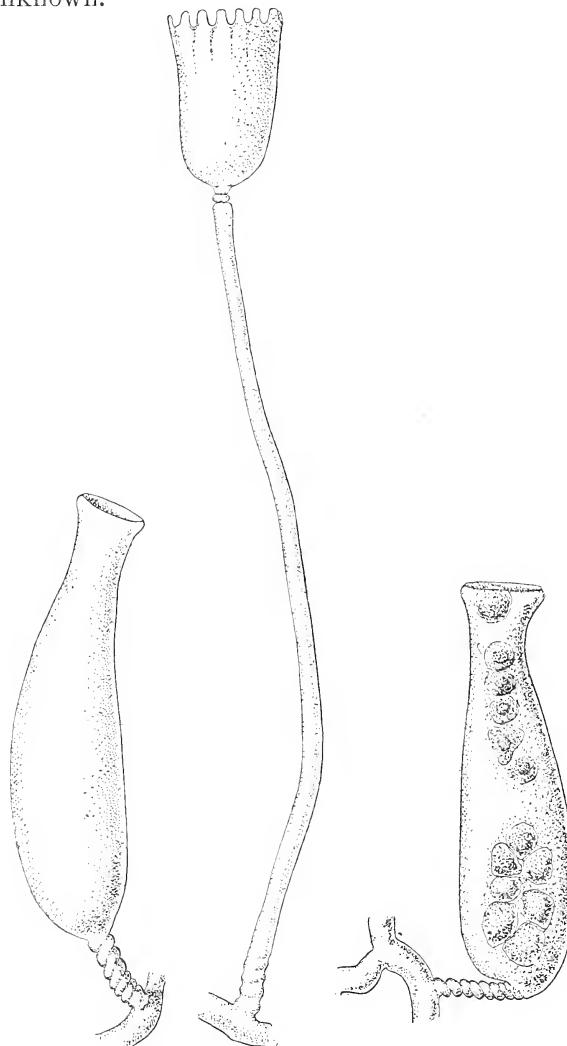


FIG. 5.—*Campanularia laevis*, trophosome and female gonangia. $\times 11.$

Dimensions.—

Stolon, diameter	0·29-0·33 mm.
Pedicel of hydrotheca, length . .	up to 11 mm.
diameter . .	0·2-0·27 mm.
Hydrotheca, length	2·2-2·7 mm.
diameter . .	1·3-1·5 mm.
Gonangium, length	5·0-5·5 mm.
greatest diameter . .	1·3 mm.
length of stalk . .	1 mm.

Localities.—Bay (east of Cape Royds), 7-8 fathoms, June 1908. M'Murdo Sound, 25-50 fathoms, 6th July 1908—rare on stems of *Ophioyes arboreus*. Off Cape Royds (south), 30-60 fathoms, 15th August 1908, on stems of *Ophioyes arboreus*.

Distribution.—*Campanularia laevis* is confined to moderately high latitudes in the southern hemisphere, within which the records, though scanty, show that it is widely distributed. Previous records are:—Calbuco, Chile (Hartlaub, 1905); M'Murdo “Bay,” 20 fathoms, and Flagon Point (Hickson and Gravely, 1907); and Gauss Station, 385 metres (Vanhöffen, 1909).

Campanularia lobata, Vanhöffen.

Campanularia verticillata, var. *grandis*, Hickson and Gravely, 1907, p. 23, pl. iv. fig. 25.

Campanularia lobata, Vanhöffen, 1909, p. 294, fig. 15.

The specimens collected by the *Nimrod*, while they are very fragmentary, are sufficient to confirm Vanhöffen's description of the peculiar, large, four-lobed hydrotheca, which here also is difficult to observe owing to the collapse of the hydrothecal walls. There can be little doubt that the variety of *Campanularia verticillata* described by Hickson and Gravely belongs to this species, for the dimensions of the hydrotheca of that form agree with my measurements, although both are somewhat less than those of Vanhöffen's specimens. The fact that the present specimens and those of the *Discovery* were obtained from approximately the same neighbourhood, a distance of only some twenty miles separating the two stations in M'Murdo Sound, converts the likelihood of the identity into all but demonstrable certainty.

The gonosome was not represented.

Dimensions.—

Hydrotheca, depth	0·67-0·74 mm.
greatest diameter	0·33-0·39 mm.
Pedicel, length	0·77-1·19 mm.
diameter	0·12-0·13 mm.

Localities.—Bay (east of Cape Royds), 7-8 fathoms, June 1908. Off Cape Royds (south), 20-30 fathoms, July 1908.

Distribution.—*Campanularia lobata* is known only from Antarctic seas, the earlier records being from M'Murdo “Bay,” 20 fathoms, and Flagon Point, 20 fathoms (Hickson and Gravely), and from Gauss Station, 385 metres (Vanhöffen).

Campanularia volubilis (Linnæus), var. *antarctica*, nov.

Trophosome.—Springing from a creeping stolon which is for the greater part smooth, although lengths here and there show strongly marked twists, are closely set hydrocauli and gonangia. A hydrocaulus is generally straight, with a diameter about half that of the stolon, marked by a few (three to five) twists at the base, which die away, leaving the remainder of the perisarc smooth. Rarely do additional undulations appear

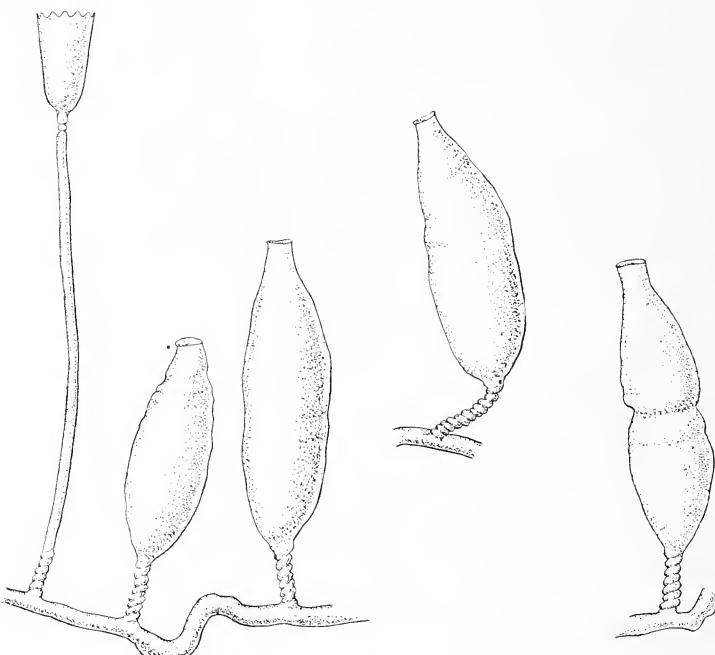


FIG. 6.—*Campanularia volubilis*, var. *antarctica*, trophosome and gonangia. The gonangium on the right shows an abnormal constriction. $\times 15$.

on the outline of the hydrocaulus except where a stalk has been truncated and a new portion, which possesses the usual basal twists, has been regenerated from the place.

The hydrothecæ are of the *volubilis* type, twice as deep as wide, with walls that, in profile, scarcely diverge from each other, and with ten to thirteen rounded marginal teeth. The hydrotheca has a prolonged base, from which it is separated by a diaphragm, and which rests upon a very definite spherical internode intervening between hydrotheca and hydrocaulus. The hydrothecæ are slightly attached and in most cases have been broken off.

Gonosome.—Gonangia spring abundantly from the stolon. They are large and flask-shaped, gradually widening from below to a median

cylindrical portion which tapers upwards into a neck terminated by a small aperture. They vary much in size and shape—especially as regards the length and development of the terminal neck, which is frequently almost obsolete. Faint undulations in the outline were rarely observable on the distal half, and a few abnormal specimens exhibited a strong median constriction, calling to mind similar abnormalities noticed in the gonangia of *Thuiaria tenera* from the Firth of Clyde (Ritchie, 1911, p. 219, fig. 4). The gonangia stand erect, supported on moderately long pedicels, which bear strongly marked twists, usually about six, but varying from five to ten in number.

The soft parts, both of trophosome and of gonosome, were unfit for examination.

Dimensions.—

Stolon, diameter	0·13-0·15 mm.
Hydrotheca, height	0·86-0·93 mm.
diameter at mouth	0·40-0·50 mm.
Hydrocaulus, length	2·0-3·6 mm.
diameter	0·065-0·105 mm.
Gonangium, length	1·8-2·5 mm.
greatest diameter	0·68-0·74 mm.
length of pedicel	0·45-0·50 mm.

Localities.—Bay (east of Cape Royds), 7-8 fathoms, and 7-20 fathoms, June 1908, growing on *Campanularia lobata*, rare. Cape Royds (south), 20-30 fathoms, August 1908.

Distribution.—*Campanularia volubilis* has been recorded as a boreal species, widely distributed within the Arctic Circle and in the North Atlantic, and occurring also in the North Pacific, on the west coast of North America. It has not hitherto been found in the southern hemisphere.

Remarks.—These specimens differ from typical *Campanularia volubilis* in their large size, in possessing almost smooth hydrocauli, in the length of the stalk of the gonangium, and in the shortness and poor development of the terminal neck of the gonangium itself. The trophosome resembles that of *Campanularia tincta*, but the gonangia of the two species are different.

It is interesting to note that large typical specimens of this species have been described by Marktanner-Turneretscher from within the Arctic Circle—Bastian Island, in the neighbourhood of East Spitsbergen (Marktanner-Turneretscher, 1895, p. 406).

Family CAMPANULINIDÆ.

Campanulina belgiceæ, Hartlaub.*Campanulina belgiceæ*, Hartlaub, 1904, p. 10, pl. i. figs. 8, 9.*Idem*, Vanhöffen, 1909, p. 308, fig. 28.*Campanulina A*, Hickson and Gravely, 1907, p. 31.

Variability in the details of structure appears to be the rule in this form. Only about a dozen hydrothecæ have been observed on a fragment of *Sertularella cumberlandica* from a single locality (a solitary stalk occurred at another dredging ground), yet here there are examples with the slightly bulging profile described by Hartlaub, and represented in some of the examples figured by Vanhöffen (1909, p. 308, fig. 28); and others with the straight, gradually widening outline of Hartlaub's fig. 9 (left individual), characteristic also of Hickson and Gravely's *Campanulina A* from M'Murdo Bay. The majority have peduncles with a distinct spiral twist, but in some the peduncles are almost smooth, although the base is generally ornamented by a few annulations. Here, as in Vanhöffen's specimens, there is a tendency in some of the hydrothecæ, owing to the special development of opposite opercular flaps, for the operculum to bear a faint resemblance to the roof-shape typical of species of *Stegopoma*.

The operculum has about eleven or twelve flaps. The hydranth has a mound-like hypostome, surrounded by tentacles which vary in number from twelve to seventeen.

There was no trace of gonosome.

Dimensions.—

Stolon, diameter	0·04 mm.
Hydrocaulus, length	0·25–0·49 mm.
diameter	0·04–0·07 mm.
Hydrotheca, length, from diaphragm	0·38–0·49 mm.
greatest diameter	0·11–0·15 mm.
length of operculum	0·10–0·14 mm.

Locality.—Growing on *Sertularella cumberlandica*, from Bay (east of Cape Royds), 7–20 fathoms, June 1908; very rare. A single stalk was seen on material collected in Bay, 7–8 fathoms, June 1908.

Two smaller but badly damaged hydrothecæ, with the chitinous basal diaphragm incompletely developed or altogether awanting, I regard as miniature specimens of this species. They occur upon the hydrorhiza of *Eudendrium ramosum* from M'Murdo Sound, 25–50 fathoms, 6th July 1908; very rare.

Distribution. — *Campanulina belgicae* is known only from the Antarctic: various stations between 70° and 71° 19' S. lat., and 80° to 89° 14' W. long. (Hartlaub, 1904); M'Murdo Bay (Hickson and Gravely, 1907); Gauss Station (Vanhöffen, 1909).

It seems to me that the characters of the trophosome of Hickson and Gravely's *Campanulina A* fall within the range of variation of *Campanulina belgicae*, with which species I therefore range it.

Lafoëina longitheca, Jäderholm.

Lafoëina longitheca, Jäderholm, 1904, p. 4; and 1905, p. 20, pl. viii. figs. 1 and 2.

Idem, Hickson and Gravely, 1907, p. 29, pl. iv. fig. 31.

Jäderholm distinguishes his sub-antarctic species from the sub-arctic *Lafoëina tenuis* of Sars, to which it is most closely related, on account of the narrower and very long tube-shaped hydrothecæ, furnished with a markedly everted aperture margin, and also on account of the very short sarcothecæ of the former. Specimens which I assign to Jäderholm's species modify these distinctions and bring the Antarctic form into even more apparent affinity with the Arctic.

The hydrothecæ of the examples collected by the *Nimrod* are much shorter than the average length given by Jäderholm, and on the contrary are wider than his examples. In these points they agree rather with the specimens obtained by the *Discovery*, as the following table shows. The proportions of both sets, however, fall practically within the limits of variation recorded by Jäderholm. On the other hand, Jäderholm emphasises the shortness of the sarcothecæ of *Lafoëina longitheca* as compared with those of *Lafoëina tenuis*, and his figures and measurements are confirmed by those of Hickson and Gravely. In the specimens before me, however, the sarcothecæ are not of uniform height, and are in general much longer, their height, in some cases, almost equalling that of the neighbouring hydrothecæ. And, judging from Sars's figures, it occasionally transcends even that of the exceptionally lengthy sarcothecæ of *Lafoëina tenuis*.

The distinction between the two species is less evident than was supposed, but I still consider them distinct, on account of the shape and dimensions of the hydrotheca, which in *Lafoëina longitheca* is longer (*cf.* measurements below), has an everted margin round the aperture, and lacks the strong and abrupt constriction that separates the base of the hydrotheca of *Lafoëina tenuis* from the stolon, having instead a gradually narrowing basal portion ornamented by five or six spiral twists.

The gonosome was not observed.

For comparison, I give in tabular form the dimensions of the Antarctic and Arctic species:—

	<i>Lafoëina longitheca.</i>			<i>Lafoëina tenuis.</i>	
	Present specimens.	Jäderholm's specimens.	Hickson and Gravely's specimens.	Sars's figures (1873).*	Jäderholm's figures (1909).*
Stolon, diameter . . .	mm. 0·06-0·07	mm. ...	mm. ...	mm. ...	mm. ...
Hydrotheca, length . . .	0·60-0·98	0·7-1·7	0·7-1·25	0·43-0·52	0·32
diameter . . .	0·14-0·18	0·08-0·13	...	0·14	0·12
Sarcotheca, length . . .	0·11-0·55	0·10-0·25	0·12-0·20	0·20-0·44	0·22
diameter of pedicel	0·025-0·032	0·02	0·02
" head . . .	0·042-0·048
length of head . . .	0·062

* These dimensions, calculated from magnified figures, must be taken as approximate only.

Locality.—Bay (east of Cape Royds), 7 to 8 fathoms, June 1908, on a Polyzoon.

Distribution.—Recorded from both sides of the Antarctic continent: various localities in South Georgia, 64-250 metres (Jäderholm, 1905); M'Murdo Bay, 5 to 20 fathoms (Hickson and Gravely, 1907); and present record.

Family LAFOËIDÆ.

Filellum (?) expansum, Levinsen.

Filellum (?) expansum, Levinsen, 1893, p. 30, pl. vii. fig. 6, 7. *Idem*, Jäderholm, 1909, pp. 21, 77, pl. vii. fig. 12.
Lafoëa expansa, Billard, 1907, p. 179.

I record this species, because it has been regarded by several workers as a Hydroid, without being at all convinced that it belongs to this group. Here, as in other cases, only the empty tube is present, and the critical animal has again escaped detection. Our specimens are on the whole larger than those which occur in northern waters, judging by Billard's measurements (1907, p. 180).

Dimensions.—

Hydrotheca, length free . . .	0·15-0·20 mm.
,, adnate . . .	0·19-0·24 mm.
diameter (at mouth) . . .	0·07-0·08 mm.

Localities.—M'Murdo Sound, 25 to 50 fathoms, July 1908. Specimens were also observed on a Polyzoon from Cape Royds, 30 to 80 fathoms, August 1908.

Distribution.—The species has been recorded from the Arctic and North Atlantic Ocean—from Spitsbergen and Greenland, south to the southwards of the Gulf of Cadiz. But the present occurrence is very far removed from this area.

Family SERTULARIDÆ.

Sertularella cumberlandica, Jäderholm.

Sertularella cumberlandica, Jäderholm, 1905, p. 27, pl. x. figs. 8-11; referred to in Vanhoffen, 1909, p. 327.

Several dredgings revealed the presence of this species, which did not occur in the *Discovery* collections. The finest colony was obtained between

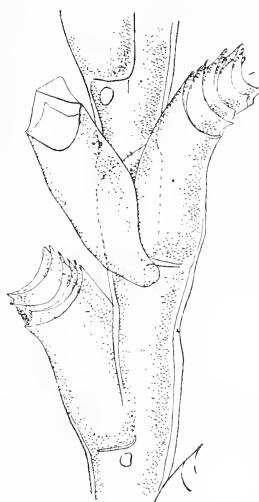


FIG. 7.—Part of branch of *Sertularella cumberlandica*, with abnormal free hydrotheca. $\times 50$.

10 and 20 fathoms in the Bay—a straggling individual with stout fascicled stem 1·5 mm. in thickness at the base, and 5 cm. high, although it is incomplete. The stem bears numerous long, straggling branches, one 7 cm. in length, which exhibit an arrangement approximating to alternate. These primary branches, which are fascicled for a great part, bear many smaller monosiphonic branchlets, but it is seldom that individuals of tertiary degree occur. The expanse of this colony from side to side is about 10 cm.

There is nothing to be added to Jäderholm's description of the detailed characters of the species.

A curious abnormality occurs on several of the branches—where, from beneath a hydrotheca on the anterior of the branch, a second hydrotheca arises exactly in the position usually occupied by a gonangium (fig. 7). This abnormal hydrotheca is quite free, except for the narrow neck which

unites it to the stem, and it stands upright at right angles to the plane in which the normal hydrothecæ lie, thus resembling the hydrotheca of some epizoic species of *Thyrosocyphus*. The resemblance to *Thyrosocyphus* is still more pronounced where, as in one case from Cape Royds, a stolon-like fascicle-tube gives rise to these free hydrothecæ; but the real condition is betrayed by the presence of a normal *Sertularella cumberlandica* branch which springs from the same tube.

The branches of the colony from the Bay are laden with elegant gonangia.

Dimensions.—

Internode, length *	.	.	0·46–0·80 mm.
diameter	0·18–0·30 mm.	
Hydrotheca, portion adnate	0·30–0·32 mm.	
„ free	0·24–0·29 (0·45) † mm.	
diameter at mouth	0·16–0·18 mm.	
Gonangium, length ‡	1·9 –2·2 mm.	
greatest diameter	0·8 –0·9 mm.	

Localities.—Bay (east of Cape Royds), 10–20 fathoms, May 1908, single fine colony with many gonangia; 7–20 fathoms, June 1908, fragments.

M'Murdo Sound, 25–50 fathoms, 6th July 1908; not common.

Off Cape Royds (south) 20–50 fathoms, July 1908, not common; 30–60 fathoms, 15th August 1908; 30–80 fathoms, August 1908, with gonangia.

Distribution.—*Sertularella cumberlandica* has been regarded only as a sub-antarctic species; the above records add it to the Antarctic fauna, but on the opposite side of the Antarctic Continent. The only previous records are those of Jäderholm, who describes it as occurring at various stations in Cumberland Bay, South Georgia.

Remarks.—Vanhöffen (1909, p. 327) identifies Jäderholm's *Sertularella cumberlandica* with *Sertularella subdichotoma*, Kirchenpauer. In so doing, however, he has failed to take account of the great difference in habit which characterises colonies of the two species. Kirchenpauer describes the type specimens of *S. subdichotoma* as being "sehr feine, schwach verästelte Stämmchen, etwa 3 cm. lang, Stamm, Äste und Zweige alle von gleicher Dicke" (Kirchenpauer, 1884, p. 46). Hartlaub writes of the same species: "meistens vielfach mit einander durchklammernde verbundene, mono-

* From any point on an internode to the corresponding point in its predecessor or successor.

† The figures given are those of primary hydrothecæ; those within the parentheses show to what length reduplication of the margin may prolong the free portion.

‡ Excluding terminal tube, which measures 0·02 mm. long by 0·015 mm. diameter at the mouth.

siphone Stämme" (Hartlaub, 1900, p. 33). And Vanhöffen himself regards his own specimens as "sehr ähnlich der vorigen [*Sertularella glacialis*] in Wuchs" (Vanhöffen, 1909, p. 326). But the stem of *Sertularella glacialis* is monosiphonic, and many colonies are aggregated in an entangled mass. The habit of *Sertularella cumberlandica*, with fascicled stem and branches, is very different from that of the species described in the above quotations, as a glance at Jäderholm's figures shows. Further, the free portions of the hydrothecæ of *Sertularella cumberlandica* lie more closely against the internode than do those of *Sertularella subdichotoma*, and the axis of the distal half is seldom at a pronounced angle with that of the proximal, as is the rule in the latter species.

I retain, therefore, Jäderholm's *Sertularella cumberlandica*, regarding it as a species distinct from *Sertularella subdichotoma*.

Sertularella glacialis, Jäderholm.

Sertularella glacialis, Jäderholm, 1904, p. 9; 1905, p. 26, pl. x. figs. 3-7; but probably not *Sertularella glacialis*, Vanhöffen, 1909, p. 325, fig. 40. (See fig. 10, p. 32.)

From various localities near Cape Royds come intertwined clumps of colonies with simple stems and branches, and very pronounced nodes, which seem to me to belong to this species. Unfortunately, there is no gonosome present, but, for the rest, the lax habit of growth, the structures of internodes and hydrothecæ, the proportions of the latter—"half or more than half free from the internode,"—the colour of the colonies as preserved in alcohol—"brown to dark brown,"—these characters agree with those described by Jäderholm, as do our specimens with his figures (see fig. 10, p. 32). Our specimens, however, are slightly larger in size than those from the Graham Region, as comparison of Jäderholm's measurements with those given below shows.

Dimensions.—

Internode, length	0·64-1·15 mm.
diameter	0·14-0·16 mm.
Hydrotheca, length adnate	0·20-0·27 mm.
" free	0·24-0·30 mm.
diameter at mouth	0·17-0·20 mm.

Localities.—Cape Royds, 10 to 25 fathoms, July 1908, rare; 30 to 60 fathoms, 15th August 1908; 30 to 80 fathoms, August 1908, rare.

Distribution.—*Sertularella glacialis* is known only from Antarctic seas, within which it has now been found on both sides of the Antarctic continent. Previous record: Graham Region, Seymour Island to Cape Seymour, 150 metres (Jäderholm).

In discussing *Sertularella pectilis*, following, I have given the reasons which lead me to regard the specimens identified by Vanhöffen (1910) as *Sertularella glacialis*, as belonging not to this species but to *Sertularella pectilis*. There is little to choose between the characters of the trophosomes of *Sertularella glacialis* and of *Sertularella subdichotoma*, Kirchenpauer.

Sertularella pectilis, Hickson and Gravely.

Sertularella pectilis, Hickson and Gravely, 1907, p. 20, pl. iii. fig. 21.

Sertularella glacialis, Vanhöffen, 1910, p. 325, fig. 40; but not of Jäderholm, 1904, p. 9, and 1905, p. 26, pl. x. figs. 3-7.

Colonies associated in the inextricable matted tangle which characterises this species have been found at several stations. At the base of each

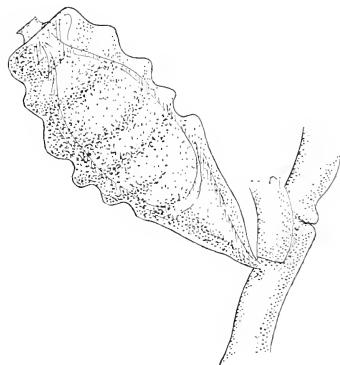


FIG. 8.—Part of branch of *Sertularella pectilis*, with male gonangium. $\times 50$.

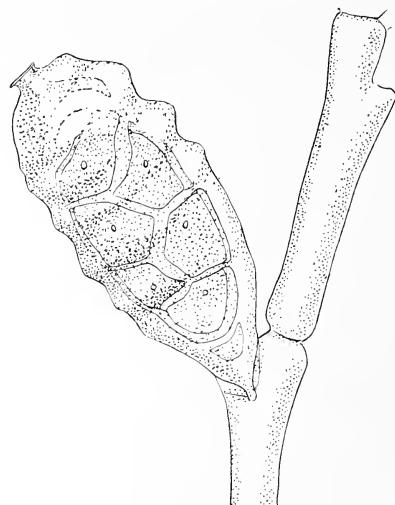


FIG. 9.—Part of branch of *Sertularella pectilis*, with female gonangium. $\times 50$.

branch there is a single twist, and rarely short sections of stem or branches bear loose corrugations.

The trophosome of our specimens agrees with that described by Hickson and Gravely from Ross Island, but the hydrothecæ of the latter are slightly larger. It also agrees very closely with the descriptions and figures by Vanhöffen of a species which, while admitting its identity with that of Hickson and Gravely, that author ascribes to *Sertularella glacialis*, Jäderholm; and the dimensions of parts are similar to those of Vanhöffen's examples. Mature gonangia were not observed by Hickson and Gravely, but Vanhöffen has described and figured an example, the sex of which is not mentioned. On our specimens both male and female gonangia occur

(figs. 8 and 9). They arise from the side of the bracket on which a hydrotheca is placed, have no distinct stalk, and gradually increase in diameter upwards. At the summit they narrow again more suddenly, and terminate in a short but very evident cylindrical tube, the margin of which is exserted. The general shape of the profile of a gonangium varies from almost oval to obovate, and the walls in the upper two-thirds bear five or six distinct flowing ridges. The gonangia with male products are similar in shape to the female gonangia, but are slightly less in size and possess more marked ridges. Neither attain the diameter recorded by Vanhöffen. The female examples are well developed, and each contains a narrow blastostyle and a large ellipsoidal gonophore which contains large ova from three to seven in number. (See fig. 9.)

Dimensions.—

Internode, length	0·52-0·89 mm.
diameter	0·18 mm.
Hydrotheca, length adnate	0·08-0·10 mm.
" free	0·28-0·31 (0·38)* mm.
diameter	0·10-0·13 mm.
Gonangium, length . . . (male) 0·80-0·90 mm., (fem.) 0·97-0·03 mm.	
greatest diameter , 0·31-0·46 mm., , 0·44-0·45 mm.	

Localities.—Bay (east of Cape Royds), 7-8 fathoms, June 1908, rare, with male gonangia. M'Murdo Sound, 25 fathoms, rare. Cape Royds (south), 10-25 fathoms, July 1908, common, with rare female gonangia; 20-30 fathoms, July 1908, rare, female gonangia; 50-80 fathoms, 20th August 1908, common, with female gonangia.

Distribution.—*Sertularella plectilis* is known only from Antarctic seas, from far-separated localities on the east side of the Antarctic continent:—M'Murdo " Bay," 20-130 fathoms (Hickson and Gravely); Gauss Station, 385 metres (Vanhöffen); and the recent records.

Note on Sertularella plectilis, Hickson and Gravely, and Sertularella glacialis, Jäderholm.—Despite the similarity which exists between their gonangia, the only apparent difference here being that the gonangial aperture is borne on a longer neck in *Sertularella glacialis*, I cannot agree with Vanhöffen in regarding these species as identical. The trophosomes are sufficiently distinct. The colours of the two are different—in *Sertularella glacialis*, brown to very dark brown; in *Sertularella plectilis*, pale straw colour to the pale transparent yellow of clover honey. This character

* The figures within the bracket give length attained when reduplicated margins are included in the measurement.

is not so trivial as may seem at first glance, for dark colour in the chitin of Hydroroids is frequently associated with robustness of minute structure. It is so in this case, for while the hydrothecæ of *Sertularella glacialis* are sturdy and perfect in form, where they have not been broken off by force, those of *Sertularella pectilis* have practically all collapsed, in spite of the fact that they do not show any other signs of injury. Hickson and

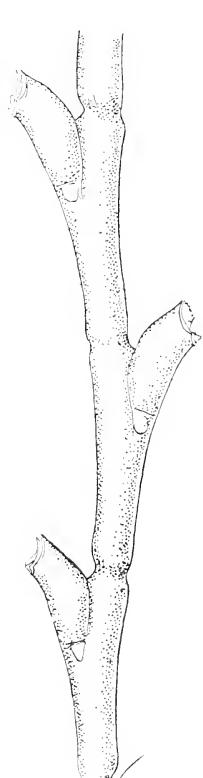


FIG. 10.—Part of branch of *Sertularella glacialis*, with hydrothecæ. $\times 30$.

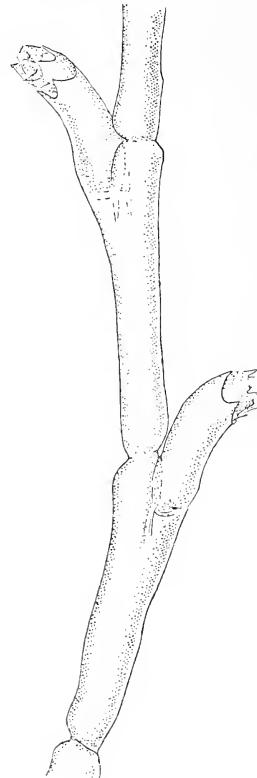


FIG. 11.—Part of branch of *Sertularella pectilis*, with hydrothecæ. $\times 50$.

Gravely also noticed that in their specimens the walls of the hydrothecæ were "extremely thin."

The shapes of the hydrothecæ differ in the two species: in *Sertularella glacialis* a hydrotheca is narrow at the base and widens upwards; in *Sertularella pectilis* the reverse is the case, for the aperture is of less diameter than is the basal portion. The proportions of the hydrothecæ also differ: in *Sertularella glacialis* the hydrotheca is more robust, for the length is less relatively to the breadth than in *Sertularella pectilis*; moreover, while in the former only half or very slightly more than half is free, in the latter, according to Hickson and Gravely, two-thirds is free, although

my measurements (see table above) indicate that the proportion free is even greater—rather over three-fourths. Lastly, in *Sertularella plectilis* the embayments between the teeth on the margin of a hydrotheca are much deeper, and the teeth therefore appear much longer and more pointed than do those of *Sertularella glacialis* (*cf.* figs. 10 and 11). These points, at least in the aggregate, definitely separate the two species concerned.

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(Issued separately February 13, 1913.)

III.—The Absorption of Light by Inorganic Salts. No. VII.:
 Aqueous Solutions of Iron Salts. By John S. Anderson,
 M.A., B.Sc., 1851 Exhibition Research Scholar (Glasgow). Communicated by Dr R. A. HOUSTOUN.

(MS. received October 18, 1912. Read January 6, 1913.)

THE absorption spectra of aqueous solutions of ferric chloride have been examined qualitatively by Jones and Anderson,* and quantitatively by Ewan,† but Ewan's measurements were confined to very dilute solutions. It was thought, therefore, that quantitative measurements might profitably be carried out on more concentrated solutions of ferric chloride and on solutions of other salts of iron.

The iron salt which has received most attention from investigators is ferric chloride, the chief researches carried out on this salt being mainly chemical. The hydrolysis of ferric chloride has engaged the attention of a great number of workers. Péan de St Gilles ‡ examined the decomposition of ferric salts which is produced when aqueous solutions of the salts are heated. Graham § showed how to prepare colloidal ferric hydroxide by dialysis. Krecke || studied the amount of hydroxide formed in aqueous solutions of ferric chloride of different concentrations and at different temperatures. He found that the amount of decomposition increases with the temperature and with increase in dilution. Ewan † showed that the hydroxide formed in solutions of ferric chloride containing less than .005 grammme-molecules per litre contains no chlorine. He found that the absorption spectrum of ferric hydroxide obtained by dialysis is not the same as that of hydroxide formed when ferric chloride is dissolved in very large quantities of water. He explains this fact as being probably due to the former hydroxide consisting of larger molecular groups than the latter. Malfitano,¶ using a collodion membrane to dialyse colloidal solutions of ferric chloride, found that the membrane, after being stained a deep red

* Jones and Anderson, "The Absorption Spectra of Solutions," Carnegie Institution of Washington, Publication No. 110, 1909.

† Ewan, "On the Absorption Spectra of Dilute Solutions," *Proc. Roy. Soc.*, lvi. p. 286, 1894; lvii. p. 117, 1895.

‡ Péan de St Gilles, *Ann. Chim. Phys.* (3), xlvi. p. 47, 1856.

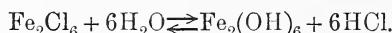
§ Graham, *Phil. Trans.*, p. 209, 1861.

|| Krecke, *Journ. prakt. Chem.* (2), iii. p. 286, 1871.

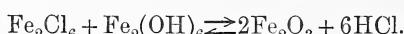
¶ Malfitano, *Compt. rend.*, cxlii. p. 1277, 1906.

colour, allowed those granules less rich in chlorine to pass through it. He suggests that the original colloidal solution contains a mixture of granules formed of from 1 to 4 molecules of $\text{Fe}_2(\text{OH})_6$ grouped round the H ion of a HCl molecule, the smaller granules being able to pass through the membrane. Goodwin* examined the hydrolysis of ferric chloride, making measurements of electrical conductivity and of freezing-point lowerings. He came to the conclusion that the colloidal reaction taking place in hydrolysis serves to put the FeOH^- ions out of the solution. Jones, Getman, etc.,† have made measurements of electrical conductivity and of freezing-point lowerings in the case of ferric chloride and nitrate, and have calculated the number of molecules of water in combination with each molecule of salt in the case of the nitrate. The conclusion drawn by Malfitano and Michel ‡ from their observations on the hydrolysis of ferric chloride is that the products of hydrolysis are not constant in composition, and that the difficulty with which these products are re-converted into ferric chloride by the hydrochloric acid present, increases as the hydrolysis proceeds. The hypothesis is put forward that the colloid is composed of complex ions.

The result of all the work carried out on this subject points to the fact that, when ferric chloride is dissolved in water, the following reversible action takes place:—



The amount of hydroxide formed depends, amongst other things, on the concentration of the solution and on the temperature. Besides this, a secondary reaction seems to take place according to the formula



If the hydrochloric acid is neutralised by the addition of ammonia, a flocculent brown precipitate of Fe_2O_3 is formed. The constitution of the colloidal hydroxide is very probably $\text{Fe}_2\text{Cl}_6x\text{Fe}_2\text{O}_6\text{H}_6$, where x is a large number, as shown by Magnier de la Source § and Sabanejew.||

In the present investigation the salts examined were ferric chloride, bromide, nitrate, and sulphate, and ferrous chloride and sulphate; they were all obtained from Kahlbaum. Ferric chloride consisted of dry yellow

* Goodwin, "Ueber die Hydrolyse des Eisenchlorids," *Z. phys. Chem.*, xxi. p. 1, 1896.

† Jones, Getman, etc., "Hydrates in Aqueous Solution," Carnegie Institution of Washington, Publication No. 60, 1907.

‡ Malfitano and Michel, *Compt. rend.*, cxlv. p. 185 and p. 1275, 1907.

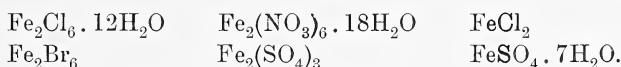
§ Magnier de la Source, *Compt. rend.*, xc. p. 1352, 1880.

|| Sabanejew, *Chem. Centralbl.*, i. p. 11, 1891.

lumps, somewhat hygroscopic, and becoming deep golden in colour on absorbing moisture from the air. Ferric bromide was a dark red powder covered with a greenish sheen where exposed to the air; it was exceedingly deliquescent. When making solutions, the salt was placed in a weighed beaker and a current of hot air was blown on to it in order to drive off moisture before it was weighed. The salt on dissolving in water gave out a great amount of heat, and the solution being filtered, a considerable brown residue was obtained. Ferric nitrate consisted of pale purplish, almost colourless crystals, which became somewhat moist after standing in the bottle for some time. The salt had a distinct odour of nitric acid. Ferric sulphate was a dry white powder which dissolved very slowly in water. The colours of the aqueous solutions of the chloride, nitrate, and sulphate varied with diminishing concentration from reddish brown to light yellow, but the more concentrated solutions of the bromide were very dark brown, almost black, in colour.

Ferrous chloride consisted of pale green hexagonal plates which became darker in colour, brown in parts, on standing for some time. Ferrous sulphate consisted of clear blue-green crystals, not very soluble in water. The aqueous solutions of the ferrous salts were green, but not deep in colour. They decomposed, owing to oxidation, soon after being made up, a flocculent brown precipitate of Fe_2O_3 forming in them.

The formulae of the different salts examined were taken as:



A solution of concentration $c=1$ was taken as one in which the number of grammes of the salt dissolved in 1 litre of solution was that represented by the corresponding formula.

The apparatus used for making measurements in the visible spectrum was the same as has been described in the second of this series of papers.* Measurements in the infra-red were made with the mirror spectrograph described and illustrated in the fifth article of the series.† Preliminary measurements of absorption in the ultra-violet were made in the case of the ferric salts with the aid of the apparatus used in the case of the cobalt and nickel salts.‡ It was found, however, that the absorption increases so rapidly in the ultra-violet that it could not be measured at all accurately except for very dilute solutions and for very small thicknesses of solution. Quantitative measurements in the ultra-violet were therefore not carried out.

* *Proc. Roy. Soc. Edin.*, xxxi. p. 530, 1911.

‡ *Ibid.*, xxxi. p. 547, 1911.

† *Ibid.*, xxxii. p. 41, 1912.

The results of the measurements made in the visible and infra-red regions of the spectrum are given below in Tables IA. and IB. The numbers

TABLE IA.—VALUES OF A OBTAINED WITH THE SPECTROPHOTOMETER.

λ .	Fe ₂ Cl ₆ ·12H ₂ O.		Fe ₂ Br ₆ .			Fe ₂ (NO ₃) ₆ ·18H ₂ O.		Fe ₂ (SO ₄) ₃ .		FeCl ₂ .		FeSO ₄ ·7H ₂ O.	
	$c=1\cdot0$.	$c=1\cdot1$.	$c=1\cdot0$.	$c=1\cdot5$.	$c=1\cdot1$.	$c=1\cdot0$.	$c=1\cdot1$.	$c=1\cdot0$.	$c=1\cdot1$.	$c=4\cdot0$.	$c=1\cdot0$.	$c=1\cdot0$.	$c=1\cdot0$.
·434 μ	...	37·2	105	7·3	10·6	6·9	9·5	·75	·30	·22	
·444	...	25·1	89	...	9·6	3·3	6·3	·46	·26	·20	
·453	...	16·5	71	5·5	8·6	2·1	4·7	·31	·21	·16	
·463	...	10·6	58	...	7·5	1·7	3·9	·19	·17	·14	
·475	...	7·8	43	3·6	6·5	1·1	3·0	·13	·15	...	
·486	...	6·2	34	...	5·1	·91	2·5	·076	·13	...	
·499	11·4	4·3	24	2·25	4·04	·76	2·2	·056	·11	...	
·514	4·8	3·0	15	...	2·81	·71	2·1	·041	·085	·098	
·529	1·8	2·0	7·8	1·03	1·59	·62	1·9	·028	...	·085	
·547	·82	1·15	4·0	·70	·82	...	1·6	·025	·063	·058	
·563	·46	·76	...	14·5	2·2	·40	·50	·51	1·3	·021	
·582	·33	·42	1·05	·26	·17	...	·73	·026	...	·054	
·602	·29	5·47	·73	·16	...	·31	·49	·032	...	·066	
·625	·26	2·96	...	·12	...	·23	·29	·042	·069	·080	
·653	·22	·19	13·9	·14	...	·16	·21	·060	·086	·106	
·687	·18	·23	6·4	·17	...	·19	...	·091	·11	·16	
·717	·16	...	4·2	·115	·15	·22		

TABLE IB.—VALUES OBTAINED WITH THE THERMOPILE.

·677	·16	...	6·0	1·74	·41	·16	...	·19	·25	·10	·14	·16	
·710	·15	·31	3·9	1·70	·47	·19	·28	·24	·28	·12	·17	·20	
·746	·15	·41	...	1·66	·58	·25	·37	·33	·37	·17	·20	·25	
·788	·17	·49	2·8	1·77	·72	·33	·50	·43	·49	·27	·28	·39	
·837	·22	·46	...	2·10	·94	·32	·45	·51	·64	·50	·50	·70	
·900	·23	·39	4·1	2·48	1·26	·23	·31	·42	·54	·89	·84	·1·26	
·968	·19	·28	...	2·66	1·22	·13	·21	·29	·41	1·10	1·00	1·54	
1·062	·12	·12	4·8	2·38	1·11	·04	·10	·13	·18	1·13	1·02	1·62	
1·175	·06	1·75	·89	·01	·04	·04	·05	1·00	·95	1·39	
1·300	1·9	1·21	·64	·84	·82	...	1·14	

given are the values of the molecular extinction coefficient A, defined by the equation

$$I = I_0 10^{-Acd},$$

where I₀ and I represent the intensities of light before and after passing through a thickness d of solution, of which c is the concentration expressed in gramme-molecules per litre; each number is the mean of two or more values. In the case of the ferric salts, solutions of concentrations $c=1\cdot0$ and $c=1\cdot1$ were examined. The solutions of the ferrous salts of concentration $c=1\cdot0$ were almost colourless, and could not therefore be examined with

the apparatus at my disposal. The solution of the sulphate of concentration $c=1\cdot0$ was nearly at the limit of solubility, but solutions of concentrations $c=4\cdot0$ and $c=1\cdot0$ were experimented with in the case of the chloride. Various thicknesses of solution were used, varying from $d=1\cdot4$ cm. to $d=2\cdot0$ cm.

In order to facilitate comparison, the chief results are shown in graphical form in figs. 1, 2, and 3. It may be observed that with the ferric salts the bands at the violet end of the visible spectrum commence at about $\cdot560\mu$. Hiebendaal * examined the absorption spectrum of ferric chloride in alcohol, and found bands at $\cdot680$, $\cdot620$, $\cdot595\mu$, and complete absorption from about $\cdot555\mu$. The first three bands do not occur in the aqueous solutions. We see also from the curves that in the case of ferric chloride, nitrate, and sulphate the infra-red absorption band increases in height and the maximum point of the band shifts somewhat towards the visible spectrum as we pass from the concentration $c=1\cdot0$ to the concentration $c=1$, whereas with ferric bromide the shift of the maximum of the band is in the same direction, but the height of the band decreases with dilution. The bromide bands are much higher than the corresponding bands of the other three ferric salts; this may probably be due to the presence of free bromine in solution. Further, it may be observed that the ferrous chloride infra-red band does not alter much with change in concentration.

The effect of concentration on the absorption was next more fully investigated. In the case of the ferric salts, solutions of concentrations $c=1\cdot0$, $\cdot7$, $\cdot4$, $\cdot1$, and $\cdot01$ were examined for three wave-lengths in the visible spectrum, namely, $\cdot453$, $\cdot529$, and $\cdot653\mu$. Solutions of ferrous chloride of concentrations $c=4\cdot0$, $2\cdot0$, $1\cdot0$, and $\cdot5$, and of ferrous sulphate of concentrations $c=1\cdot5$, $\cdot75$, and $\cdot5$, were also examined for the same three wave-lengths. The measurements were made on these solutions when they had been freshly made up, and were repeated in the case of the ferric salts after a period of six weeks in order to see if any change in the absorption could be detected.

The results of the measurements are given in Table II. below. The numbers in the first column in each case are the values of the molecular extinction coefficient A for freshly made-up solutions, while the numbers in the second column are the values for A for the same solutions after six weeks.

Most of the above results are shown in graphical form in figs. 4, 5, and 6. The dotted curves represent the measurements on the six-weeks-old solutions, while the other curves represent the measurements on the fresh

* Hiebendaal, "Onderzoek over eenige absorbiespectra," *Academ. Proefschrift*, 1873.

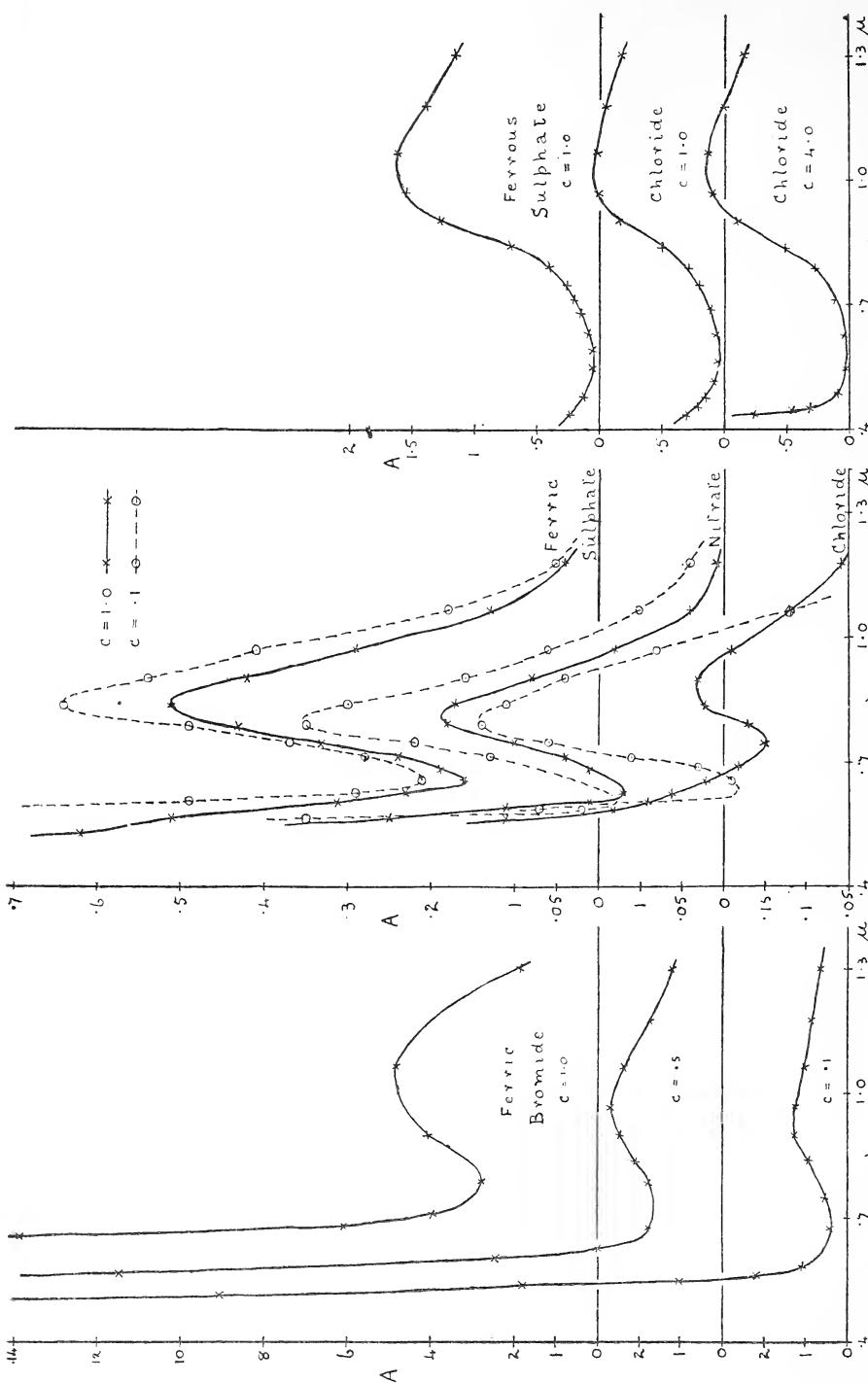


FIG. 3,

FIG. 1,

FIG. 2.

solutions. It may be seen that the absorption in the visible spectrum increases with dilution in the case of ferric nitrate and sulphate, but decreases with dilution in the bromide. With ferric chloride the absorption at first decreases with dilution and then slightly increases. This increase with dilution at low concentrations may probably be due to the formation of colloidal hydroxide. As seen from Table II., the absorption by the ferric chloride solution of concentration $c=0.01$ increased greatly when the

TABLE II.—VARIATION OF A WITH c .

$\lambda.$	$c.$	$\text{Fe}_2\text{Cl}_6 \cdot 12\text{H}_2\text{O}$		Fe_2Br_6		$\text{Fe}_2(\text{NO}_3)_6 \cdot 18\text{H}_2\text{O}$		$\text{Fe}_2(\text{SO}_4)_3$		FeCl_2		$\text{FeSO}_4 \cdot 7\text{H}_2\text{O}$	
		Fresh.	Old.	Fresh.	Old.	Fresh.	Old.	Fresh.	Old.	$c.$...	$c.$...
453μ	1.0	5.57	4.63	2.41	1.81	4.0	.39	1.5	.074
	.7	5.95	4.87	2.55	2.72	2.0	.18	.75	.074
	.4	6.48	5.48	3.37	2.71	1.0	.16	.5	.12
	.1	14.6	14.7	63.4	58.0	10.49	8.77	5.61	4.87	.5	.22
	.01	14.9	58.4	22.6	54.7	13.15	12.75	12.4	13.7
529	2.0	6.85	4.73	4.0	.033	1.5	.032
	1.5	3.53	2.52	2.0	.051	.75	.048
	1.0	1.72	1.25	1.28	.85	.68	.63	1.0	.077
	.7	1.26	1.02	1.21	.98	.77	.75	.5	.097
	.4	1.26	1.04	35.3	27.5	1.35	1.03	.99	.90
	.1	1.68	1.62	7.59	5.90	1.92	1.73	1.92	1.95
	.01	3.49	17.5	4.90	9.88	2.37	3.84	6.53	5.90
653	2.0	.26	.23	4.0	.057	1.5	.10
	1.5	.22	.21	2.0	.067	.75	.094
	1.0	.17	.18	11.8	8.01	.14	.13?	.18	.14	1.0	.082	.5	.11
	.7	.15	...	2.52	1.91	.15	.09?	.20	.15	.5	.12
	.4	.1366	.56	.16	.16?	.23	.22
	.137	.33
	.01	...	4.2

solution had been standing for six weeks. The solution was then no longer clear even after being filtered through ordinary filter paper, but contained a great number of fine colloidal particles. After standing for a year the solutions of the ferric salts of concentrations $c=1.0$, .7, and .4 were found to be quite clear; the solutions of concentration $c=.1$ contained fine colloidal particles, and those of concentration $c=.01$ contained large particles, which in the case of the nitrate and sulphate had deposited themselves at the bottom of the bottles in which they were kept, leaving the solutions practically colourless.

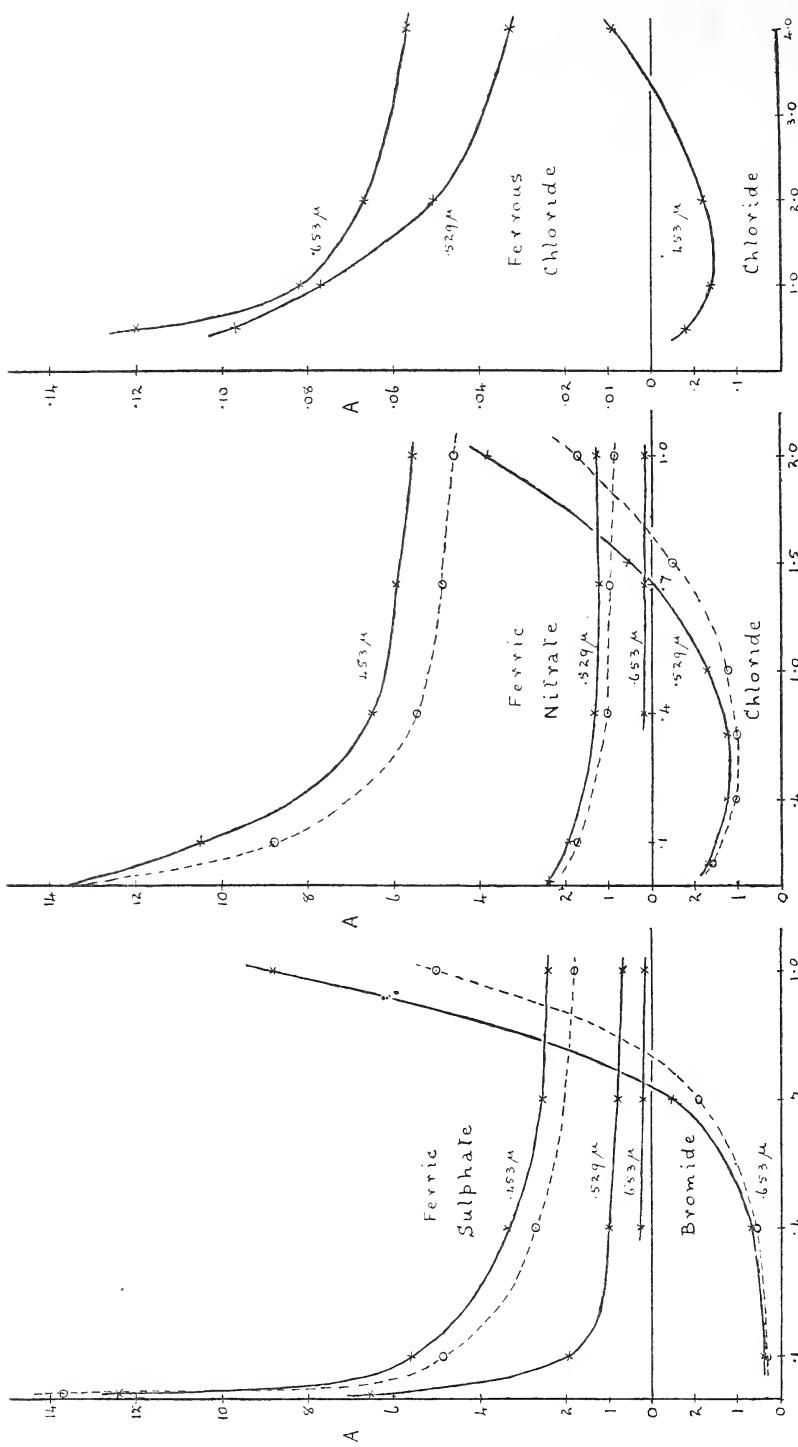


FIG. 4.

FIG. 6.

FIG. 5.

The values of pe/m calculated from the formula *

$$\frac{pe}{m} = \frac{3170 A_0 (\lambda_1 - \lambda_0)}{\lambda_0^2}$$

are given below in Table III. p represents the number of electrons or ions per molecule of salt causing the absorption band considered, e being the charge on the electron or ion, and m being its mass. A_0 is the value of A at the maximum of the band, λ_0 is the position of the maximum, and λ_1 is the wave-length for which $A = A_0/2$.

TABLE III.—VALUES OF pe/m .

Salt.	$c.$	$A_0.$	$\lambda_0.$	$\lambda_1.$	$pe/m.$
$Fe_2Cl_6 \cdot 12H_2O$	1·0	·24	880μ	$1\cdot300\mu$	4·1 10^2
	·1	·50	·800	1·150	8·7 10^2
Fe_2Br_6	1·0	4·9	1·050	1·350	4·2 10^3
	·5	2·7	·970	1·500	4·8 10^3
	·1	1·3	·920	1·600	3·3 10^3
$Fe_2(NO_3)_6 \cdot 18H_2O$	1·0	·34	·810	1·200	6·4 10^2
	·1	·51	·800	1·300	1·3 10^3
$Fe_2(SO_4)_3$	1·0	·51	·840	1·300	1·1 10^3
	·1	·64	·840	1·300	1·3 10^3
$FeCl_2$	4·0	1·17	1·020	·570	1·6 10^3
	1·0	1·07	1·010	·570	1·5 10^3
$FeSO_4 \cdot 7H_2O$	1·0	1·65	1·020	·570	2·3 10^3

If m is the mass of the iron atom, e/m should be about 178.

Further researches on the iron salts require to be undertaken before a more exact knowledge of their structure can be obtained. The study of the iron salts is somewhat complicated from the point of view of absorption spectra by the hydrolytic action which takes place, more especially in the more dilute solutions. The systematic study of freshly made-up aqueous and other solutions may, however, shed light on the complex question of molecular structure.

The main part of the work connected with this paper was carried out in the Natural Philosophy Department of the University of Glasgow when I was the holder of a Carnegie Research Scholarship. I beg to express my thanks to Dr R. A. Houstoun, at whose instigation the work was entered upon, and to Professor Zsigmondy, Göttingen, Germany, to whom I am indebted for my present knowledge of colloidal chemistry.

* R. A. Houstoun, "On the Mechanism of the Absorption Spectra of Solutions," *Proc. Roy. Soc., lxxxii. A*, p. 606, 1909.

IV.—The Absorption of Light by Inorganic Salts. No. VIII.
Alcoholic Solutions of Copper, Cobalt, and Nickel Salts in the
Ultra-Violet. By Alex. R. Brown, M.A., B.Sc., Carnegie Research
Scholar in the University of Glasgow. *Communicated by* Dr R. A.
HOUSTOUN.

(MS. received October 18, 1912. Read January 6, 1913.)

THE salts experimented on in this research were the chlorides, bromides, nitrates, and sulphates of copper, cobalt, and nickel. They were obtained from Kahlbaum. Jones and his co-workers* have already investigated qualitatively the absorption of alcoholic solutions of some of these salts. Also quantitative measurements of the absorption at a few points in the visible spectrum have been made by Vaillant† for some of the hydrated salts. In the present case solutions of the anhydrous salts were examined.

The spectrograph and methods used were those adopted by R. A. Houstoun and John S. Anderson in a previous research, and described in the fourth article of this series.‡

In the first place it was found necessary to determine the absorption of the ethyl alcohol used as a solvent. This has been discussed by Hartley, and by Soret and Rilliet. Also in Uhler and Wood's *Atlas of Absorption Spectra* there is a plate showing the absorption spectrum of ethyl alcohol. The latter authors state: "This liquid transmitted all the strong ultra-violet lines, but it absorbed the continuous background from 20μ to about 275μ ." The present investigation would tend to show that even beyond 275μ there is slight absorption which must be taken into account in quantitative work with alcoholic solutions. The table on p. 45 gives the value of A, the molecular extinction coefficient, for five wave-lengths. Intermediate wave-lengths have intermediate values of A.

Of the copper salts, the sulphate is nearly quite insoluble in absolute alcohol; a saturated solution was made for which c , the concentration in grm.-mols. per litre of solution, was only 0.012 . The chloride and bromide were easily soluble, and at moderate concentrations gave brownish-green and brown solutions respectively. All solutions showed very great absorption in the ultra-violet, a photographic plate which was exposed in the

* Carnegie Institution of Washington, Publication No. 110.

† *Ann. Chim. Phys.* (7), xxviii. p. 213, 1903.

‡ *Proc. Roy. Soc. Edin.*, xxxi. p. 547, 1911.

usual manner behind a solution of copper chloride of 1 mm. thickness and concentration $c=50$ being unaffected by rays of wave-length beyond 490μ .

$\lambda.$	A.
$290\mu\mu$	0·0
257	·078
250	·223
243	·368
233	·481

In order to obtain measurements, alcohol was added to the solutions of chloride and bromide until both reached a concentration given by $c=0·0010$ and were practically colourless. In working with such dilute solutions

VALUES OF A FOR SOLUTIONS IN ETHYL ALCOHOL.

$\text{CuSO}_4,$ $c=0·0012.$		$\text{CuCl}_2,$ $c=0·0010.$		$\text{CuBr}_2,$ $c=0·0010.$		$\text{CoCl}_2,$ $c=0·085.$		$\text{CoBr}_2,$ $c=0·015.$		$\text{NiCl}_2,$ $c=0·026.$		$\text{NiBr}_2,$ $c=0·040.$	
λ	A	λ	A	λ	A	λ	A	λ	A	λ	A	λ	A
$\mu\mu$		$\mu\mu$		$\mu\mu$		$\mu\mu$		$\mu\mu$		$\mu\mu$		$\mu\mu$	
278	7300	290	10000	316	10000	268	73	266	500	272	110	265	280
279	6100	300	8300	325	8700	273	63	273	440	300	18	268	240
282	4300	310	5400	337	6200	359	14	292	330	323	14	273	180
292	2500	320	1800	345	3300	397	11	299	145	355	8	305	110
319	850	357	1200	375	1300	420	7	302	62	364	10	311	48
375	0	366	970	397	1500	450	9	307	50	397	22	320	28
...	...	375	400	414	1500	313	35	416	24	333	23
...	...	385	170	415	1300	318	13	437	17	335	17
...	...	418	0	417	1100	338	0	444	5	337	9
...	426	840	374	11
...	431	530	383	19
...	437	280	385	25
...	387	30
...	427	30
...	428	25
...	430	19
...	443	11
...	449	5

there is a greater probability than usual of the results being affected by the small fraction of water inseparable from the alcohol, but this result is unavoidable.

The curves here obtained bear a considerable resemblance to the curves

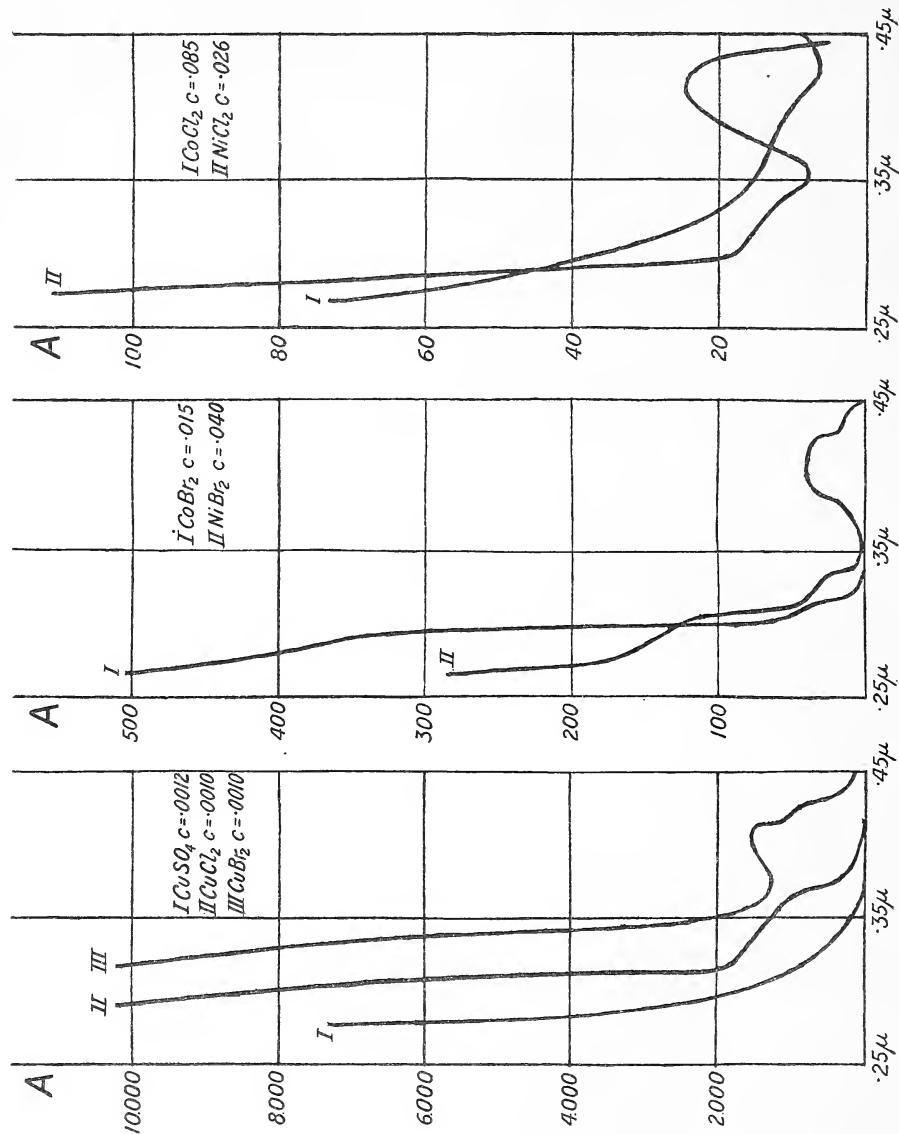
VALUES OF Λ FOR ALCOHOLIC SOLUTION OF COPPER, COBALT, AND NICKEL SALTS.

FIG. 1.

for aqueous solutions of the same salts given in the fifth article, but also indicate that the copper band in the extreme ultra-violet has increased very much in height and breadth.

The sulphates of nickel and cobalt were so nearly insoluble that no additional absorption could be noticed for alcohol in which the anhydrous salts had been immersed for days. The nickel salts were much less soluble than the cobalt salts. The original solutions of cobalt chloride and bromide were blue and purple-blue respectively, and after necessary dilution the former became purple and the latter almost transparent. The nickel chloride solution was very pale yellow, and the bromide solution pale

VALUES OF A FOR SOLUTIONS OF NICKEL CHLORIDE.

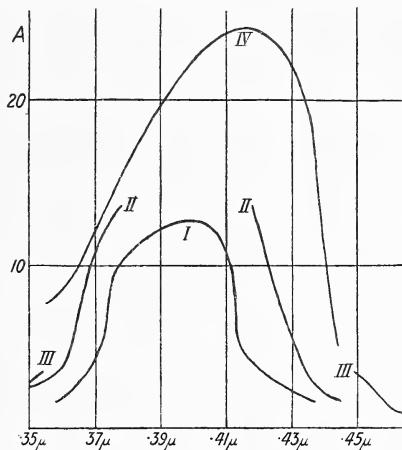


FIG. 2.

yellow-green. In the figures the curves for the chlorides and bromides are grouped according to the acid radicles.

It was intended to make alcoholic solutions of the anhydrous nitrates. Unfortunately these salts seem to dissociate before complete dehydration takes place, and no satisfactory results could be obtained.

In the curves shown there is no very distinct feature peculiar to either acid radicle, but the curves of the chlorides have more in common than those of the bromides have. It is to be noticed that in the case of the nickel salts there is an absorption band with its maximum about $\lambda = 410\mu$.

This band also occurs in the aqueous solutions of nickel salts, as has been shown in the fourth article of the series. I made an effort to trace the effect of concentration on this band in the case of an aqueous solution of nickel chloride. As it was not possible with the cells at my disposal to work with a thickness of solution less than 1 mm., the height and position

of the maximum of the band could not be determined for aqueous solutions of high concentration. In fig. 2 curves are given showing the absorption of aqueous solutions I., II., III., of concentrations $c = 10$, $c = 1.00$, $c = 4.00$ respectively, and of one alcoholic solution, IV., of concentration $c = 0.26$. From these curves it may be inferred that the nickel chloride in solution undergoes a steady continuous change as we pass from dilute aqueous solutions through saturated aqueous solutions to alcoholic solutions. This phenomenon bears a great resemblance to that observed in solutions of cobalt chloride and discussed in the sixth article of this series, and it is probably to be ascribed to the same cause, viz. the change of the chloride in solution from a more to a less hydrated phase.

(Issued separately February 13, 1913.)

V.—The Theory of Axisymmetric Determinants from 1857
to 1880. By Thomas Muir, LL.D.

(MS. received June 24, 1912. Read November 18, 1912.)

My last communication in reference to the history of axisymmetric determinants dealt with the period 1841–1859 (*Proc. Roy. Soc. Edin.*, xxvii. pp. 135–166). The present paper continues the history up to the year 1880, but in addition contains an account of one contribution belonging to the previous period, namely, by d'Arrest (1857).

d'ARREST, H. (1857, August).

[Beobachtungen des Cometen iii., 1857. *Astron. Nachrichten*, xlvi., col. 17–19.]

From consideration of three special cases d'Arrest ventures on the generalisation that if in the application of the Method of Least Squares the normal equations be

$$\left. \begin{array}{l} (aa)x + (ab)y + (ac)z + \dots = 0 \\ (ab)x + (bb)y + (bc)z + \dots = 0 \\ (ac)x + (bc)y + (cc)z + \dots = 0 \\ \dots \dots \dots \dots \end{array} \right\}$$

then the weights of x, y, z, \dots are

$$\frac{\Delta}{[aa]}, \frac{\Delta}{[bb]}, \frac{\Delta}{[cc]}, \dots$$

where Δ is the determinant of the set of equations and $[aa]$ is the cofactor of (aa) in Δ . In the same serial in 1866 (vol. lxvii. pp. 174–175) a proof of the proposition is given but unaccompanied by any author's name.

FERRERS, N. M. (1861).

[An Elementary Treatise on Trilinear Co-ordinates (chap. iii. pp. 58–71). xii + 154 pp. London.]

One of the results given at the close of the short chapter on determinants is

$$\left| \begin{array}{ccccc} . & 1 & 1 & 1 & \dots \\ 1 & . & a+b & a+c & \dots \\ 1 & b+a & . & b+c & \dots \\ 1 & c+a & c+b & . & \dots \\ \dots & \dots & \dots & \dots & \dots \end{array} \right| = abc \dots \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \dots \right).$$

It is readily seen that the left-hand member

$$= \begin{vmatrix} . & 1 & 1 & 1 & \dots \\ 1 - 2a & . & . & . & \dots \\ 1 & . & -2b & . & \dots \\ 1 & . & . & -2c & \dots \\ \dots & \dots & \dots & \dots & \dots \end{vmatrix}$$

and that therefore the right-hand member ought to be

$$(-1)^n 2^{n-1} abc \dots \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \dots \right).$$

where n is the number of variables.

BOOLE, G. (1862, June).

[On the theory of probabilities. *Philos. Transac. R. Soc. London*, clii. pp. 225–252.]

An important part of Boole's investigation turns upon the solution of a peculiar set of linear equations, and he consequently devotes considerable space (pp. 235–240) to an examination of the determinant of the set.

As the determinant is axisymmetric he begins by establishing the proposition which results, in regard to such determinants, from subtracting λ times the j^{th} row from the i^{th} row, and thereafter λ times the j^{th} column from the i^{th} column. This operation does not do away with the axisymmetry, and the diagonal element a_{ii} is thereby changed into

$$a_{ii} - 2\lambda a_{ij} + \lambda^2 a_{jj}.$$

Following this lemma and made in part dependent on it comes the rather notable proposition that if an axisymmetric determinant have all its elements of the form

$$\lambda a + \mu b + \nu c + \dots,$$

and the coefficients λ, μ, ν, \dots in all the diagonal elements be positive, and generally be such that all those joined to any one of the variables in any row be in order proportional to the coefficients of the same variable in any other row, then the final development of the determinant will contain only positive terms. The proof is disappointingly lengthy, occupying very nearly three pages (pp. 226–238).

The other proposition, which is a deduction from this, is to the effect that if F be a rational integral function of n variables x_1, x_2, \dots, x_n , having no powers of them above the first and having all its terms positive, then the final development of

$$\begin{vmatrix} F & x_1F_1 & x_2F_2 & \dots & x_nF_n \\ x_1F_1 & x_1F_1 & x_1x_2F_{12} & \dots & x_1x_nF_{1n} \\ x_2F_2 & x_2x_1F_{21} & x_2F_2 & \dots & x_2x_nF_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_nF_n & x_nx_1F_{n1} & x_nx_2F_{n2} & \dots & x_nF_n \end{vmatrix},$$

where F_r, F_{rs} stand for $\partial F/\partial x_r, \partial^2 F/\partial x_r \partial x_s$ respectively, will contain nothing but positive terms. For the case of two variables the result is

$$\begin{vmatrix} axy + bx + cy + d & axy + bx & axy + cy \\ axy + bx & axy + bx & axy \\ axy + cy & axy & axy + cy \end{vmatrix} = abcx^2y^2 + abdx^2y + acdxy^2 + bcdxy :$$

and for the development in the case of three variables we are referred to a similar memoir * of the year 1857, where we find the expression

$$\begin{aligned} & (d+h+e+f)(abc+aeg+abg+beg) \\ & + (a+b+c+g)(dhe+dhf+def+hef) \\ & + (ac+bg)(df+dh+ef+eh) \\ & + (ag+bc)(df+eh+de+fh) \\ & + (ab+cg)(de+dh+fe+fh) \\ & + 4agfe + 4bcdh, \end{aligned}$$

and infer that it is the equivalent of

$$\begin{vmatrix} a+b+c+\dots+h & a+c+d+e & a+b+d+f & a+b+c+g \\ a+c+d+e & a+c+d+e & a+d & a+c \\ a+b+d+f & a+d & a+b+d+f & a+b \\ a+b+c+g & a+c & a+b & a+b+c+g \end{vmatrix}.$$

SIEBECK, F. H. (1862).

[Ueber die Determinante deren Elemente die Quadrate der sechzehn Verbindungslienien der Eckpunkte zweier beliebigen Tetraeder sind. *Crelle's Journ.*, lxii. pp. 151-159.]

The determinant in question is brought forward as a companion to Sylvester's of the year 1852, being, in fact, the complementary minor of the zero element in the latter. It is shown that the ratio of the one to the other is

$$2r\rho \cos \phi$$

where r, ρ are the radii of the spheres circumscribing the tetraedra and ϕ the angle of intersection of the said spheres. The interest of the paper, like Sylvester's, is mainly geometrical.

* Boole, G., "On the Application of the Theory of Probabilities to the Question of the Combination of Testimonies or Judgments," *Trans. Roy. Soc. Edin.*, xxi. pp. 597-652 (see p. 648).

FREUCHEN, P. (1863).

[To Determinanter af nte Grad. *Math. Tidsskrift*, v. p. 42.]

The two determinants are the cases of Ferrers' of the year 1855, in which $a_1 = a_2 = a_3 = \dots$.

ROBERTS, M. (1864, March).

[Question 694. *Nouv. Annales de Math.* (2), iii. pp. 139–140. Solutions by L. Ferrara, G. Torelli, in *Giornale di Mat.*, ii. pp. 95–96, p. 191: solutions by A. Smet-Jamar, M. Cornu, H. Picquet, in *Nouv. Annales de Math.* (2), iii. pp. 395–399, and by “Mirza-Nizam” in (2), iv. pp. 500–504.]

Roberts' result is essentially the same as Ferrers' second, but is expressed more suggestively, namely,

$$\begin{vmatrix} a_1 & -1 & -1 & \dots & -1 \\ -1 & a_2 & -1 & \dots & -1 \\ -1 & -1 & a_3 & \dots & -1 \\ \dots & \dots & \dots & \dots & \dots \\ -1 & -1 & -1 & \dots & a_n \end{vmatrix} = f(1) - f'(1), \quad \text{where } f(x) = (x + a_1) \dots (x + a_n).$$

Ferrara and Smet-Jamar develop the determinant in Cayley's manner 1847, obtaining $f(1) - f'(1)$ in the form

$$(1-n) + (2-n) \sum a_1 + (3-n) \sum a_1 a_2 + \dots + a_1 a_2 \dots a_n;$$

Cornu and Picquet subtract the last column from each of the others and obtain the development in the form

$$(1+a_1)(1+a_2) \dots (1+a_n) \cdot \left[1 - \left(\frac{a_1}{1+a_1} + \frac{a_2}{1+a_2} + \dots + \frac{a_n}{1+a_n} \right) \right]$$

Torelli, after generalising the determinant by putting $-x$ everywhere for -1 , writes it in the form

$$\begin{vmatrix} (x+a_1)-x & 0-x & 0-x & \dots \\ 0-x & (x+a_2)-x & 0-x & \dots \\ 0-x & 0-x & (x+a_3)-x & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}$$

which, if expressed as a sum of determinants with monomial elements, gives

$$(x+a_1)(x+a_2) \dots (x+a_n) - x \sum (x+a_2)(x+a_3) \dots (x+a_n) \\ \text{i.e. } f(x) - xf'(x);$$

and “Mirza Nizam” obtains this wider result of Torelli's by following Cornu and Picquet.

WALKER, J. J. (1865, 1868).

[Properties of the discriminant of the quaternary quadric form. *O. C. and D. Messenger of Math.*, iv. pp. 25-31, 189-190.]

[Question 2730. *Educ. Times*, xxi. p. 139: or *Math. from Educ. Times*, xi. pp. 107-108.]

The subject here is mainly the vanishing of the primary minors of any axisymmetric determinant of the fourth order, the final result being that *all the primary minors will vanish when three of them vanish, provided that two, and not more than two, of the three be taken from the same rows of the adjugate.* The number *three* is Sylvester's $\frac{1}{2}(4-3+1)(4-3+2)$ of the year 1850.

In proving a simple case of the well-known theorem regarding a minor of the adjugate he incidentally finds

$$\begin{vmatrix} a & e & f & g \\ e & b & h & i \\ f & h & c & j \\ g & i & j & d \end{vmatrix} = \begin{vmatrix} e^2 - ab & ef - ah & eg - ai \\ ef - ah & f^2 - ac & fg - aj \\ eg - ai & fg - aj & g^2 - ad \end{vmatrix} \div a^2,$$

and thence later shows that when $|a_1b_2c_3d_4|$ is axisymmetric and vanishes we have

$$|a_1b_2| \cdot |a_1c_3d_4| - |a_1b_3| \cdot |a_1c_2d_4| + |a_1b_4| |a_1c_2d_3| = 0.$$

Here, however, we must note that for this last identity axisymmetry is not necessary, and that the other condition may also be dispensed with if in place of 0 we put $a_1|a_1b_2c_3d_4|$.

CALDARERA, F. (1871).

[Nota su talune proprietà dei determinanti, in ispecie di quelli a matrici composte con la serie dei numeri figurati. *Giornale di Mat.*, ix. pp. 223-232.]

The main determinant dealt with is

$$\begin{vmatrix} a & a+\delta & a+2\delta & a+3\delta & \dots \\ a & 2a+\delta & 3a+3\delta & 4a+6\delta & \dots \\ a & 3a+\delta & 6a+4\delta & 10a+10\delta & \dots \\ a & 4a+\delta & 10a+5\delta & 20a+15\delta & \dots \\ \dots & \dots & \dots & \dots & \dots \end{vmatrix}_n \quad \text{or, say, } D(a, \delta),$$

it being first shown to be independent of δ , and consequently to be equal to

$$\alpha^n \left| \begin{array}{cccccc} 1 & 1 & 1 & 1 & \dots & \\ 1 & 2 & 3 & 4 & \dots & \\ 1 & 3 & 6 & 10 & \dots & \\ 1 & 4 & 10 & 20 & \dots & \\ \dots & \dots & \dots & \dots & \dots & n \end{array} \right|$$

The evaluation of the cofactor of α^n Calderara effects with much trouble (pp. 226–231) by means of his (or, rather, Chio's) condensation-theorem, not observing that a series of similar steps can be made by simply diminishing each row by the row immediately preceding: for example, when $n=4$, we have

$$\frac{D(\alpha, 0)}{\alpha^4} = \left| \begin{array}{ccc} 1 & 2 & 3 \\ 1 & 3 & 6 \\ 1 & 4 & 10 \end{array} \right| = \left| \begin{array}{cc} 1 & 3 \\ 1 & 4 \end{array} \right| = 1,$$

the theorem repeatedly used in performing the subtractions being

$$C_{n,r} - C_{n-1,r} = C_{n-1,r-1}.$$

The same process applied directly to $D(\alpha, \delta)$ gives the value α^n with equal ease, showing at one and the same time that $D(\alpha, \delta) = D(\alpha, 0)$, and that the axisymmetric cofactor of α^n in the latter is equal to 1.

SCHULTZE, E. (1871, Sept.).

[Ueber die aus einer symmetrischen Determinante $\Delta_n = \sum \pm a_{11} \dots a_{nn}$ gebildete Reihe $\Delta_n, \Delta_{n-1}, \dots, \Delta_o$. Sch. Progr. 22 pp. Berlin.]

The title is misleading, the subject really being the linear transformation of a quadric, whose discriminant is Δ_n , into an aggregate of multiples of squares. As, however, one of the modes of transformation referred to is that resuscitated by Brioschi in 1856,* but originally due to Lagrange (1759†), namely, that which changes

$$a_{11}x_1^2 + a_{22}x_2^2 + \dots + a_{nn}x_n^2 + 2a_{12}x_1x_2 + \dots$$

into

$$\Delta_1 y_1^2 + \frac{\Delta_2}{\Delta_1} y_2^2 + \frac{\Delta_3}{\Delta_2} y_3^2 + \dots + \frac{\Delta_n}{\Delta_{n-1}} y_n^2$$

by a transformation of the form

* BRIOSCHI, F., "Sur les séries qui donnent le nombre de racines réelles des équations algébriques à une ou à plusieurs inconnues," *Nouv. Annales de Math.*, xv. pp. 264–286; or *Zeitschrift f. Math. u. Phys.*, ii. pp. 209–222; or *Opere Mat.*, v. pp. 127–143.

† LAGRANGE, J. L., "Recherche sur la méthode de maximis et minimis," *Miscell. Taurinensis*, i. p. 18; or *Oeuvres*, i. pp. 3–20. See also Gauss (1823), *Werke*, iv. pp. 27–53, and Jacobi (1847), *Crelle's Journ.*, liii. pp. 265–270.

$$\left. \begin{array}{l} x_1 = y_1 + c_{12}y_2 + c_{13}y_3 + \dots + c_{1n}y_n, \\ x_2 = y_2 + c_{23}y_3 + \dots + c_{2n}y_n, \\ \dots \dots \dots \dots \dots \dots \dots \\ x_n = y_n \end{array} \right\}$$

the series $\Delta_n, \Delta_{n-1}, \dots, \Delta_0$ does come up for consideration. The only property established (p. 7) is: *If Δ_n and all its coaxial minors of the $(n-1)^{th}$ order vanish, then every other coaxial minor of the said order will also vanish.* This is easily arrived at by considering any two-line coaxial minor of the adjugate.

We may note that in dealing with a second transformation—the orthogonal—Schultze formulates another proposition regarding the evanescence of minors, but that this is identical with the first of Dodgson's pair of 1867.

HUNYADY, E. (1872, January).

[Question 979 (proposée par H. Brocard). *Nouv. Annales de Math.* (2), xi. pp. 39-44.]

In seeking to solve anew Lagrange's interpolation-problem, namely, to obtain the values of the A's in the set of equations

$$A_1 \cos \frac{1r \cdot \pi}{n+1} + A_2 \cos \frac{2r \cdot \pi}{n+1} + \dots + A_n \cos \frac{nr \cdot \pi}{n+1} = y_r \quad \left. \right\}_{r=1}^{r=n}$$

Hunyady finds that

$$\left| \cos \frac{1 \cdot 1 \cdot \pi}{n+1} \cos \frac{2 \cdot 2 \cdot \pi}{n+1} \dots \cos \frac{n \cdot n \cdot \pi}{n+1} \right| = \begin{cases} \frac{1}{4} \left(\frac{n+1}{2} \right)^{n-2} & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd.} \end{cases}$$

WILLIAMSON, B. (1872): BUCHWALD, E. (1872).

[Condition for a maximum or a minimum in a function of any number of variables. *Quart. Journ. of Math.*, xii. pp. 48-51: or his *Differential Calculus*, 1st edition pp. 340-343, 2nd edition pp. 363-367, etc.]

[Question 3683. *Educ. Times*, xxiv. p. 296, xxv. p. 18: or *Math. from Educ. Times*, xvii. pp. 66-68.]

[Betingelsen for at den algebraisk rationale homogene Function af anden Grad af n variable er positiv for alle Værdier af de variable eller negativ for dem alle. *Tidsskrift for Math.* (3), ii. pp. 20-25.]

The real object here is to obtain the conditions that must be fulfilled in order that a quadric may remain positive for all real values of the variables.

The investigation, due in essence to Lagrange (1759),* is based on a simple transformation, three examples of which we may write for ourselves in the form

$$\begin{aligned} a_1 \cdot \frac{x}{a_1} \frac{y}{a_2} \left| \begin{array}{c} x \\ b_1 \\ b_2 \\ y \end{array} \right. &= (a_1x + a_2y)^2 + |a_1b_2|y^2, \\ a_1 \cdot \frac{x}{a_1} \frac{y}{a_2} \frac{z}{a_3} \left| \begin{array}{c} x \\ b_1 \\ b_2 \\ b_3 \\ c_1 \\ o_2 \\ c_3 \\ z \end{array} \right. &= (a_1x + a_2y + a_3z)^2 + \frac{y}{|a_1b_2|} \frac{z}{|a_1b_3|}, \\ a_1 \cdot \frac{x}{a_1} \frac{y}{a_2} \frac{z}{a_3} \frac{w}{a_4} \left| \begin{array}{c} x \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \\ w \end{array} \right. &= (a_1x + \dots + a_4w)^2 + \frac{y}{|a_1b_2|} \frac{z}{|a_1b_3|} \frac{w}{|a_1b_4|}, \end{aligned}$$

where, it must be borne in mind, the square arrays are axisymmetric.† By dividing both sides by a_1 it is seen that in the case of the binary quadric the conditions are

$$a_1 > 0, \quad |a_1b_2| > 0.$$

In the next case it is equally evident that they are $a_1 > 0$ and the like conditions for the quadric in y, z : and as by the previous case the latter are

$$|a_1b_2| > 0, \quad \begin{vmatrix} |a_1b_2| & |a_1b_3| \\ |a_1c_2| & |a_1c_3| \end{vmatrix} > 0,$$

we obtain in all for the ternary quadric

$$a_1 > 0, \quad |a_1b_2| > 0, \quad |a_1b_2c_3| > 0.$$

Similarly for the quaternary quadric we must have $a_1, |a_1b_2|, |a_1b_2c_3|, |a_1b_2c_3d_4|$ all positive: and so on, the last determinant in each case being the discriminant of the quadric. It is casually added that if the 1st, 3rd, 5th, . . . of the series be negative and the others positive, the quadric will be negative for all real values of the variables.

As might be expected from the connection with Lagrange, it is also pointed out that, calling these determinants $\Delta_1, \Delta_2, \Delta_3, \dots$, we can by repeated applications of the fundamental transformation change the n -ary quadric into the form

$$\Delta_1 U_n^2 + \frac{\Delta_2}{\Delta_1} U_{n-1}^2 + \frac{\Delta_3}{\Delta_2} U_{n-2}^2 + \dots + \frac{\Delta_n}{\Delta_{n-1}} U_1^2,$$

* See immediately preceding footnote.

† This restriction may be done away with if we alter the squared expressions on the right hand into

$$(a_1x + a_2y)(a_1x + b_1y), \quad (a_1x + a_2y + a_3z)(a_1x + b_1y + c_1z), \quad \dots$$

that is to say, into an aggregate of multiples of squares of linear functions of the variables: for example,

$$\begin{aligned} & ax^2 + by^2 + cz^2 + 2dyz + 2exx + 2fxy \\ = & a\left(x + \frac{f}{a}y + \frac{e}{a}z\right)^2 + \frac{ab - f^2}{a}\left(y + \frac{ad - ef}{ab - f^2}z\right)^2 + \frac{abc + 2def - \dots}{ab - f^2}z^2. \end{aligned}$$

Finally, as the quadric is invariant to a variety of sets of interchanges, there must be a corresponding variety of sets of conditions: and, as these latter sets must be all coexistent, there follows an interesting theorem which we may formulate for ourselves thus: *If the axisymmetric determinant $|a_1b_2c_3d_4|$ and its coaxial minors $|a_1b_2c_3|$, $|a_1b_2|$, a_1 be positive, then all the other coaxial minors are positive also.*

Having had in the foregoing to transform

$$\left| \begin{array}{cc} |a_1b_2| & |a_1c_3| \end{array} \right|, \quad \left| \begin{array}{ccc} |a_1b_2| & |a_1c_3| & |a_1d_4| \end{array} \right|, \quad \dots$$

into

$$a_1|a_1b_2c_3|, \quad a_1^2|a_1b_2c_3d_4|, \quad \dots$$

Williamson proposed the general problem in the *Educ. Times*, where solutions by Townsend, Laverty, and himself duly appeared. Chio's more general result of 1853 was apparently unknown.

Buchwald's paper is closely on the lines of Williamson's.

RITSERT, E. (1872).

[Die Herleitung der Determinante für den Inhalt des Dreiecks aus den drei Seiten. *Zeitschrift f. Math. u. Phys.*, xvii. pp. 518–520.]

This merely gives a variant on the usual way of deducing the four-line determinant in question from the determinant which involves the co-ordinates of the vertices.

CAYLEY, A. (1874, April).

[On the number of distinct terms in a symmetrical or partially symmetrical determinant. *Monthly Notices R. Astron. Soc.*, xxxiv. pp. 303–307, 335 : or *Collected Math. Papers*, ix. pp. 185–190.]

Denoting by $\phi(m, n)$ the number of distinct terms in a determinant of the $(m+n)^{\text{th}}$ order having an m -line coaxial minor which is axisymmetric,

Cayley, by considering two different forms of development—Bezout's of 1764 and his own of 1847—arrives at the equations

$$\begin{aligned}\phi(m, n) &= m\phi(m-1, n) + n\phi(m, n-1), \\ \phi(m, 0) &= \phi(m-1, 0) + m\phi(m-2, 0) + \frac{1}{2}m(m-1)\phi(m-3, 1),\end{aligned}$$

and thence obtains

$$\left. \begin{array}{c} \phi(0, 0) \\ \phi(1, 0), \quad \phi(0, 1) \\ \phi(2, 0), \quad \phi(1, 1), \quad \phi(0, 2) \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \end{array} \right\} = \left\{ \begin{array}{ccc} & 1 & \\ & 1 & 1 \\ 2 & 2 & \\ \cdot & \cdot & \cdot \end{array} \right\}$$

Then, confining himself to $\phi(m, 0)$, he shows that the differential equation of the generating function of $\phi(m, 0)/m!$ is

$$2\frac{du}{dx} - u - xu = \frac{u}{1-x},$$

and consequently that

$$u = \frac{e^{\frac{1}{2}x+\frac{1}{4}x^2}}{(1-x)^{\frac{1}{2}}}.$$

This readily leads to similar results for $\phi(m, 1), \phi(m, 2), \dots$

As an afterthought (p. 335) he puts the said differential equation in the form

$$2(1-x)\frac{du}{dx} = (2-x^2)u,$$

and thence deduces the equation of differences

$$u_n = nu_{n-1} - \frac{1}{2}(n-1)(n-2)u_{n-3}.$$

CUNNINGHAM, A. (1874, April).

[An investigation of the number of constituents, elements, and minors of a determinant. *Quart. Journ. of Sci.* (2), iv. pp. 212–228.]

Cunningham devotes §§ 8–10 (pp. 220–224) to axisymmetric determinants. There is an oversight, however, in his reasoning, and his results are correct only as far as the fifth order.

ROBERTS, S. (1874, May).

[Question 4392. *Educ. Times*, xxvii. pp. 45, 66: or *Math. from Educ. Times*, xxi. pp. 81–83.]

Roberts' difference-equation for the number of distinct terms is

$$u_n - u_{n-1} - (n-1)^2u_{n-2} + \frac{1}{2}(n-1)(n-2)\{u_{n-3} + (n-3)u_{n-4}\} = 0,$$

and Cayley establishes it by taking his own, namely,

$$u_n - nu_{n-1} + \frac{1}{2}(n-1)(n-2)u_{n-3} = 0 \\ \text{or, say, } E_n = 0,$$

and showing that the other is

$$E_n + (n-1)E_{n-1} = 0.$$

The first eight values of u_n he finds to be

$$1, 2, 5, 17, 73, 388, 2461, 18156, \dots$$

Roberts himself identifies the problem with that of finding “the number of distinct ways in which $2n$ things, two of a sort, can be made into parcels of 2.”

SEELIGER, H. (1875).

[Bemerkungen über symmetrische Determinanten, und Anwendung auf eine Aufgabe der analytischen Geometrie. *Zeitschrift f. Math. u. Phys.*, xx. pp. 467-474.]

With the help of an unwieldy multiple-sigma representation of the elements of a power-determinant Seeliger arrives at Sylvester's unproved proposition of 1852 regarding the p^{th} power of an axisymmetric determinant. To this he adds the statement that the four modes of performing the multiplication lead to the same result. Another proposition is that *the p^{th} power of a vanishing two-line determinant is axisymmetric*.

He next investigates the consequences of the simultaneous vanishing of a determinant and one of its primary minors, say the determinant $|a_1 b_2 c_3 d_4|$ and B_3 . Since *all* the two-line minors of the adjugate must vanish, he has of course

$$\begin{aligned} 0 &= |A_1 B_3| = |A_2 B_3| = |A_4 B_3| \\ &= |C_1 B_3| = |C_2 B_3| = |C_4 B_3| \\ &= |D_1 B_3| = |D_2 B_3| = |D_4 B_3| \end{aligned}$$

$$\begin{aligned} \text{and } \therefore 0 &= A_3 B_1 = A_3 B_2 = A_3 B_4 \\ &= C_3 B_1 = C_3 B_2 = C_3 B_4 \\ &= D_3 B_1 = D_3 B_2 = D_3 B_4 \end{aligned}$$

from which it is easy to see that

$$\begin{aligned} \text{either } 0 &= A_3 = C_3 = D_3, \\ \text{or } 0 &= B_1 = B_2 = B_4. \end{aligned}$$

The general result is that *if a determinant and one of its primary minors, M, vanish, then the other primary minors which are in the same*

row of the adjugate with M must also vanish, or those which are in the same column. As a corollary it is added that *when in addition Δ is axisymmetric and M is coaxial, there is no alternative.*

Following on this is a proposition less readily acceptable, namely: *If an n-line axisymmetric determinant and n-2 of its primary coaxial minors simultaneously vanish, then all the other primary minors vanish also.* It is at once seen that after the application of the preceding corollary the only elements of the adjugate that require consideration are those of the last two-line coaxial minor; in other words, that the adjugate must be of the form

$$\begin{vmatrix} 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & X & Y \\ 0 & 0 & \dots & 0 & Y & Z \end{vmatrix}.$$

Further, it may be agreed that the data give us

$$\left. \begin{array}{l} xX + yY = 0 \\ yY + zZ = 0 \\ XZ - Y^2 = 0 \end{array} \right\};$$

but the deduction made therefrom, namely, that

$$X = Y = Z = 0,$$

is clearly not the only one. A test case that we may put to ourselves is

$$\begin{vmatrix} a & b & b \\ b & c & c \\ b & c & c \end{vmatrix}.$$

Finally, Seeliger considers the primary minors of the product-determinant, showing that they must all vanish if the one factor and its primary minors vanish and the other factor does not vanish.

GLAISHER, J. W. L. (1874, 1878).

[On the solution of the equations in the method of least squares.
Monthly Notices R. Astron. Soc., xxxiv. (1874), pp. 311-334.]

[Questions 4418, 5530. *Educ. Times*, xxvii. p. 69: xxxi. p. 21. Solution by E. J. Nanson in *Math. from Educ. Times* (2), ii. pp. 95-96.]

What is given in the *Educ. Times* is an expression in the form of a series for d'Arrest's ratio of an axisymmetric determinant to one of its

coaxial primary minors. A perusal of the paper on the “Method of Least Squares” will show how the expression originates * (see §§ 5, 6). When freed of awkward notations it is, in the case of the fourth order,

$$\frac{|\alpha_1 b_2 c_3 d_4|}{|\alpha_1 b_2 c_3|} = d_4 - \frac{\alpha_4^2}{\alpha_1} - \frac{|\alpha_1 b_4|^2}{\alpha_1 |\alpha_1 b_2|} - \frac{|\alpha_1 b_2 c_4|^2}{|\alpha_1 b_2| |\alpha_1 b_2 c_3|}.$$

We may note for ourselves that the corresponding identity when axisymmetry is not insisted on is

$$\frac{|\alpha_1 b_2 c_3 d_4|}{|\alpha_1 b_2 c_3|} = d_4 - \frac{\alpha_4 d_1}{\alpha_1} - \frac{|\alpha_1 b_4| |\alpha_1 d_2|}{\alpha_1 |\alpha_1 b_2|} - \frac{|\alpha_1 b_2 c_4| |\alpha_1 b_2 d_3|}{|\alpha_1 b_2| |\alpha_1 b_2 c_3|},$$

and that a very instructive way of establishing it is to combine the first two terms on the right into one, then in similar manner combine the result thus obtained with the third term, and so on, the successive “approximations,” so to speak, being

$$d_4, \quad \frac{|\alpha_1 d_4|}{\alpha_1}, \quad \frac{|\alpha_1 b_2 d_4|}{|\alpha_1 b_2|}, \quad \frac{|\alpha_1 b_2 c_3 d_4|}{|\alpha_1 b_2 c_3|}.$$

The series is thus seen to be one of those that close up telescopically.

GLAISHER, J. W. L. (1876).

[Theorem relating to the differentiation of a symmetrical determinant.
Quart. Journ. of Math., xiv. pp. 245-248.]

The theorem is to the effect that, *If* $u = 1/\Delta^t$ where Δ stands for

$$\left| \begin{array}{cccc} 2a_{11} & a_{12} & a_{13} & \dots \\ a_{21} & 2a_{22} & a_{23} & \dots \\ a_{31} & a_{32} & 2a_{33} & \dots \\ \dots & \dots & \dots & \dots \end{array} \right| a_{rs} = a_{sr},$$

then any derivative of u with respect to one or more of the a 's is equal to any other such derivative having the same number of occurrences of each suffix-number. For example,

$$\frac{\partial^2}{\partial a_{12}^2} \frac{\partial}{\partial a_{23}} \frac{\partial}{\partial a_{34}} u = \frac{\partial}{\partial a_{11}} \frac{\partial}{\partial a_{22}} \frac{\partial}{\partial a_{24}} \frac{\partial}{\partial a_{33}} u = \frac{\partial^2}{\partial a_{13}^2} \frac{\partial}{\partial a_{22}} \frac{\partial}{\partial a_{24}} u = \dots$$

* Other papers on the same subject are—

GEER, P. VAN, “Over het gebruik van determinanten bij de methode der kleinste kwadraten,” *Nieuw Archief voor Wiskunde*, i. (1875), pp. 179-188.

CATALAN, E., “Remarques sur la théorie des moindres carrés,” *Mém. de l'Acad. . . . de Belgique*, xlili. (1878), pp. 24-33.

It is an inference from a property of the multiple integral

$$\left(\int_{-\infty}^{\infty} \right)^n e^{-\frac{1}{2}Q} dx_1 dx_2 \dots dx_n$$

where Q is the n -ary quadric

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & \dots & \\ \hline 2a_{11} & a_{12} & a_{13} & \dots & x_1 \\ a_{21} & 2a_{22} & a_{23} & \dots & x_2 \\ a_{31} & a_{32} & 2a_{33} & \dots & x_3 \\ \dots & \dots & \dots & \dots & \end{array}$$

TRZASKA, W. (1876, Dec.).

[Question 201. *Nouv. Corresp. Math.*, ii. p. 401. Solution by Even, iii. pp. 91–92.]

Exactly in Cayley's manner it is shown that higher-order determinants formed like his third of 1841 (*Hist.*, ii. p. 110) vanish also.

JAMET, V. (1877); LONGCHAMPS, G. DE (1877).

PAIGE, C. LE (1879); WOLSTENHOLME, J. (1879).

[Sur une application des déterminants. *Nouv. Annales de Math.* (2) xvi. pp. 372–373.]

[Des fractions étagées (p. 325). *Giornale di Mat.*, xv. pp. 299–328.]

[Question 514. *Nouv. Corresp. Math.*, v. p. 452. Solution by Jamet, vi. pp. 92–93.]

[Question 6038. *Educ. Times*, xxxii. pp. 243, 315: or *Math. from Educ. Times*, xxxii. p. 91.]

None of these is of any moment, the first and third being but instances of the square of an oblong array, the fourth a simple instance of the multiplication of two determinants, and the second a reproduction of Ferrers' first result of 1855.

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(Issued separately March 15, 1913.)

VI.—The Precipitation of Salts by the Corresponding Acids. By
 Irvine Masson, M.Sc. (1851 Exhibition Scholar of the University
 of Melbourne). Communicated by Professor JAMES WALKER, F.R.S.

(MS. received December 16, 1912. Read same date.)

It is well known that salts such as the chlorides of the alkali metals are less soluble in water containing the corresponding acid than in water alone ; and for sparingly soluble salts, in weakly acid solution, the decrease of solubility has been shown to follow quantitatively the course predicted from simple ionic notions. With concentrated solutions of strong electrolytes, however, the causes which operate are still obscure ; and, indeed, much has yet to be discovered as to the facts in such cases. There may be said to be two methods of studying the isothermal phenomena in question. The first, which has received attention from numerous observers, consists in determining the solubility of a salt in water containing the acid in varying concentrations. This may be termed the "solubility" method. The second method seeks to ascertain the minimum concentration of aqueous acid which must be used in order to cause deposition of salt, when the acid is added in small quantity to the saturated aqueous salt solution. This we shall refer to as the "precipitation" method ; it was adopted by Gibson and Denison, and formed the subject of a communication by them to this Society (*Proc.*, vol. xxx., 1910, p. 562).

The objects of the present paper are : first, to show the manner in which the results of the two methods are quantitatively related and may be compared, and secondly, to inquire how far the conclusions arrived at by users of the two different methods are in accord.

The change in the solubility of a given salt in water containing varying concentrations of the corresponding acid can be plotted, using solubility and acidity as co-ordinates. The data available include numerous determinations by Engel at 0° (*Compt. rend.*, 1886, **102**, 619 ; 1887, **104**, 433 ; *Ann. Chim. Phys.*, 1888 [vi.], **13**, 132), and also measurements carried out by the author for certain chlorides in hydrochloric acid and nitrates in nitric acid, at 30° (*Trans. Chem. Soc.*, 1911, **99**, 1132). In the last-mentioned paper it is shown that the whole course of the curve is, in each case, expressed by the empirical equation

$$-\frac{da}{db} = k_0 + k \frac{a}{b} \quad . \quad . \quad . \quad . \quad (1)$$

where b = the solubility of the salt corresponding with acidity = a , and k_0 and k are constants for the particular case. It was further stated that the same expression holds for Engel's data.

The factor to which attention is to be directed is k_0 , which, as is evident, expresses the initial slope of the curve, *i.e.* the slope at low values of a , when

$$-\frac{da}{db} = k_0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The limits within which these curves retain their initial slopes—that is to say, are straight lines—were found to be wide. Within these limits, then, equation (2) gives

$$\frac{a}{B - b} = k_0 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where B = the initial solubility in pure water.

Now turn to the cases in which the precipitation method is used to study the same salt. Let a *small* volume, v , of acid whose concentration = A be added to unit volume of the saturated aqueous salt solution. The concentrations will then be $A\frac{v}{1+v}$ and $B\frac{1}{1+v}$ respectively. But the solubility of the salt in this particular strength of acid, as follows from equation (3), is $B - \frac{1}{k_0} \cdot A\frac{v}{1+v}$. If this be less than $\frac{B}{1+v}$, precipitation will occur; if it be greater, the solution will become unsaturated; and the critical condition is, therefore, that

$$\frac{B}{1+v} = B - \frac{1}{k_0} \cdot A\frac{v}{1+v},$$

or

$$A = k_0 B \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equation (4), then, gives the desired relationship between the two methods; the "critical" concentration of acid which just fails to precipitate salt when it is added to the pure saturated salt solution (as found by the precipitation method) is equal to the product of the aqueous solubility of the salt and the initial slope of the solubility-acidity curve (as found by the solubility method).

There might, however, be some initial fluctuation in the solubility-acidity curve, which would render interpolation doubtful. Hence some experiments were carried out to test the relationship directly, precipitation trials being made with salts whose solubility-acidity curves had previously been determined.

The method was as follows:—A saturated solution of the salt at 30° was

carefully prepared, and was kept in a thermostat at $30^{\circ} \pm 0.02$. From it, 20 c.c. samples were withdrawn by means of warm pipettes provided each with a glass-wool filter-cap, and were quickly transferred to a series of "Pasteur" tubes in a rack in the bath. The rack was then put into a bath at 40° to 50° , and to each tube 0.5 c.c. of acid was added, a different concentration of acid being used for each tube. One sample was not mixed with acid, and served as a control. The ground caps of the tubes, which fitted perfectly when lubricated, were replaced, and the whole rack was completely immersed in the thermostat, where after a while it was rotated for some hours at a time, if necessary during two days. If crystals were formed, they were very easily seen on inverting the tube under the water of the bath, when they rolled down to the narrowed end of the cap; it was possible in this way to detect quite minute deposits.

This procedure alone might lead to erroneous conclusions owing to supersaturation; therefore, after the above examination was completed, the narrowed ends of the tubes, containing some of the solution, were allowed to project above the surface of the thermostat-water, and were cooled till local crystallisation set in; the tubes were then submerged again and rotated for a considerable time, and subsequently examined for undissolved crystals.

By two or more repetitions of these experiments, the value of A, the critical concentration of added acid, could be determined to within about 0.2 gram-equivalents per litre. The three test cases, shown in Table I., are those of sodium chloride, barium chloride, and barium nitrate; and it is seen that the agreement is excellent in the first two cases. With the nitrate, the amounts of precipitate were small and therefore difficult to observe, and the discrepancy between found and calculated result may be due to this cause. It appears, however, that for the chlorides interpolation is valid.

TABLE I.

Salt.	NaCl.	$\frac{1}{2}\text{BaCl}_2 \cdot 2\text{H}_2\text{O}$.	$\frac{1}{2}\text{Ba}(\text{NO}_3)_2$.	NH ₄ Cl.	KCl.
Previously found $\left\{ \begin{array}{l} B \\ k_0 \end{array} \right.$	5.40	3.49	0.85
	0.95	0.85	1.06
A _{calc.} = k ₀ B	5.1	3.0	0.9
A found	max.	5.2	3.0	1.4	5.4
	min.	5.0	2.7	1.2	5.0

We may now examine Engel's solubility data in the light of what has been said. Table II. shows values of k_0 , of B, and of their product k_0B .

TABLE II. Chlorides at 0°.

Metal.	Li.	$\frac{1}{2}$ Mg.	$\frac{1}{2}$ Ca.	NH ₄	$\frac{1}{2}$ Sr.	Na.*	K.*	$\frac{1}{2}$ Ba.
k_0	. .	.96	.98	.99	1.11	.90	.84	.94
B	. .	12	10	9.3	4.6	5.2	5.47	3.45
$k_0B = A_{\text{calc.}}$	11.5	9.8	9.2	5.1	4.6	4.6	3.2	2.6

* In connection with this table, it is necessary to observe that for the chlorides of sodium and potassium the solubilities found by Engel are in excess of the accepted values, which are 5.39N for sodium chloride, and 3.33N for potassium chloride; this is quite sufficient to make a considerable error in k_0 , hence the figures tabulated in these two instances are certainly incorrect. The other values are correct.

The special point to which attention is drawn is that k_0 is approximately the same for all the chlorides, and is nearly unity. This was practically the claim put forward by Engel.

It should further be mentioned that change of temperature seems to have little effect on the value of k_0 .

As a result, it follows that the critical concentration of acid in precipitation-tests with a given chloride ($=k_0B$) is roughly equal to the concentration of the chloride in saturated aqueous solution ($=B$).

It is noteworthy that chlorides which are deposited in the hydrated condition are, from the standpoint of the course of their curves as expressed by the equation, in nowise different from those which are anhydrous—a fact which is in harmony with the generally accepted notion of the hydration of all salts in aqueous solution.

Furthermore, the only chlorides whose solubility-acidity curves have not an initial slope roughly = 1, and with which this equation does not seem to agree, are those such as thallous chloride, which are sparingly soluble and for which simple ionic notions suffice, and those such as ferric chloride, which are known to form complexes with the hydrochloric acid.

It therefore appears to be likely that, since the same equation (with practically the same initial constant), expresses the course of the solubility-acidity curves of ordinary chlorides, whether hydrated or not, therefore the processes controlling the solubility changes are the same in all these cases.

Reference may now be made to the precipitation experiments of Gibson and Denison (*loc. cit.*; *Trans.*, 1911, **48**, 117). These authors, investigating

the properties of hydrochloric acid solution of maximum specific conductivity, found that acid of approximately this concentration would just fail to cause precipitation of certain chlorides from their saturated solutions ; that is to say, that what we have called the "critical" strength of acid, A, is approximately the same for all the salts studied by them.

In the light of equation (4), we should expect to find that the salts examined by these authors should have roughly equal solubilities (B). And such proves to be the case with three of the four salts, whose values are : sodium chloride, 5·4N ; ammonium chloride, 5·7N ; rubidium chloride, 5·9N. Since the critical acidity, A, is stated to be between 5·0N-HCl and 5·7 N-HCl, it will be seen that these salts behave in exactly the same way as do many other chlorides in having volumes of k_0 nearly=1. The fourth chloride, that of potassium, evidently has a slightly higher value for k_0 than is usual.

The conclusions which would appear to be drawn by Gibson and Denison are that hydrochloric acid has a tendency towards assuming the condition of maximum conductivity ; that this fact can explain why the chlorides of sodium, potassium, ammonium, and rubidium are precipitated from their saturated solutions only by acid of strength greater than this "maximal" strength (*Proc., loc. cit.*, p. 563; Denison, *Trans. Faraday Soc.*, vol. viii., part i. (1912), p. 27) ; and that the chlorides of lithium, magnesium, calcium, strontium, and barium do not behave in the same way, possibly because of hydration or of complex-formation (*Proc., loc. cit.*, p. 568).

From what has been said, however, it is clear that the last-named group of chlorides in no way differs from the first-named when judged by the proper standard, namely, the course of the solubility-acidity curves ; and that for the first-named group the approximation of the critical acidities to that of maximal acid is an accidental coincidence which depends upon the solubilities of the salts concerned.

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LONDON.

(Issued separately March 24, 1913.)

VII.—Preliminary Observations on Magnetic Induction in Ferric Oxide at Various Temperatures and in Different States, and on the Possible Chemical Changes indicated by the Observations. By G. E. Allan, D.Sc., Lecturer on Electricity, Pure and Applied, and Assistant to the Professor of Natural Philosophy in the University of Glasgow; and John Brown, M.A., B.Sc., Houldsworth Research Scholar in the University of Glasgow. *Communicated by Professor A. Gray, F.R.S., LL.D.*

(Read July 1, 1912. MS. received December 8, 1912.)

MAGNETIC INDUCTION IN FERRIC OXIDE.

IN a previous paper by one of us,* it was shown that a certain part of rock magnetism was variable and uncertain under the influence of raised temperature. In some instances the rock specimen became permanently more magnetic after it was heated in air, whereas, in the majority of cases, heating caused great loss of magnetic quality. It was suggested that the increase of magnetism observed was caused by the transformation of haematite into a more magnetic iron oxide.

In order to obtain material which might support this assumption, and also to amplify the work of previous investigators, experiments were undertaken with artificial and natural ferric oxide, in which the transformation of ferric oxide from a non-magnetic into a magnetic condition was observed, while some of the conditions which assist or impede this change were also studied.

The magnetic properties of crystalline haematite have been described by Westman and Kunz, and Abt has given an account of those of the compact ore.

J. Westman,† dealing with specular iron ore (*Eisenglanz*), subjected it to magnetic fields up to 70 C.G.S., and found that the mineral possesses considerable coercive force, also that the magnetic property is confined chiefly to the base plane of the crystal. The susceptibility of crystalline haematite amounts to only one- to two-thousandths of that of ordinary iron.

J. Kunz,‡ from tests of over a hundred specimens from different sources,

* *Phil. Mag.*, April 1909.

† *Upsala Universitets Årsskrift*, 1896; *Mat. och Naturv.*, ii. 1, 1897.

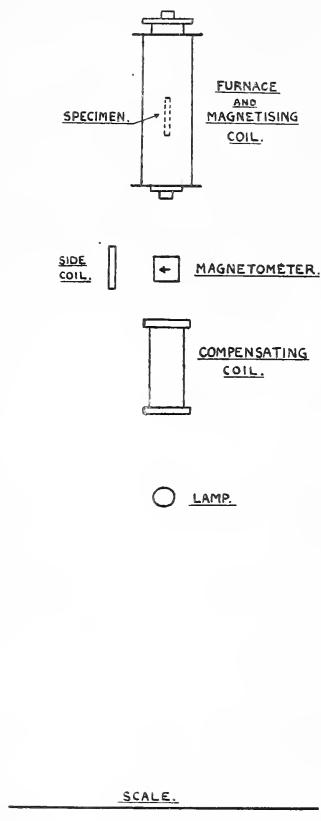
‡ *Neues Jahrb. f. Mineralogie*, vol. i. pp. 62–88, 1907.

classed the crystals in two groups. Those in the first group exhibit little hysteresis and do not change when heated. The crystals of the second group have great hysteresis, and suffer some deterioration on heating. The critical temperature of the members of the first group is 645°, and that of the second group 660° C.

A. Abt's tests* were made on bars cut from blocks of massive haematite of great purity, and they relate chiefly to the remanent magnetism of the

mineral after subjection to fields of increasing value. The coercive force measured was 150 C.G.S., and the bars were apparently unsaturated in a field of 1000 C.G.S.

So far as is known to the authors, no continuous observation of the magnetic behaviour of the ordinary pure artificial ferric oxide when heated in air or in other gases has yet been made, and the present paper deals with this phase of the subject.



METHOD AND APPARATUS.

The method and apparatus were practically the same as employed on former occasions.† For the purpose of this paper, the method may be shortly recapitulated.

The bar to be tested is placed in a combined magnetising coil and electric furnace (see figure); a current is passed in series through the magnetising and compensating coils, and through a small side coil intended to correct want of alignment in the other two coils.‡

The specimen when in the furnace can be tested either by passing it through a heat cycle, in which the temperature is gradually raised from that of the room to 1000° C. and then brought back to room temperature again, or by keeping it at a constant high temperature. In both cases magnetometer readings are taken at intervals, such readings indicating the magnetic state of the bar at the time. The temperature of the bar is taken by means of a thermo-couple with its junction placed at the middle

* *Wied. Ann.*, lxviii. pp. 658–673, 1899.

† *Phil. Mag.*, Jan. 1904 and April 1909.

‡ *Ibid.*, April 1909, p. 574.

of the bar. The magnetometer readings are taken by noting the position of the spot of light when a given magnetic field is applied to the specimen and when the same field is reversed. The difference of the two readings, called the double deflection, is taken as a measure of the magnetic state of the bar at the particular time or temperature. This is considered sufficient in the case of such weakly magnetic substances.

MATERIAL.

The ferric oxide employed was in the form of :

- (1) The ordinary dark-red artificial ferric oxide.
- (2) A dark-brown ferric oxide obtained from Merck.
- (3) A yellow-brown ferric oxide, also from Merck.
- (4) Blocks of haematite, obtained from cargoes of Spanish ore discharged at Glasgow, containing about 60 per cent. of iron.

The ferric oxide mentioned under (2) and (3) was of the purest obtainable commercial form.

The ores tested were known as Lucinena, red Seriphos, brown Seriphos, Agulas, and Bacares. Of these, the Lucinena was a soft ore, and brown Seriphos a very hard one.

TREATMENT OF THE MATERIAL.

The amorphous powder specified under (1), (2), and (3) was tested in the form of :

- (a) Loose, dry powder held in a glass or copper tube open at both ends.
- (b) Bars formed by the use of (i.) alcohol as a cement; (ii.) water as a cement; (iii.) compression in a Bramah press.

Bars were also cut from pieces of the haematite ore.

None of the material underwent any previous heating or drying treatment.

HEAT TREATMENT.

Three heat processes were employed, which were as follows :—

1. *Heat Cycle*.—In general the bars were first placed in the furnace-magnetometer and passed through a temperature cycle from the air temperature up to 1000° C. and back again, readings being taken usually at intervals of 50°, sometimes at smaller intervals. Observation of the bar during this treatment showed whether any magnetic change took

place, and also what was the optimum temperature for this change. Further, the cooling curve indicated the final state of the bar. A cycle was seldom completed, as a rule, under two hours, and often required three to four hours.

2. *Constant High Temperature*.—The bar was placed in the furnace-magnetometer, and its temperature was raised to a given value and kept constant for one, two, or more hours, during which its magnetic condition was noted at five-minute intervals by taking the double deflection due to the reversal of a known magnetic field.

3. *Heating in Gas Furnace*.—In this method the bar was placed in a gas furnace, raised to a temperature of 850° C., and maintained at this temperature for ten minutes. It was then cooled, and afterwards examined in the magnetometer when cold. This treatment was repeated many times until the magnetic behaviour of the bar became steady.

Weiss and Foëx,* when preparing artificial magnetite from ferric oxide, enclosed the oxide in platinum, and heated it in an oxyhydrogen flame, and afterwards sealed the platinum in the flame to prevent re-oxidation on cooling.

In our preliminary experiments it was observed that when a bar of haematite ore was heated to 850° in a gas furnace and cooled, there was always a change from the non-magnetic to the magnetic state, although no precautions had been taken to prevent chemical change. It was therefore considered that a series of heatings for short intervals might enable us to note the progressive changes which took place in the magnetic state of the bar of haematite.

EXPERIMENTAL RESULTS.

These are given entirely in the form of curves.

I. Ferric Oxide Bars heated in Air.† (Curves 1 to 6.)

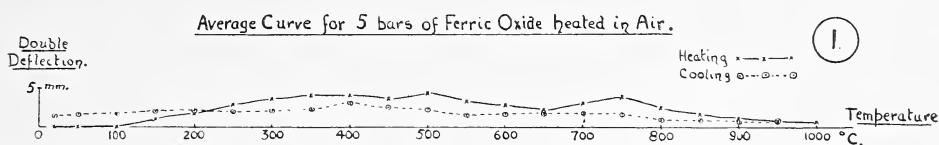
1. Heat Cycles. (See Curve 1.)

In the case of five "bars" of ferric oxide heated in air to 1000° C. the deflections obtained were small. All the bars were initially non-magnetic at air temperature in a field of 43 C.G.S., and the deflection began to grow at a temperature of 100° C. for three of the bars, and at 200° for the other

* *Archives des Sciences*, xxxi. p. 89, Feb. 1911.

† The air here referred to means the air of the furnace in which the bars were heated. Usually the electric furnace was stopped at one end with wool and obstructed at the other by the thermo-couple. The gas furnace was open to the air at both ends.

two. On the average, there was a maximum at 500° , a minimum at 650° , another maximum at 750° , and a minimum at 1000° , but these are by no



means distinct, as the whole effect is very small. The average curve for these five bars is given. The bars were made up as follows:

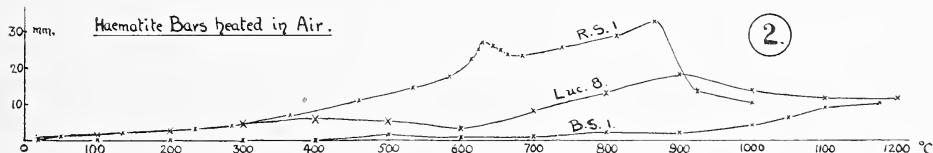
- (1) Yellow oxide mixed with alcohol and dried.
- (2) Red oxide mixed with alcohol and dried.
- (3) Bar of compressed red oxide.
- (4) Red oxide powder in glass tube.
- (5) Brown oxide powder in copper tube.

As a rule, the bars were non-magnetic when cooled down after heating; but the majority of them showed a slight maximum when cooling through 400° .

Bars of Hæmatite Ore. (See Curves 2 and 3.)

When heated in air, bars of hæmatite ore containing about 60 per cent. of iron also exhibited an increase of magnetic quality.

Curves 2 contain the results for heating fresh bars of red Seriphos, Lucinena, and brown Seriphos in the furnace-magnetometer. The behaviour of these bars seems irregular, but it may be noted that all three curves agree in passing first through a low maximum and afterwards through a



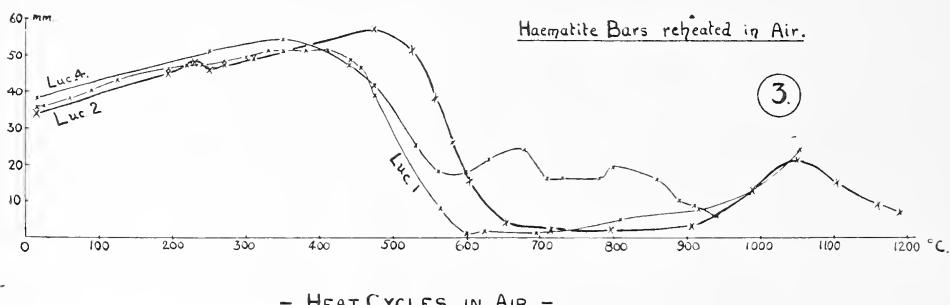
higher one, and it would appear that the same succession of changes had gone on in all three bars, but at different rates. No importance is placed on the absolute value of the deflections at present, because the three bars were of different weights and were heated for different lengths of time.

When the hæmatite bars were cooled in the furnace-magnetometer, they followed a course similar to that given for Luc. 1 in Curves 3; but no accurate data are at hand to illustrate this behaviour, the cooling curves having been affected by furnace trouble and by coil effect.

The magnetic property acquired by heating in air persists when the bar is cooled, and under further heat testing the bar gives a heating curve which resembles that for natural magnetite.

The set of Curves 3 is for three bars of Lucinena ore which had been previously heated for various periods in the gas furnace in air, and were then passed through heat cycles in the electric furnace.

Lucinena No. 1 was heated for 100 minutes at 850° in the gas furnace, Lucinena No. 2 for 30 minutes at 600° (with little effect) and then an additional 10 minutes at 850°, and Lucinena No. 4 for 10 minutes at 850°,



the heat cycle being made thirteen days after the heating in the gas furnace. The slight maximum in Luc. 2 at 230° is no doubt of the same nature as that in Curve 2, R.S. 1, at 630°, and that referred to later on in other curves. It appears to be characteristic of the magnetic change in ferric oxide.

The curves coincide to a remarkable extent; they follow the heating curve of natural magnetite* up to 600°. It may therefore be concluded that haematite ore, even when containing a considerable amount of impurity (pure haematite contains 70 per cent. of iron), exhibits, after being heated to a high temperature in air, the magnetic characteristics of magnetite.

2. Constant High Temperature. (See Curve 4.)

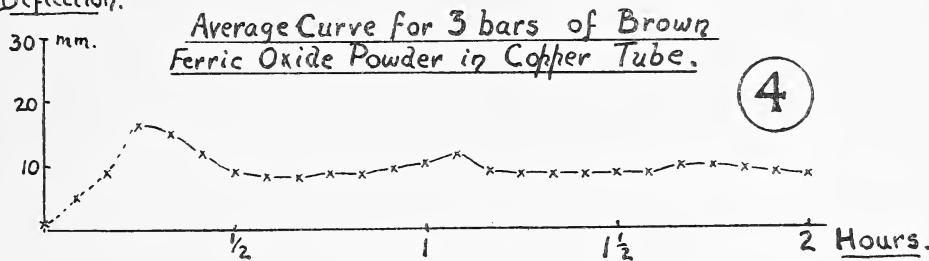
The full line in the curve begins when the bar had reached a temperature of 500°.

When heated in air at 500° C., the ferric oxide, in the form of powder held in a copper tube, did not exhibit uniform behaviour; but in each of the three specimens tested there was considerable gain in magnetic quality with time. In one case the deflection dropped to zero after about 2½ hours' heating. The average curve given for the three specimens is similar in

* See *Phil. Mag.*, Jan. 1904, curve 15.

form to that obtained for the oxide when heated in hydrogen. But the curves differ in one particular. The magnetic property appeared sooner, and at a lower temperature, in those bars of ferric oxide which were heated

Double Deflection.



in air than in the bars heated in hydrogen. This holds both for the curves obtained at constant temperature and for those taken during continuous rise of temperature (compare Curve 4 with Curve 8, also Curve 1 with Curve 7). This result was entirely unexpected.

3. Heating in Gas Furnace.

The specimen was placed in a Fletcher gas furnace about one foot long, provided with ten Bunsen jets over its whole length. The bar did not come into contact with the flame, but was protected from it by means of a porcelain or copper tube. A thermo-couple with its junction at the middle of the bar was read every minute after the temperature had risen to 850° . When in the furnace, the bar lay east and west. After 10 minutes' heating at 850° , the gas was turned out and the bar cooled down to air temperature.

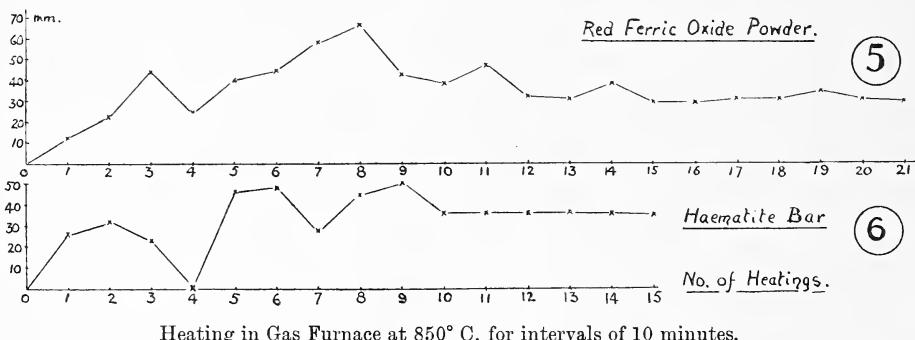
Examination in the Magnetometer after Heating in Gas Furnace.

(See Curves 5 and 6.)

Some time after the surface of the specimen had become cold, it was removed to the magnetometer, and the deflection was noted which was caused by the bar on reversal of a given magnetic field. The above procedure of heating in the gas furnace for 10 minutes, followed by examination in the magnetometer after cooling, was repeated many times with each specimen, and the results were plotted in the following way:—The abscissa is divided into intervals which measure the 10-minute periods of heating. At the end of each 10-minute of abscissa, the height of the ordinate measures the deflection given by the bar after it had cooled down and was put into the magnetising coil. The curves given are for a bar of

red oxide powder held in a copper boat, and for a bar of Bacares ore. This method of treatment appears to develop greater fluctuations in the magnetic state of the bar than the second method, in which the bar is not raised above the critical temperature of magnetite. There is a certain amount of similarity in the course of the two curves, and the magnetic state tends to become steady after two hours' total heating at 850°. It is remarkable

Double Deflection



Heating in Gas Furnace at 850° C. for intervals of 10 minutes.

that the first minimum for the haematite bar is on the zero line; also that in each of the curves the first maximum is followed by a higher one, as in Curves 2, 3, 5, 6, 7, and 9. The method is unlikely to lead to numerical results, but it exhibits the gradual change in the magnetic state of a bar subjected to short intervals of heating at a temperature above the critical point of iron.

II. Ferric Oxide Bars heated in Hydrogen.

The hydrogen used in this part of the investigation was produced by the action of hydrochloric acid on granulated zinc, and the gas was bubbled through strong sulphuric acid, to remove moisture, before it was led into the furnace.

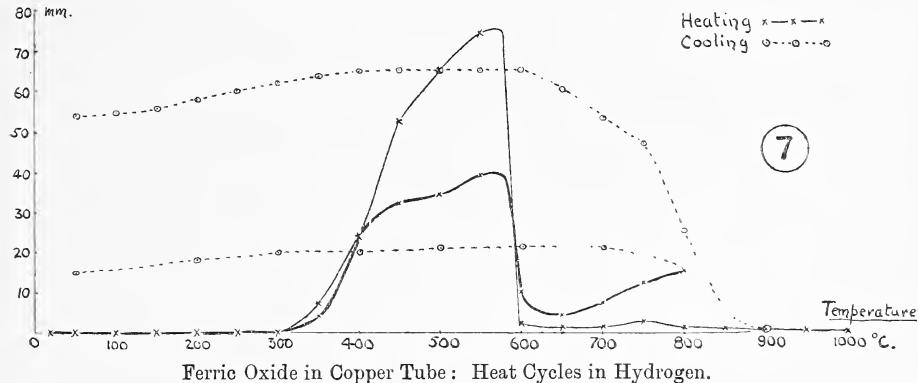
1. Heat Cycles. (See Curve 7.)

The two curves given refer to the behaviour of a specimen of red oxide and one of brown oxide, each held in a copper tube. From the curves it may be seen that magnetic quality begins to appear a little above 300°, increases to a high maximum at 575°, and falls to a minimum at 600° or 625°. In one case the heating was discontinued at 800°. In the second case another minimum was reached at 900°. In both cases the powder was magnetic during cooling, and the cooling curves point to a separation of

iron during heating, since they have the minimum point of iron and not that of magnetite.

The above curves differ from those for heat cycles in air in their extent of coincidence. There is here a definite magnetic effect which repeats itself when the test is repeated with a fresh bar. The magnetic effect is large, and is confined within comparatively narrow temperature limits, viz. 300°

Double Deflection.



Ferric Oxide in Copper Tube: Heat Cycles in Hydrogen.

to 600° C. As these curves and their underlying changes appeared to follow more closely than any of the others the lines of chemical research on this subject, a new series of tests was instituted on the heat cycles in hydrogen, the details of which we hope to publish later. This new series has brought out with distinctness a small maximum at 425° and a higher one at 575°, followed by the minimum at 625° C. We consider this first maximum to be the same as that referred to under Curves 2 and 3; it also appears distinctly in Curve 5.

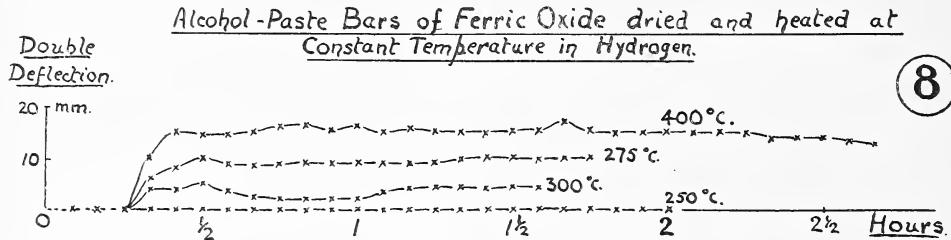
2. Constant High Temperature in Hydrogen. (See Curves 8 and 9.)

The two groups of curves given show, in one case, the effect of heating ferric oxide bars made with alcohol and dried, and in the second case the effect of heating bars of Bacares ore.

With the artificial oxide the rise of the curve began 15 minutes after heating had commenced, and a moderately steady state was soon reached. No magnetic effect was obtained until the temperature rose above 250°.

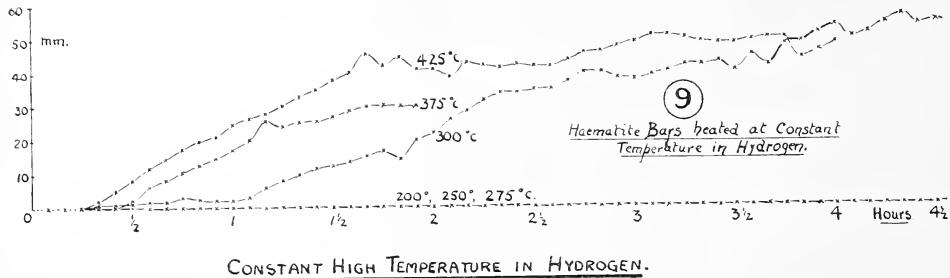
With the haematite bars the rise was more gradual, but it reached a higher maximum. This was due in part to the mass of the haematite bars being roughly ten times as great as that of the bars of ferric oxide. In

other respects the behaviour of the natural oxide was similar to that of the artificial material. Little or no effect was observed in these at temperatures below 275° C. Practically no magnetic effect was observed in a bar of



haematite ore which was heated in hydrogen at 275° for 3½ hours; whereas, at the same temperature, a notable amount appeared in the bars of ferric oxide, and in a much shorter time than the above interval.

A great change of volume and of colour takes place at temperatures above 500° C. in a bar of ferric oxide as the effect of heating in hydrogen, alcohol vapour, or air. The bars contracted in some cases to less than half their



initial volume, and hardened into compact, rigid bars. At the same time their dark red colour changed to a dark grey-black.

DISCUSSION OF RESULTS.

The observed numbers, from which the curves have been drawn, are liable to be affected by certain causes, the chief of which are: (a) coil effect, (b) shortening of the bar as a result of heating, and (c) errors of temperature measurement.

(a) *Coil Effect.*—Errors of observation may arise through displacement of the magnetising or compensating coil during the course of a prolonged heating and after compensation has been effected. We have called the deflection thus introduced the coil effect. It makes its appearance on a very slight motion of either coil, since these coils have to be set as close to

the needle as possible on account of the somewhat unmagnetic nature of the working material.

It is impossible to test for coil effect during the course of a heat cycle; it can be done only at the beginning and end of the cycle. When the coil effect was small, say 2 mms., it was distributed uniformly over the curve, and the curve was re-drawn. All curves shown are either devoid of coil effect or have been corrected as described. When the coil effect at the end of a test was large, the heat cycle was repeated. These repeated tests have shown that the coil effect does not take place at any particular instant, but appears gradually, and a curve vitiated by coil effect exhibits the same maxima and minima as one in which no coil effect has been noted. The difference between the two curves lies only in the absolute value of the deflections.

(b) *Shortening of the Specimen while under Test.*—Observation showed that shortening took place at temperatures higher than 500°, and chiefly in bars which had been moulded by mixing the ferric oxide with alcohol; one such bar shortened by about 20 per cent. of its original length.

The effect of change of length of the bar on the deflection which the bar produces may be calculated as follows:—

The force F , due to the bar at a point on its axis at distance r from its centre is given by

$$F = 2Mr/(r^2 - l^2)^2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where M is the magnetic moment of the bar, and $2l$ its pole distance.

Suppose that, by the action of heat or otherwise, the bar, of length $2L$, undergoes a change of length $-2\delta L$. Suppose also that the permeability of the bar does not change as a result of shortening alone, and that the pole strength remains the same.

Taking logarithms of equation (1) and differentiating, or taking the variation of the equation so formed, we obtain

$$\frac{\delta F}{F} = \frac{\delta M}{M} + \frac{\delta r}{r} - 2 \frac{\delta r + \delta l}{r+l} - 2 \frac{\delta r - \delta l}{r-l} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since $M = 2ml$, and, on the usual assumption, $l = 4L/5$,

$$\frac{\delta M}{M} = \frac{\delta l}{l} = \frac{\delta L}{L} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and shortening implies that all three quantities are negative. By substituting from (3) in (2), using negative signs for δl and δL , we get

$$\frac{\delta F}{F} = -\frac{\delta L}{L} + \frac{\delta r}{r} - 2 \frac{\delta r - \delta l}{r+l} - 2 \frac{\delta r + \delta l}{r-l} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The limiting values of (4) could be obtained by putting

$$\delta r = \pm \delta L.$$

In the course of our research, however, no case was observed in which shortening took place from one end while the other end remained fixed. In work with solid haematite bars, no shortening whatever was noted. In other cases in which ferric oxide was held in a copper tube, there was a change of volume without change of length. But with ferric oxide moulded by means of alcohol and held in a copper mould or boat, shrinking took place at both ends, to an equal extent so far as could be judged. Hence experiment points to the value

$$\delta r = 0,$$

and equation (4) becomes

$$\frac{\delta F}{F} = -\frac{\delta L}{L} + \frac{2\delta l}{r+l} - \frac{2\delta l}{r-l} \quad \quad (5)$$

or, substituting for l and δl ,

$$\frac{\delta F}{F} = -\frac{\delta L}{L} \left\{ 1 - \frac{8L}{5r+4L} + \frac{8L}{5r-4L} \right\} \quad \quad (6)$$

This equation expresses the proportional change of magnetic force at the needle due to the shortening $\delta L/L$; and if we substitute for r and L their usual values in our experiments, viz. $r=33.5$ cms., $L=4$ cms., the equation becomes

$$\frac{\delta F}{F} = -1.037 \frac{\delta L}{L} \quad \quad (7)$$

If θ is the angular deflection of the needle in the earth's field H ,

$$F = H \tan \theta = H \frac{d}{2D} \text{ nearly,}$$

where d is the scale deflection and D the distance from needle to scale.

Hence

$$\frac{\delta F}{F} = \frac{\delta d}{d},$$

and

$$\frac{\delta d}{d} = -1.037 \frac{\delta L}{L} \quad \quad (8)$$

From (8) we see that the shortening of the bar *in situ* causes a reduction of the deflection, the amount of which may be determined, if $\delta L/L$ is known, and added to the observed deflection.

In order to use this result properly, one would require to know the shortening of the bar at each stage of the heating—a matter of considerable difficulty and not within the scope of the present experimental series.

How far this effect concerns the given curves may be judged from the following particulars :—

Of the five bars which went to make Curve 1, two were moulded with alcohol and one was compressed, and the shortening in each was 14 per cent. In the other two bars, which consisted of oxide held in a copper tube, no shortening took place. Thus the final correction for Curve 1 amounts to about 9 per cent., or an increase of less than $\frac{1}{2}$ mm. Alcohol was also employed in forming the bars from which Curves 8 were obtained, but the temperature in this instance did not rise above 500° C., and no shortening was observed. In all other cases, viz. solid haematite bars and dry ferric oxide held in a copper tube, there was no shortening.

From the foregoing it may be seen that this effect, although large in bars formed with alcohol, does not seriously affect the given curves. No correction has been applied for it.

(c) *Errors in Temperature Measurement.*—The moving-coil temperature indicator used in conjunction with the thermo-couple had been standardised by means of a series of fixed points ranging from the ice-point up to the melting-point of potassium carbonate at 883° C. The scale divisions of the indicator had a value of about 7° , and were estimated to tenths by the eye. Temperatures could therefore be read to about 1° C., but we have not taken our readings to less than 5° , partly because of a variation of temperature from end to end of the furnace, and partly because, in work with the solid haematite bars, of the lag of temperature within the bar. The former error was removed in certain cases by the use of the copper tube.

Temperatures above 883° C. were extrapolated. It has to be remarked that the temperatures in Curves 2 and 3, which rose to 1200° C., were measured by means of a platinum thermometer.

Of the results submitted, those obtained by heating ferric oxide and haematite in air, although somewhat irregular, all agree on the point that magnetism appears with heating. This is contrary to the effect to be expected when ferric oxide is heated in an oxidising atmosphere. The less permanent nature of the effect in the case of the powdery ferric oxide as compared with that in the more solid haematite bars indicates that the effect takes place only at parts of the test specimen which are inaccessible to the air surrounding it. In the first case, the magnetism must be caused by an iron oxide; and in the second, the curves for haematite, when compared with those for magnetite already published, indicate that ferrosoferric oxide is present. This has, however, not yet been verified by means of chemical analysis.

The outstanding result of the heat cycles in hydrogen is the definiteness

of the temperature at which the magnetic effect reaches a maximum, viz. near 550° C. Further, there are signs of three maxima on each curve. These occur at the temperatures 425°, 575°, and 750° (800°) respectively, and are supposed by us to correspond with the appearances of Fe_3O_4 , FeO , and Fe , or possibly with the sequence: magnetic ferric oxide, Fe_3O_4 , and Fe . The cooling curves are both iron curves, and not magnetite, since they proceed from a minimum at about 900° C.

The reduction of volume of the powder which takes place when the ferric oxide is heated above 500° C. in hydrogen or air makes it impossible to add a true scale of intensity or susceptibility to the curves.

SUMMARY OF RESULTS.

The experimental results may be briefly summarised as follows:—

1. When pure artificial ferric oxide and impure haematite ore were heated in air in a magnetic field of 43 C.G.S., they became magnetic with rise of temperature, and were magnetised, more or less permanently, when cooled down from 1000° C. The seat of magnetisation in the case of the ore, as indicated by magnetic methods, was ferrosoferric oxide. (Curves 1, 2, 3.)

2. When ferric oxide was heated in hydrogen, the magnetic effect usually appeared later and at a higher temperature than when air was employed. Also, the temperature at which magnetism began to appear was more definite, viz. 300° C., and the intensity of magnetisation was much greater than with air. (Curves 1 and 7.)

3. The temperature-deflection curves for ferric oxide heated in hydrogen have maxima at temperatures 425°, 575°, and 750° C.,* and the cooling curve is that for iron. These three temperatures are therefore regarded as the optimum temperatures for the production of Fe_3O_4 , FeO , and Fe respectively, or for the sequence: magnetic ferric oxide, Fe_3O_4 , and Fe . (Curve 7.)

4. The experiments, so far as they go, support the view that magnetic oxide may be formed at comparatively low temperatures in rocks which contain haematite.

Note.—In a short appended statement, references are made to the work of Moissan and others on the chemical transformation of ferric oxide when continuously heated in a reducing gas. The results seem to the authors to have an intimate bearing on the interpretation of the experimental results described above.

* See p. 77.

The above experiments were carried out in the Natural Philosophy Institute of Glasgow University, and the authors desire to thank Professor Gray for his aid and encouragement during the progress of the work.

CHEMICAL TRANSFORMATION OF FERRIC OXIDE WHEN CONTINUOUSLY HEATED IN A REDUCING GAS.

Moissan * appears to have been the first to make a complete investigation of this question. He states that when ferric oxide is heated in pure dry hydrogen or carbon monoxide it is reduced, the changes following the order: ferric or sesqui-oxide, Fe_2O_3 —magnetic or ferrosoferric oxide, Fe_3O_4 —ferrous or protoxide, FeO —metallic iron. The change from Fe_2O_3 to Fe_3O_4 takes place between 350° C. and 440° C. , and the magnetic oxide obtained is an allotropic form of that produced at high temperatures, *e.g.* that found in meteoric dust. The magnetic oxide when further heated at 500° is reduced to ferrous oxide. This reduction is more complete when done in carbon monoxide than when done in hydrogen, and the protoxide obtained is free from magnetic oxide and from metallic iron—a result difficult to attain when hydrogen is employed. When the protoxide is heated at 600° to 700° in hydrogen, it is reduced to metallic iron.

If the temperature is kept constant, then, under the same conditions, the protoxide is formed at 440° in about 6 hours, and metallic iron after 12 hours.

Hilpert,† who repeated some of Moissan's work, found that the temperature of reduction from Fe_2O_3 to Fe_3O_4 depended in great measure on the temperature to which the ferric oxide had been previously heated, and that reduction began at temperatures which varied between 280° and 350° . The reduction does not proceed at the same speed throughout the material, and preparations may be obtained in which all stages of reduction are present, from unchanged red oxide to metallic iron.

Other observers ‡ have found the reduction of ferric oxide to metallic iron to take place at temperatures as low as 370° .

In addition to these observed differences in chemical behaviour, all three iron oxides have been described by chemists as existing in two allotropic

* *Ann. Chim. et Phys.*, v. 21, pp. 199–255, 1880.

† *Ber. d. chem. Ges.*, xlvi. 4, pp. 4575–4581, 1909.

‡ Siewert, *Jahresber. Chemie*, pp. 265–266, 1864; Glaser, *Zs. anorg. Chemie*, xxxvi. p. 21, 1903.

forms. The ordinary dark-red ferric oxide is non-magnetic in fields of about 50 C.G.S., but a magnetic sesqui-oxide has been described by Malaguti,* and has also been mentioned by Hauser † and Hilpert. Moissan has distinguished two allotropic forms of ferrosferric oxide which are produced in hydrogen at 350° and about 1500°; the former is black, has a density 4·86, and is turned into sesqui-oxide when heated on a platinum plate in air. The other form is also black; its density is 5 to 5·09, and when heated in air it does not change into the sesqui-oxide. Moissan has also described two forms of ferrous oxide. That obtained at 600° has a great affinity for oxygen, with which it combines with incandescence to form sesqui-oxide. The other form of ferrous oxide is obtained at 1000° C., and is not inflammable.

* *Ann. Chim. et Phys.*, iii. 69, pp. 214–224, 1863.

† *Ber. d. chem. Ges.*, xl. 2, pp. 1958–1960, 1907.

(Issued separately March 24, 1913.)

VIII.—Studies on Periodicity in Plant Growth. Part I.: A Four-Day Periodicity and Root Periodicity. By R. A. Robertson, M.A., F.L.S., and Rosalind Crosse, B.Sc., Carnegie Research Scholar, Botanical Department, The University, St Andrews. (With Three Plates.)

(MS. received June 12, 1912. Read November 18, 1912.)

THE immediate purpose of this research has been:—

1. To investigate whether a presumably spontaneous growth variation amounting to a four-day rhythm which had been observed in certain organs of limited growth was an individual idiosyncrasy or was of general occurrence in plants.
2. To examine the growth rhythm in roots and ascertain if any correlation existed between the respective rhythms of root and shoot in the same plant.

The ultimate aim is to determine by an examination of the rhythm phenomena in higher and lower plants whether a fundamental rhythm exists based essentially on periodic chemical changes taking place in the living matter of the cell, that is, whether processes of anabolism and catabolism of rhythmical recurrence find expression cumulatively in variations of macroscopic magnitude.

The classic researches of Sachs (*Arb. Bot. Inst.*, Würzburg, 1874, Bd. 1, pp. 99 *et seq.*) and of Baranetzsky (*Mém. Acad. Imp.*, Petersb., 1879, 7 Ser., vol. xxvii. pp. 1 *et seq.*) laid the scientific foundations of our knowledge of the phenomena of periodicity.

Sachs introduced precision apparatus, and first demonstrated experimentally the spontaneous regular variation constituting the grand period of growth, the induced variations associated with the regular alternation of day and night, known as the daily periodicity, as well as spontaneous irregular variations dependent on causes inherent in the cell itself. He further showed that the daily periodicity—the retardation and acceleration of growth in length associated with the regularly recurring alternation of day and night—was not a simple light-variation effect, but was partly a resultant reaction to a combination of stimuli consisting of variations of light-intensity, of temperature and of humidity, in so far as the latter influenced the turgor of the cells.

In his monograph Sachs gives hourly and three-hourly statistics of twelve experiments on shoots, green as well as etiolated, some of limited, others of unlimited growth. These observations extended over periods varying from four to twelve days, and were carried on during the months of April, May, and June. Each experiment was worked for a fixed twelve-hours day and twelve-hours night, irrespective of the length of day and night to which under natural conditions the plant would have been exposed at the time of experimentation.

Baranetzsky improved on Sachs' apparatus, confirmed the main points of the latter's research, and generally extended our knowledge of the daily periodicity. His monograph contains statistics of thirty-five experiments under more varied conditions than those of Sachs. The majority of these consist of observations on *Gesneria tubiflora* and *Gesneria cardinalis*, the remainder on green and etiolated shoots of *Helianthus tuberosus*, and etiolated *Brassica rapa*. His periods of observation are short, varying from two to six days, and only in few instances extended to seven, nine, or ten. Presumably the optimum temperature for each plant species under observation was determined, and the temperature during experimentation kept below the optimum: the same species in different experiments is subjected to temperatures varying from 21° to 29° C., and in some cases in the same experiment a variation of temperature of 2° to 4° is scheduled. The experiments were carried on during the months of April, May, June, and July, and for uniform day and night periods of twelve hours similarly to those of Sachs.

While Sachs was of opinion that an autonomic variation underlay the daily periodicity, Baranetzsky considered the latter an induced phenomenon. He found that it disappeared in three days in shoots of *Gesneria tubiflora* kept in continuous darkness, and in *Helianthus tuberosus* after a somewhat longer period (fourteen days), being succeeded in each case by irregular variations of varying amplitude. Further, in etiolated shoots of the latter grown from the tuber in darkness, the daily periodicity was not manifested. On the other hand, in shoots of *Brassica rapa*, produced from the tuber in darkness, a periodicity of growth similar to the daily one was found. This Sachs regarded as evidence for the autonomic nature of the variations, while Baranetzsky regarded it as a phenomenon of correlation, where the tendency to periodicity was transferred as an after-effect from the tuber to the stem. On the other hand, this might be explained as an inherited tendency to periodicity in the organ.

Pfeffer is of opinion that heredity does not enter into this question.

MacMillan and Golden (*Amer. Nat.*, 1891, vol. xxv. p. 462) found a daily periodicity in the potato tuber, and ascribed it to a correlative effect between tuber and aerial shoots.

The factors which have been by different investigators regarded as variously influencing periodicity are as follows:—

1. Intensity of illumination. The periodic variation in the intensity of the refrangible rays reaching a maximum about midday and proportionally retarding or inhibiting growth in length.

2. Variations in temperature. According to Sachs, in order to influence growth appreciably the temperature variations must be large; in which case the growth curve in shoots follows the temperature curve. Pedersen (*Arb. Bot. Inst.*, Würzburg, 1874, p. 563) states, on the other hand, that unless the optimum is exceeded by many degrees, variations of temperature produce no marked effect on growth of roots.

3. Variations in humidity. This factor, combined with the action of light on the stomata, affects the transpiration, for the existence of a daily periodicity in which some evidence is available.

4. The daily variation in bulk of the constituent cells of the plant organ. This phenomenon, described by Kraus (*Bot. Zeit.*, 1867), is the expression of the variations of the internal tensions and constitutes a tension periodicity. Vines regards it as the cause of the daily periodicity of growth in length of growing organs.

5. Reciprocal interaction between root and shoot.

Special points characteristic of this research are:—

(1) The growth records are continuous for relatively longer periods than is usual in this kind of experimental work; of about eighty experiments, the results of which have been graphically plotted, the duration varied from ten to forty or more days in different instances.

(2) Instead of an arbitrary division of the twenty-four hours period into twelve hours day and twelve hours night, the periods of light and darkness of natural occurrence at the date of experimentation were utilised. Sachs' well-known curve representing the results of an experiment on the Dahlia performed in May is calculated on the basis of an alternation of twelve hours light and twelve hours darkness for twenty-four hours, but in nature at that period of the year the plant would be subjected to at least sixteen hours light and only eight hours darkness. This point is of some importance, because the light and darkness periods vary throughout the year, and results of experiments performed when the day is at its shortest may be expected to differ from those carried on when it is at its maximum length.

The importance in this research of the first characteristic is, that it has in consequence been possible to bring to light a hitherto unrecognised growth variation of greater extent than the diurnal one, extending on the average to about four days.

In connection with the second, it has been found that there is a tendency for the time of maximal diurnal growth to vary according to the change in the length of the period of natural illumination per twenty-four hours throughout the year.

(3) The experiments have been carried on in a special experimental greenhouse, in which it is possible to regulate the temperature within one degree of variation; to control the illumination and regulate the humidity, although not to the same degree of precision as in the case of the temperature.

For statistical purposes the following table gives the data used in calculating the relative duration of the periods of light and darkness per twenty-four hours for different months of the year. The figures are approximate, since an hour or more requires to be added in certain months for the additional illumination before sunrise and after sunset:—

	Sunrise.	Sunset.		Sunrise.	Sunset.
January . . .	8 am.	4 p.m.	July . . .	4 a.m.	8 p.m.
February . . .	7 "	5 "	August . . .	5 "	7 "
March . . .	6 "	6 "	September . . .	5 "	6 "
April . . .	5 "	7 "	October . . .	6 "	5 "
May . . .	4 "	8 "	November . . .	7 "	4 "
June . . .	4 "	8 "	December . . .	8 "	4 "

In estimating the relative day and night growth rate for an experiment during April, for example, the growth increments from 5 a.m. to 7 p.m. were summed and divided by the number of hours of daylight, that is, fourteen; for night, the sum of the growth increments from 7 p.m. to 5 a.m. divided by ten gave the night rate; while at midsummer the day and night figures are sixteen and eight respectively. In nature the retardatory factor—light—increases in magnitude, while the acceleratory factor—darkness—diminishes from spring to midsummer; while the converse is the case in the latter half of the year.

The four-day variation was first met with in organs of limited growth, but experiment has shown that it is not confined to such, and that organs of all kinds show it—leaves, shoots, roots, leaf stalks and flowering axes.

RESEARCH APPARATUS AND METHODS.

The necessary statistics for construction of the growth curves were obtained :—

- (a) From direct readings taken twice or oftener during the twenty-four hours.
- (b) From continuously recorded curves on the automatic recorder.

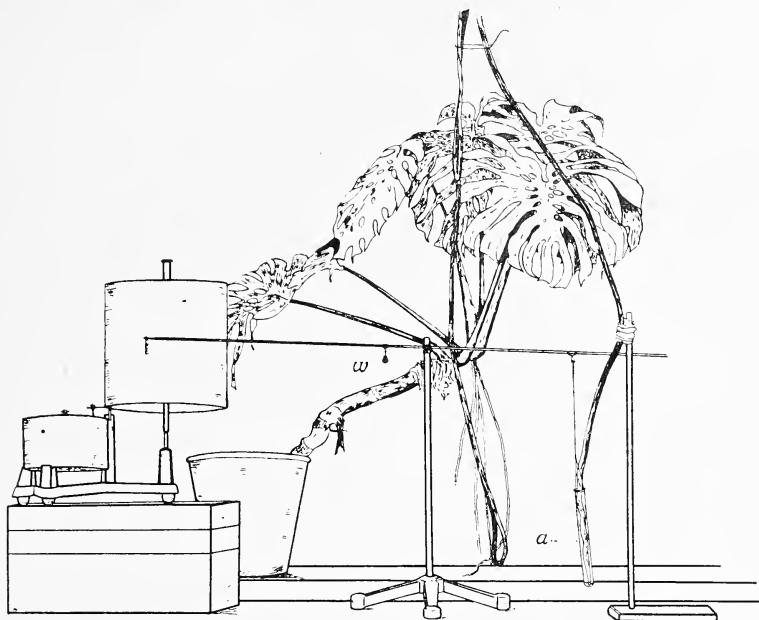


FIG. 1.—Sketch of Automatic Apparatus for measuring the growth of Aerial Roots—*Monstera deliciosa*.

a=glass tube in which root tip is growing ; *w*=weight counterpoising the glass tube.

For (a) the apparatus used was of the simplest character possible :—

1. For root growth the root was grown in water in a glass tube graduated in millimetres, and the increments read off by eye.
2. For the growth of subaerial organs the organ was arranged vertically or horizontally, with the least possible disturbance along a bar with millimetre graduations and increments read off as necessary.
3. A simple type of arc auxanometer was made use of (*Trans. Scot. Mic. Soc.*, 1908, vol. v. p. 66).

In (b) precision auxanometers were used giving continuous revolutions, one per day, or one per week, or intermittent (one movement per hour), as was found necessary in the particular case. The organ was connected by thread to one end of a light lever, the other end of which, carefully counter-

poised, carried a writing pen tracing the curve on white paper on the revolving drum.

In shoots and leaves, the thread was affixed direct to the organ by means of a small linen or worsted collar encircling the tip.

For roots, the following devices were found workable and satisfactory. In the case of stout roots, such as those of the Aroidaceæ, a light glass tube kept moist was fitted on to the root tip, and this tube was connected by thread with the lever. The apparatus is shown in use in fig. 1.

For more delicate roots a special piece of apparatus was devised (fig. 2). This is essentially a U-tube in which the curved part of the tube is replaced

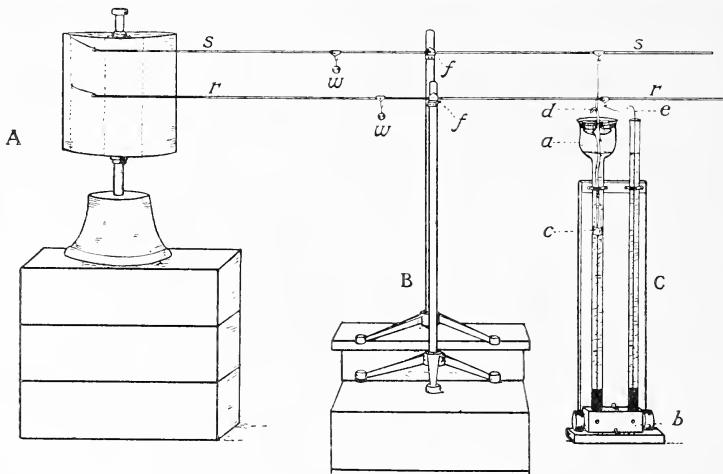


FIG. 2.—Sketch of Automatic Apparatus for measuring the growth of Root and Shoot simultaneously.

A, Automatic precision auxanometer with seven days' drum. B, Tripods and stands supporting levers:—*f*=fulcrum; *r*=root lever; *s*=shoot lever; *w*=weight. C, Special glass apparatus for measuring root growth (turned at right angles to show the parts):—*a*, thistle funnel and tube fitted with flat cork to which bean is pinned; *b*, box into which two glass tubes are sealed, fitted with two glass windows (through these the two metal pulleys are seen); *c*, cork and tube pushed down by root as it grows. Thread attached to cork passes under pulleys and up other limb; *d*, shoot with collar and thread attached to lever; *e*, thread passing to root lever.

by a metal box containing two light pulleys. The tube is filled with water; in one arm the seedling is fixed with its root depending into the water, and its tip in contact with a light float; from the float a thread passes down over the two pulleys up the other arm and thence to the distal end of the recording lever. Very accurate counterpoising is necessary. The figure shows the apparatus as arranged for recording the growth curves of root and shoot on the same drum simultaneously.

In order to bring out clearly the existence of the four-day periodicity, long continuous records are essential; short records extending over a week or so show only one phase of it. The periodicity manifests itself in a gradually accelerated growth extending over four days, when a maximum

rate is attained, then follows a fall in the rate to a minimum immediately, or occupying again about four days, and so on repeatedly.

In an organ of limited growth, the crest of successive four-day waves is progressively higher, culminating in a maximum which represents the acme of the grand period; then a series of waves of successively less height follows until the organ ceases to grow.

Frequently after the crest of the grand period is reached there follows a sudden diminution in the rate, succeeded by a long undulation, composed of a gradual acceleration to the fourth day and a gradual fall to the eighth (Plate I., fig. 1).

The maximum of the long wave may occur either during a night or a day period, and thus in the premaximal phase of the grand period the maxima of the successive long waves may occur during the day, while their position is reversed in the post-maximal period. The converse is also found.

An interesting feature brought out by the research up to the present is the widespread occurrence of the four-day periodicity—it being found alike in leaf of Cryptogam and the various organs of Phanerogams.

Further, that it occurs in roots in light and darkness indifferently, is evidence that light is not a necessary condition here any more than it appears to be for the diurnal periodicity of roots.

This present research only aims at recording the existence of this special periodicity and giving details regarding it. The extent to which it is influenced by external factors has yet to be accurately determined. On this point some knowledge may be gained from the results of experiments now in progress.

EXPERIMENTAL EVIDENCE OF FOUR-DAY PERIODICITY.

1. *Tulip Leaf* (Plate I., fig. 1).—This record was taken by means of the simple auxanometer from 25th March to 23rd April—a period of twenty-nine days. The natural condition during that period approximated to fourteen hours light and ten hours darkness per twenty-four hours period. An analysis of the curve shows the existence of five waves culminating on 29th March, 2nd-3rd April, 8th-9th May, 11th-12th May, and 17th-18th May respectively; in all but the first the wave reached its highest during the night period. The waves occupied four, five, six, five, four, four days respectively, the third including the climax of the grand period of this organ of limited growth.

During the period of slow fall in the growth rate following the maximum of the grand period, that is, during the post-maximal phase, there was frequently observed the occurrence of one or a series of waves extend-

ing over about eight days, and made up a slow rise and a slow fall instead of the usual slow rise and rapid fall characteristic of the premaximal phase. This peculiarity is seen in the part 9th to 14th April inclusive (Plate I., fig. 1).

The mean rate of growth for the whole period of observation worked out to: night rate to day rate as 1 : 0·94; whereas the relative amounts per twenty-four hours period for the same proportion of light and darkness were: night amount to day amount as 1 : 1·8. This was due to the longer duration of the illumination period, which allowed of the deficiency consequent on slower average growth being more than made good (Table of Statistics, No. 1).

2. *Bryophyllum Shoot* (Plate I., fig. 2).—This experiment was carried on during the winter from 18th November to 4th December, a period of sixteen days, when the conditions as to illumination were almost the converse of those in the preceding experiment—there being per twenty-four hours, fifteen hours darkness to nine hours illumination. The main shoot under observation here developed into a flowering axis, and its growth was therefore limited. The occurrence of the series of long undulations referred to in the post-maximal of the tulip is here very marked, and evidences that the maximum of the grand period had been passed before the record of growth was begun. The mean rate of night to day growth was 1 : 1·2; the relative amount of growth was greater by night, the figures being 1 : 0·7, the result of the longer duration of the period of darkness (Table of Statistics, No. 2).

3. *Calla Inflorescence Axis* (Plate I., fig. 3).—This experiment lasted from 26th March to 7th May, and hourly observations were taken by day during that period—forty-three days—with only occasional intermittence. The conditions as to illumination were similar to those of the tulip. The experiment represents nearly the whole course of development of the Calla flowering axis from its appearance above ground to the fading of the spathe, and therefore included the grand period. The climax of the latter occurred during night 2nd–3rd April, and was led up to by a series of waves of three to four days duration; the post-maximal phase, on the contrary, included several slow long undulations similar to what occurred in the corresponding phase of *Bryophyllum*. The mean rate of growth for the night and day periods during the forty-three days experimentation was as 1 : 0·8, i.e. greater on the average during the night period, the relative amounts of growth were night : day as 1 : 1·5, i.e. greater by day owing to the longer period of illumination (Table of Statistics, No. 3).

4. *Lygodium Climbing Leaf* (Plate I., fig. 4).—The continuous observations here extended over a period of forty-one days, from 29th April to

9th June inclusive. During the earlier part, the plant was subjected under natural conditions to fourteen hours illumination and ten hours darkness per twenty-four hours period, passing at the middle and towards the conclusion of the record into eighteen hours illumination and six hours darkness. The organ had practically ceased to grow by the time the experiment was finished. The maximum of the grand period was reached on 9th May during the daytime, preceded by the four-day waves and succeeded in the post-maximal phase by a series of undulations which can be analysed into irregular compounds of longer and shorter waves. Both the greater amount of growth and the greater rate of growth in this case were attained during the illuminated periods (Table of Statistics, No. 4).

5. *Heptapleurum petiole* (Plate II., fig. 5). Observations were commenced at the earliest possible moment, that is, as soon as the organ was sufficiently developed from the bud to allow the necessary collar being affixed for attachment to the registering apparatus, and were continued until all growth in the organ ceased, which happened about the twenty-third day. The dates were 21st November to 14th December, when the natural ratio of light to darkness was as 9:15. The readings were accidentally interrupted from 2nd-4th December.

The maximum of the grand period occurred on the eighth day of the experiment. During the premaximal phase, the greatest growth rate was by day; in the post-maximal, this was repeated except during a short period where there occurred a night maximum. The four-day undulations culminated on 24th and 28th November, and about 2nd, 3rd, and 7th December respectively, and were followed by a few more rapid oscillations of less amplitude as the growth rate slowed down to zero, when the organ attained its full size.

Owing to the shorter period of illumination the night amount was greater than that of the day, the ratio being 1:0·8, although the day growth rate was practically greater than the night rate throughout, the figures being night rate to day rate as 1:1·4 (Table of Statistics, No. 5).

6. The rate graph of the *Crocus flower axis* is given in Plate II., fig. 6. The rate was greater during the night period, and the actual amount of growth was greater during the darkness period also, owing to the greater length of the night (Table of Statistics, No. 6). The curve shows a trace of the long wave of growth in the post-maximal half, but as a whole it is too short to demonstrate the point clearly.

To ascertain whether the four-day periodicity was manifested in the material used by Sachs under equally distributed conditions of illumination and darkness, that is, when during the twenty-four hour periods the plant

was submitted to twelve hours illumination and twelve hours darkness alternately, the numerical data given by Sachs in his Statistical Tables (*loc. cit.*) were used. Those of *Polemonium*, *Calla*, and *Dahlia* were selected; the two former were under observation by Sachs during April, the latter in June. The necessary calculations were made on the basis of the natural length of the day and night for the particular months, and graphs were plotted. The results are given in Plate II., figs. 7, 8, and 9. In spite of the short duration of the experiments, which were only carried on for about six days, and the consequent paucity of numerical data suitable for this purpose, enough is represented to indicate that the phenomenon of four-day periodicity is present. In fig. 7, *Polemonium*, and in fig. 8, *Calla*, there is an obvious four-day increase from 24th–25th to 28th–29th April in the former, and from 16th to 19th April in the latter. In fig. 9 the *Dahlia* exhibits part of one of the long eight-day undulations of growth rate characteristic of the post-maximal phase of the grand period of an organ which has limited growth.

The four-day rhythm was here represented even when the plants were under observation under artificial conditions as to the length of exposure to darkness and illumination.

These results may be regarded as giving some support to the view that the long periodicity is to a certain extent autonomic, or based on internal changes in the plant, although modifiable by external conditions. Additional support is afforded for this view from two series of experiments performed on Bean plants at approximately the same stage of growth. In one series the whole plant, root as well as shoot, was subjected to normal shoot conditions, *i.e.* to the successive alternations of day and night; in the other the whole plant was kept in continuous darkness, that is, stem as well as root was subjected to normal root conditions. Results are shown in Plate II., figs. 10 and 11, where the four-day waves are equally obvious in both cases.

In Statistical Table, page 98, items 7, 8, and 9 are also of some interest. In all three the amount of growth was greater by day, being more than double that by night, although in only one, *Calla*, was this associated with a greater mean day rate. In the other two the mean day rates were identical and were less than the mean night rates. These results emphasise the necessity of basing conclusions on data derived from experiments continued over longer periods of time, in the case of organs of limited growth, including the greater part of the grand period, as well as of clearly distinguishing growth amount from growth rate by day and night respectively.

GROWTH PERIODICITY IN ROOTS AND ITS CORRELATION WITH THAT OF THE STEM.

When a shoot is exposed to the stimulus of variation in light intensity, consequent on the alternation of light and darkness and the various other accompanying conditions, it exhibits its daily periodicity of growth in those cases where the shoot is phototonic, *i.e.* where light is a tonic condition of growth. When such a shoot is placed in continuous darkness, according to Sachs and Baranetzsky, the daily rhythm disappears. This may be taken as due to two factors:—

(a) Withdrawal of the stimuli, or rather the interference with the sum of the stimulatory conditions consequent on the withdrawal of light; and

(b) Because of the inability of the mechanism to act on account of its being in a non-phototonic condition due to the withdrawal of the tonic light factor.

Experiments on such stems, when etiolated, are of questionable value from this point of view, as the material must be regarded as being in a pathological condition. It is possible, however, that such stems by adaptation might develop a periodicity in response to the sum of the external daily factors minus light, *i.e.* a special darkness-periodicity might develop. Such a periodicity, modified by correlation with the stem, is what might be expected in the case of roots, and also in those "permanently etiolated stems" of certain climbers recorded by Sachs. On the latter we have not yet made any observations in this connection.

In roots light is not a necessary condition of growth, and any periodicity they exhibit, on the analogy of the shoot, may depend on—

- (1) The sum of the external daily stimuli exclusive of light.
- (2) A correlative effect with the periodicity of the shoot.

The periodicity of the root ought to have a reactive influence on the shoot when the plant is placed in continuous darkness, after the preliminary disturbance, due to the change of the environment of the stem, has passed off. Baranetzsky's experiments on shoots show that as an after-effect periodicity continues to be manifested in the shoot for some time after it has been subjected to continuous darkness, giving way ultimately to small spontaneous variations. His observations broke off at this point. The suggestion is now made of the possibility of the appearance of a darkness-periodicity after a period of adaptation has elapsed, induced by the new set of conditions, exclusive of light, and in correlation with root periodicity.

EXPERIMENTAL EVIDENCE OF ROOT PERIODICITY.

Monstera, aerial root.—About eighteen successive experiments were made with the roots of this plant at different seasons of the year, and in all evidence of daily periodicity was obtained. Each observation was continued over prolonged periods, varying from eighteen to forty-five days. The records were taken with the automatic auxanometer. In Plate III., fig. 14, there is shown a portion of a typical record of one of those experiments which was carried on during May–June 1910. The daily periodicity is here well marked, the growth curve beginning to rise regularly about midday, attaining a maximum during the night, thereafter falling to a minimum during the forenoon. A four-day wave is also seen in the curve presented commencing on 29th May, and attaining its maximum on the night 1st–2nd June. Two other instances of the four-day wave are exemplified in Plate II., figs. 12 and 13, which represent portions of long records made during winter and summer conditions respectively. In the former, the organ was exposed to an alternation of light and darkness in the ratio of 9:15, while in the latter the conditions were reversed, the ratio being 18:6. The occurrence of the periodicity under such diverse conditions of illumination is an argument in favour of its autonomic character.

The effect of the seasonal change in the duration of the natural period of illumination is seen when the relative amounts of day and night are compared. Thus in Table of Statistics, Nos. 10 and 11 (page 98), the mean rate in both cases is greater by night, but the relative amounts diverge widely.

For the November experiment the night amount is about three times greater than that of the day, whereas, in the other case, owing to the longer June day, the growth amount for the day period is about two and half times that for the night.

Vicia Faba, hypogea roots.—The graphs of about forty experiments under various conditions as to light and darkness on the roots of the Bean were worked out, and daily periodicity was found to occur throughout. Fig. 15, Plate III., represents part of a record taken during the month of June when the duration of light to darkness was as 3:1. In this case the periodicity is evidenced by a rise in the growth curve during the day, the maximum being attained early in the evening, followed by a fall during the night period, the lowest growth occurring generally in the forenoon.

This is exactly the converse of what is recorded by Sachs in his Dahlia curve, and might be regarded as an effect, in the root, due in part to correlation with the growth periodicity of the stem, which, owing to

the retardatory effect of illumination, exhibits a daily maximum in the early morning.

Kny (*Ann. of Botany*, 1894, vol. viii. pp. 265 *et seq.*; 1901, xv. p. 613), working on different lines, by a method of amputation, is led to conclude that no correlation exists between root and shoot growth.

The validity of Kny's results has been criticised by Hering (*Jahr. f. wiss. Bot.*, 1896, xxix. p. 132) on technical grounds.

Observations on this matter are in progress, and will form Part II. of this research.

This curve also shows one of the greater four-day oscillations of growth extending from 9th-12th June.

The following conclusions have been deduced from comparisons of the various observations made up to the present:—

1. Roots exhibit a periodicity under ordinary conditions of environment which differs from that of shoots.
2. Owing to correlation, the root periodicity is affected by changes in the shoot rhythm, but to what extent has yet to be determined.

Statistics of portions of the records referred to on page 94, and figured in Plate II., figs. 10 and 11, in relation to the four-day waves are given in Table of Statistics, Nos. 12 and 13, and bear on this point of correlation.

In both cases the relative amount of growth was the same, the ratio of night amount to day amount being as 1:0·7; on the other hand, the rate ratios were different. In the first case, the day rate of the root was greater than in the second. The explanation appears to be that the growth rate of the root is accelerated as that of the stem is retarded under normal conditions, the root rate increasing as that of the stem diminishes, owing to the retardatory effect of light. Even when the root is exposed to shoot conditions, this acceleratory correlative effect persists in spite of the direct retardatory effect of light. In the second case, owing to the plant shoot being in continuous darkness, and therefore the retardatory effect of light being removed, the greater shoot rate continues, and, as a correlative effect, that of the root is to a certain extent altered.

GENERAL RESULTS.

1. There occurs in elongating plant organs a four-day periodicity apparently due in part to internal causes, but affectable by external conditions.
2. Roots exhibit a daily periodicity, and this is correlated with that of the stem.

TABLE OF STATISTICS OF EXPERIMENTS REFERRED TO IN THE TEXT.

No.	Plant.	Date.	Relative Amt. of Growth.		Mean Rate of Growth.		Remarks.
			Night.	Day.	Night.	Day.	
1	Tulip leaf . .	Mar. 25–Apr. 23	1	1·8	1	0·94	
2	Bryophyllum axis	Nov. 18–Dec. 4	1	0·7	1	1·2	
3	Calla axis . .	Mar. 25–Apr. 22	1	1·5	1	0·81	
4	Lygodium leaf .	Apr. 29–June 9	1	3·6	1	1·2	
5	Heptapleurum (leaf stalk)	Nov. 21–Dec. 14	1	0·8	1	1·4	
6	Crocus (flower stalk)	Mar. 14–20	1	0·96	1	0·7	
7	Polemonium shoot	Apr. 23–28	1	2·1	1	0·87	
8	Calla shoot . .	Apr. 16–20	1	2·5	1	1·1	
9	Dahlia shoot . .	June 19–25	1	2·7	1	0·87	
10	Monstera root . .	Nov. 6–13	1	0·36	1	0·6	
11	" " . .	June 4–11	1	2·4	1	0·82	
12	Bean root " .	Feb. 20–28	1	0·73	1	1·03	Whole plant in alternate light and darkness.
13	" " . .	" "	1	0·7	1	0·99	Whole plant in dark.

TABLES SHOWING THE MEAN AMOUNTS AND MEAN RATES OF GROWTH FOR THE NATURAL PERIODS OF ILLUMINATION AND DARKNESS RESPECTIVELY PER TWENTY-FOUR HOURS. ON THESE NUMERICAL DATA THE TABLE OF STATISTICS (PAGE 98) IS BASED, AND ALSO THE RATE GRAPHS (PLATES I. AND II.).

No. 1. Tulip Leaf (Pl. I., fig. 1).			No. 3. Calla Inflorescence (Pl. I., fig. 3).		No. 4. Lygodium Leaf (Pl. I., fig. 4).		
Time.	Amount.	Mean Rate.	Amount.	Mean Rate.	Time.	Amount.	Mean Rate.
March 25/26 (7 p.m.-5 a.m.)	.6	.06			April 29	5.25	.29
26	1.4	.1	19.6	1.4	29/30	5.88	.98
26/27	.25	.025	7.9	.8	30	7.87	.43
27	1.75	.12	11.6	.82	May 30/1 (9 p.m.-3 a.m.)	3.6	.6
27/28	1.35	.135	6.5	.6		11.62	.64
28	1.4	.1	17.5	1.2		3.63	.605
28/29	.7	.07	10.5	1.0		10.87	.60
29	6.5	.46	15.4	1.1		3.13	.52
29/30	.22	.02	10.2	1.0		11.62	.64
30	1.03	.07	15.9	1.1		2.13	.35
30/31	1.1	.11	15.8	1.5		10.12	.56
31	3.8	.27	11.2	.8		5.5	.91
April 31/1 (8 p.m.-4 a.m.)	1.5	.16	11.8	1.1		11.25	.62
1	2.5	.15	17.7	1.1	5/6	4.2	.7
1/2	3.1	.38	8.5	1.0	6	21.37	1.18
2	1.8	.11	22.5	1.4	6/7	4.63	.77
2/3	4.7	.58	13.75	1.7	7	23.62	1.31
3	2.5	.15	15.75	.98	7/8	8.25	1.37
3/4	0	0	8.4	1.0	8	27.0	1.5
4	2.75	.17	11.6	.72	8/9	2.38	.39
4/5	1.85	.23	12.0	1.5	9	34.87	1.93
5	6.9	.43	13.0	.8	9/10	7.38	1.23
5/6	.15	.01	6.75	.8	10	28.87	1.6
6	3.6	.22	18.75	1.1	10/11	6.88	1.14
6/7	1.2	.15	8.0	1.0	11	25.12	1.39
7	8.0	.5	16.0	1.0	11/12	8.0	1.33
7/8	1.0	.12	10.75	1.3	12	26.25	1.45
8	3.25	.20	21.5	1.3	12/13	7.13	1.18
8/9	4.2	.52	11.5	1.4	13	21.37	1.18
9	1.8	.11	13.5	.8	13/14	5.63	.93
9/10	.9	.11	9.25	1.1	14	25.87	1.43
10	3.35	.20	12.7	.79	14/15	8.13	1.35
10/11	2.25	.28	8.75	1.0	15	23.62	1.31
11	4.0	.25	18.75	1.17	15/16	4.88	.81
11/12	3.0	.37	14.0	1.7	16	28.12	1.56
12	5.0	.31	18.0	1.12	16/17	5.88	.98
12/13	2.7	.33	13.3	1.6	17	26.62	1.47
13	1.8	.11	13.4	.83	17/18	9.88	1.64
13/14	2.3	.28	7	.8	18	22.87	1.27
14	.7	.04	13.4	.83	18/19	5.0	.83
14/15	.66	.08	11.5	1.4	19	19.5	1.08
15	1.09	.06	13	.81	19/20	4.5	.75
15/16	1.4	.17	10.7	1.3	20	21.75	1.20

TABLES SHOWING THE MEAN AMOUNTS AND MEAN RATES OF GROWTH—*continued*.

No. 1 Tulip Leaf (Pl. I., fig. 1).			No. 3. Calla Inflorescence (Pl. I., fig. 3).		No. 4. Lygodium Leaf (Pl. I., fig. 4).		
Time.	Amount.	Mean Rate.	Amount.	Mean Rate.	Time.	Amount.	Mean Rate.
April 16 (8 p.m.-4 a.m.)	3·3	.2	15·5	.96	May 20/21 (9 p.m.-3 a.m.)	1·63	.27
16/17	2·2	.27	7·9	.9	21	31·87	1·77
17	1·8	.11	8·3	.51	21/22	6·75	1·12
17/18	2·6	.32	8·9	1·1	22	19·5	1·08
18	1·09	.06	10·1	.63	22/23	8·5	1·41
18/19	0	0	6·9	.8	23	25·5	1·41
19	.5	.03	6·8	.42	23/24	7·0	1·16
19/20	0	0	8·8	1·1	24	29·25	1·62
20	.25	.015	14·1	.88	24/25	5·13	.85
20/21	.25	.03	7·5	.9	25	22·87	1·27
21	0	0	8·7	.54	25/26	9·63	1·6
21/22	0	0	6·5	.8	26	22·62	1·25
22	.25	.015	8	.5	26/27	8·5	1·4
22/23	0	0	3·75	.4	27	30·0	1·66
23	0	0	6·5	.4	27/28	2·0	.33
23/24			5·6	.7	28	21·0	1·16
24			6·8	.42	28/29	8·13	1·35
24/25			3·1	.4	29	20·62	1·14
25			4·6	.28	29/30	9·13	1·52
25/26			2·6	.3	30	13·87	.77
26			3·6	.22	30/31	4·8	.8
26/27			1·8	.2	31	15·37	.85
27			3·9	.24	June 31/1 (9 p.m.-3 a.m.)	1·38	.23
27/28			2·2	.3	1	7·12	.39
28			4·3	.26	1/2	.5	.083
28/29			3·7	.4	2	4·5	.25
29			4·3	.26	2/3	1·25	.208
29/30			1·9	.2	3	4·5	.25
30			3·6	.22	3/4	1·08	.18
May 30/1 (9 p.m.-3 a.m.)			1·8	.2	4	4·5	.25
1			3·45	.19	4/5	1·0	.16
1/2			1·75	.3	5	1·5	.083
2			4·25	.23	5/6	1·5	.25
2/3			1·3	.2	6	0	0
3			4·9	.27	6/7	1·0	.16
3/4			2·0	.3	7	0	0
4			3·0	.16	7/8	.75	.125
4/5			.8	.1	8	0	0
5			3·4	.18	8/9	.125	.020
5/6			1·4	.2	9	.375	.020
6			3·8	.21	9/10	0	0

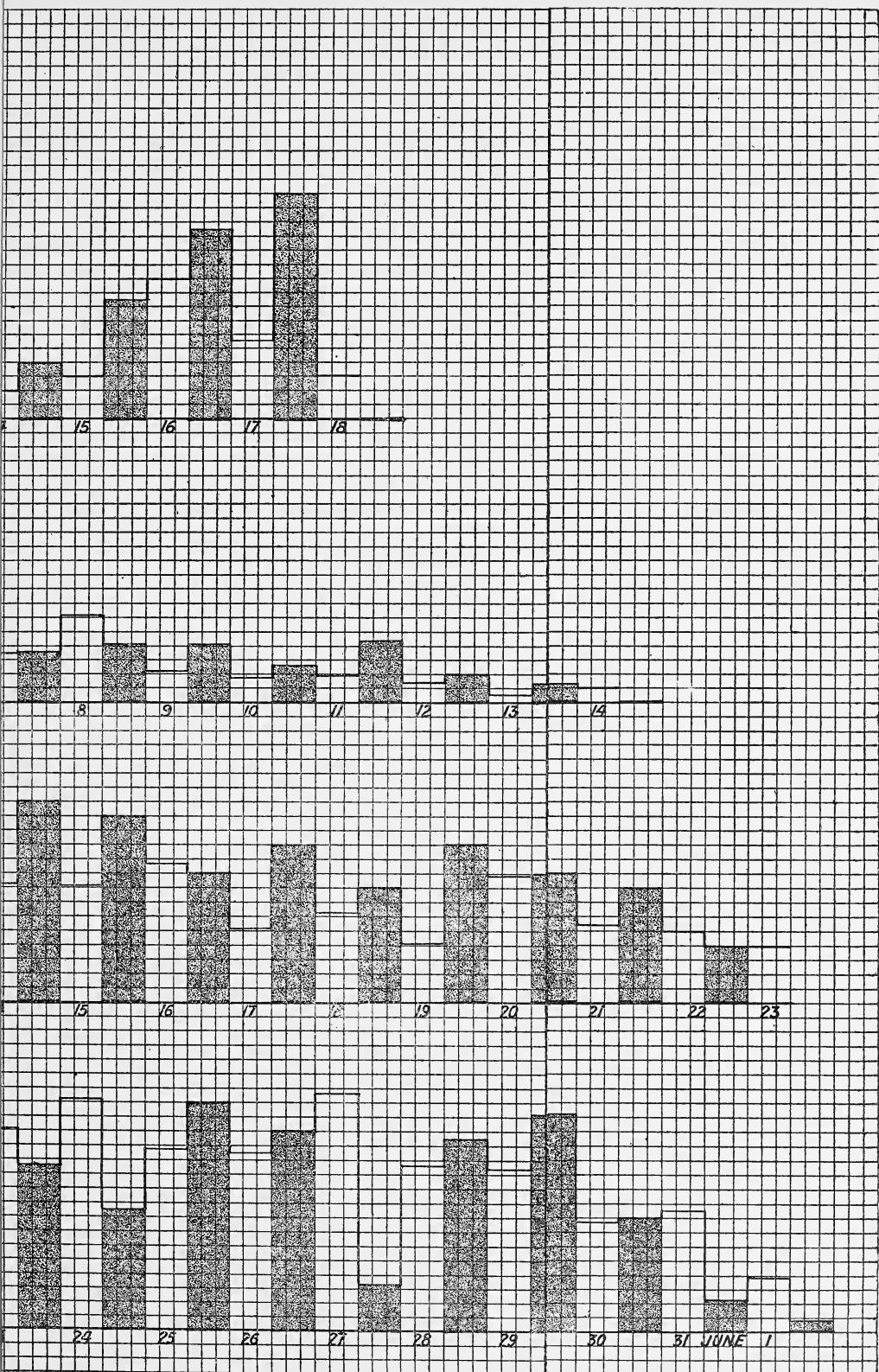
TABLES SHOWING THE MEAN AMOUNTS AND MEAN RATES OF GROWTH—*continued.*

No. 11. Monstera Root (Pl. II., fig. 13).			No. 10. Monstera Root (Pl. II., fig. 12).			No. 2. Bryophyllum Shoot (Pl. I., fig. 2).			No. 5. Hep- tapleurum Petiole (Pl. II., fig. 5).		
Time.	Amount.	Mean Rate.	Time.	Amount.	Mean Rate.	Time.	Amount.	Mean Rate.	Amount.	Mean Rate.	
June 4/5 (9 p.m.-3 a.m.)	14·0	2·33	Nov. 6/7 (4 p.m.-7 a.m.)	42·18	2·81	Nov. 18 (7 a.m.-4 p.m.)	.94	.104			
5	31·75	1·76	7	13·49	1·49	18/19	9·6	.64			
5/6	14·25	2·37	7/8	27·8	1·85	19	16·0	1·7			
6	40·5	2·25	8	11·87	1·31	19/20	8·1	.54			
6/7	15·0	2·5	8/9	43·42	2·89	20	9·55	1·06			
7	36·5	2·02	9	15·36	1·70	20/21	7·75	.516			
7/8	17·5	2·91	9/10	45·0	3·0	21	14·85	1·65	3·13	.346	
8	44·25	2·45	10	17·47	1·94	21/22	11·01	.73	2·59	.172	
8/9	15·5	2·58	10/11	47·17	3·14	22	16·9	1·87	3·76	.416	
9	24·75	1·37	11	15·83	1·75	22/23	8·22	.54	2·3	.152	
9/10	11·5	1·91	11/12	43·42	2·89	23	10·91	1·21	4·75	.526	
10	35·25	1·95	12	19·21	2·13	23/24	10·74	.71	4·21	.28	
10/11	12·75	2·12	12/13	48·1	3·2	24	14·61	1·62	6·86	.762	
						24/25	10·01	.66	6·28	.418	
						25	7·35	.816	2·17	.241	
						25/26	8·34	.55	4·37	.290	
						26	5·30	.58	5·58	.62	
						26/27	8·03	.53	3·5	.23	
						27	5·82	.64	5·75	.638	
						27/28	10·76	.71	3·05	.20	
						28	4·79	.53	8·20	.911	
						28/29	10·05	.67	9·22	.612	
						29	5·45	.60	6·73	.746	
						29/30	4·43	.29	8·63	.57	
						30	3·82	.42	6·42	.714	
						Dec. 30/1 (4 p.m.-8 a.m.)	8·66	.541	11·78	.73	
						1	3·34	.417	4·22	.52	
						1/2	11·99	.74	10·77	.67	
						2	4·31	.538	4·43	.55	
						2/3	12·12	.75			
						3	4·43	.553			
						3/4	5·86	.36			
						4	3·49	.436			
						4/5	9·47	.59	4·14	.25	
						5	1·48	.185	2·25	.28	
						5/6	13·65	.85	3·19	.19	
						6	3·55	.44	1·96	.24	
						6/7	9·68	.60	2·76	.17	
						7	2·81	.351	2·2	.27	
						7/8	5·82	.36	.907	.056	
						8	4·89	.611	.483	.06	
						8/9	6·65	.41	1·97	.12	
						9	1·79	.223	1·27	.158	
						9/10	6·58	.41	1·475	.092	
						10	1·41	.176	.520	.065	
						10/11	4·22	.26	1·447	.090	
						11	1·53	.191	1·182	.147	
						11/12	7·19	.44	.400	.025	
						12	1·16	.145	.035	.004	
						12/13	3·3	.20	.89	.05	
						13	.443	.055	.925	.115	
						13/14	2·16	.13	1·200	.075	
						14	.14	.017	.560	.070	

TABLES SHOWING THE MEAN AMOUNTS AND MEAN RATES OF GROWTH—*continued.*

Bean Root (Pl. II., figs. 10 and 11).			No. 6. Crocus (Pl. II., fig. 6).			No. 8. Calla Inflorescence (Pl. II., fig. 8).			No. 7. Polemonium (Pl. II., fig. 7).			No. 9. Dahlia (Pl. II., fig. 9).				
No. 12. In alternate Light and Dark.	No. 13. In Dark.	Mean Rate.	Time.	Amount.	Mean Rate.	Time.	Amount.	Mean Rate.	Time.	Amount.	Mean Rate.	Time.	Amount.	Mean Rate.		
Feb. 20/21 (6 p.m.-7 a.m.)	1·29	1·02	24/19	1·72	Mar. 14/15 (7 p.m.-5 a.m.)	4·66	·46	Apr. 16 (4 a.m.-8 p.m.)	27·7	2·1	Apr. 23/24	19·7	2·4	Jun. 19/20 (9 p.m.-3 a.m.)	34·4	5·7
21	6·89	·689	20·0	2·0	15/15	9·75	·696	16/17	2·0	24	45·8	2·8	20/21	122·9	6·8	
21/22	6·4	·45	28·0	2·0	15/16	13·36	1·33	17	42·4	2·6	24/25	13·8	1·7	20/21	50·4	8·4
22	4·6	·46	17·1	1·71	16	3·85	·274	17/18	18·7	2·3	25	32·2	2·0	21	162·0	9·0
22/23	7·0	·5	23·61	1·68	16/17	3·72	·37	18	43·5	2·7	25/26	30·5	3·8	21/22	60·1	10·0
23	5·63	·563	14·18	1·41	17	4·36	·31	18/19	17·9	2·2	26	74·0	4·6	22	155·1	8·6
23/24	9·03	·64	14·58	1·04	17/18	1·0	·1	19	46·9	2·9	26/27	42·0	5·2	22/23	57·8	9·6
24	9·18	·918	13·76	1·37	18	3·5	·25	19/20	17·0	2·4	27	92·5	5·7	23	130·5	7·2
24/25	10·19	·726	14·0	1·0	18/19	·66	·06				27/28	53·4	6·6	23/24	54·2	9·0
25	7·5	·75	10·84	1·08	19	1·16	·082				28	99·4	6·2	24	132·5	7·3
25/26	9·61	·68	19·82	1·41	19/20	1·83	·18				28/29	55·0	6·8	24/25	43·1	7·1
26	6·68	·668	14·18	1·41												
26/27	8·74	·62	16·9	1·2												
27	8·13	·813	11·05	1·10												
27/28	15·45	·1·1	12·24	·87												
28	10·63	1·063	7·92	·79												

(Issued separately March 26, 1913.)



25 PLATE I

FIG. 1
TULIP LEAF

20

15

10

5

MARCH 26 27 28 29 30 31 APRIL 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

FIG. 2

DRIED TULIP LEAF

SHOOT

10

5

NOVEMBER 8 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44

FIG. 3

CALLA INFLORESCENCE

15

10

5

MARCH 26 27 28 29 30 31 APRIL 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23

20 FIG. 4

ZYGODIUM LEAF

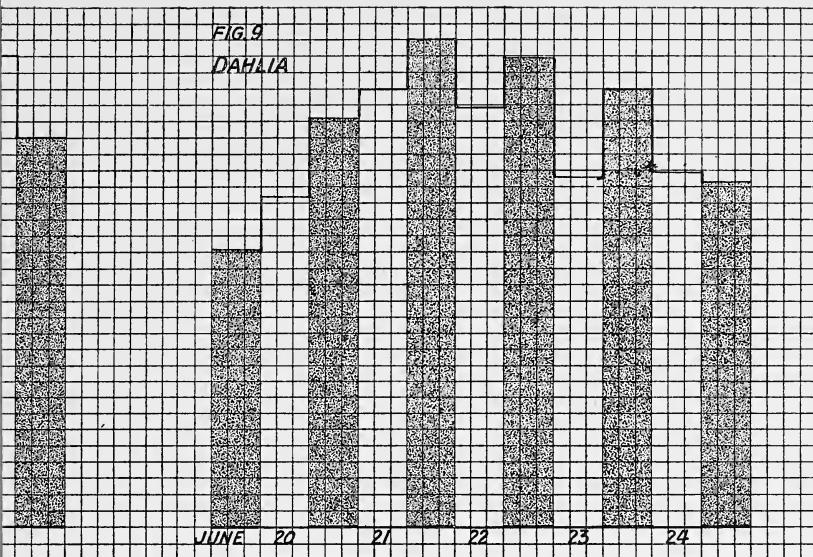
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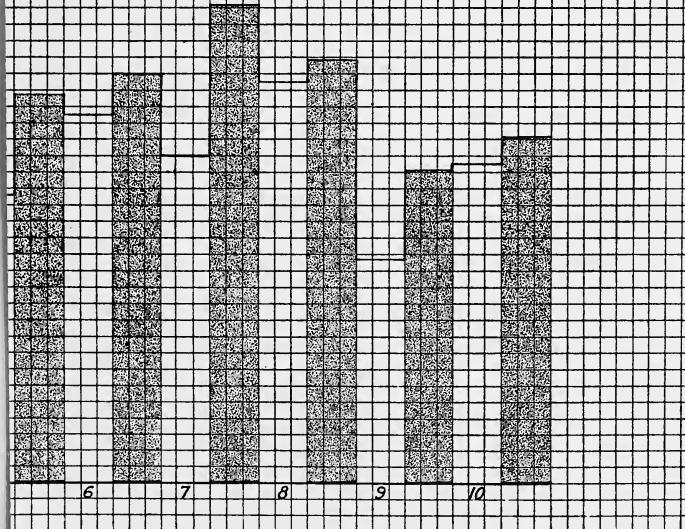
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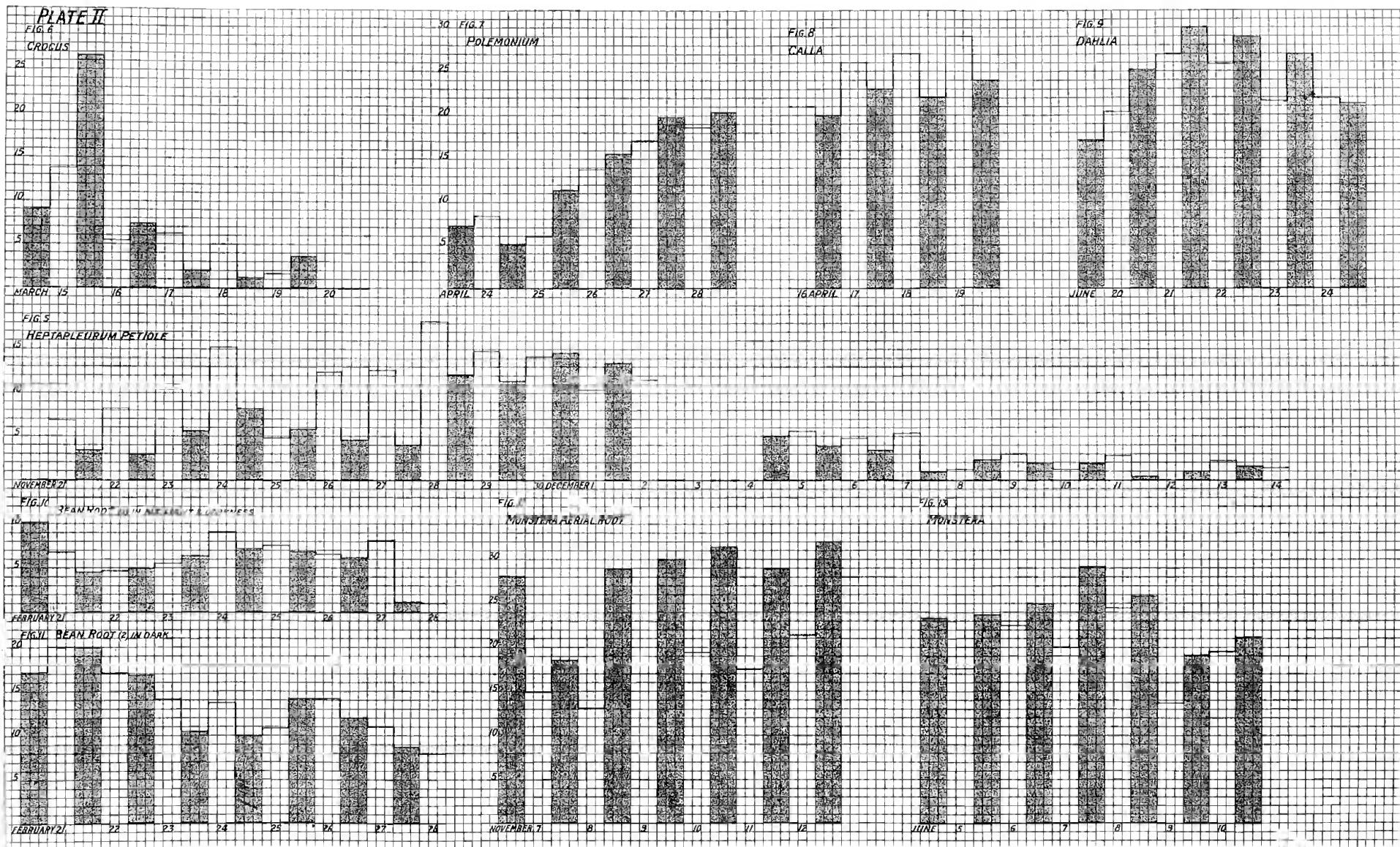
MAY 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 JUNE 1 2

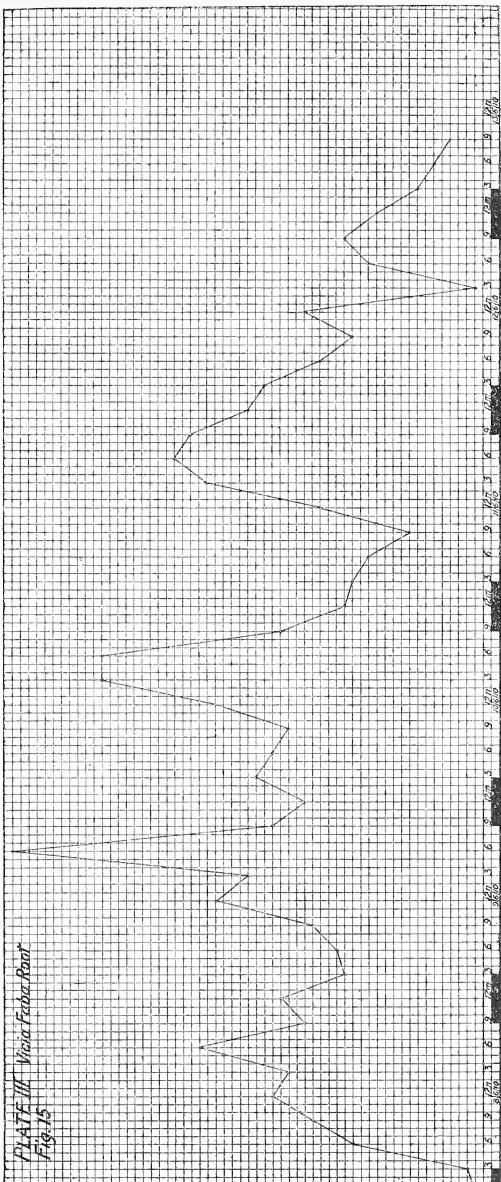
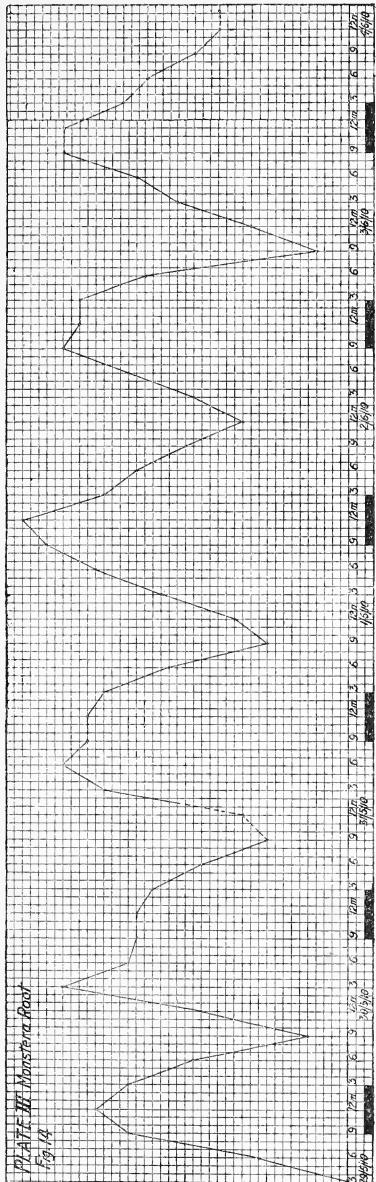
FIG. 9
DAHLIA



RA







R. A. ROBERTSON AND ROSALIND CROSSE.

IX.—The Error caused by "Lag" in a Recording Instrument: An Experimental Study. By J. R. Milne, D.Sc., and H. Levy, M.A., B.Sc., Carnegie Scholar.

(Read February 17, 1913. MS. received February 18, 1913.)

INTRODUCTION.

SOME time ago one of us was engaged in experimental work in which it was necessary to measure the total flow of air past a certain instrument exposed out of doors.* The attempt to do this by means of a Robinson cup anemometer raised the question of the accuracy of such an instrument in a gusty wind. Owing to the inertia of the moving parts the apparatus revolves too slowly at the beginning of a gust, and too fast at the conclusion of a gust; and it is hardly to be expected that the two errors will exactly compensate each other. All instruments designed to record some rapidly fluctuating quantity are liable to suffer from the effects of "lag," and much ingenuity has been exercised in minimising or overcoming the difficulty. If, for example, the differential equation of motion be known, it may be possible to subsequently correct the record of the apparatus.† That many cases, however, are insusceptible of such treatment is evident for a variety of reasons; a case in point is furnished by Professor Chrystal's investigation of "The Theory of the Leaking Microbarograph,"‡ in which it is shown that the instrument record seriously distorts even ideally simple atmospheric disturbances, and also that those which are actually observed are of a most complicated nature. In the case of other instruments it has been found possible to so arrange the design that the effects of lag are almost or altogether absent. Modern oscillographs furnish a striking example of this,§ as to a less degree do certain types of seismographs.|| Now, no doubt an instrument might be constructed to record rapidly fluctuating

* "On Atmospheric Cooling: An Experimental Investigation," J. R. Milne, D.Sc., *Jour. Scot. Met. Soc.* (3), vol. xvi., No. xxix. p. 9, 1912.

† In regard to this see, e.g., Prof. Hopkinson's tract on *Vibrations of Systems having One Degree of Freedom*, chap. ii.

‡ *Proc. Roy. Soc. Edin.*, vol. xxviii., part vi., p. 437, 1908.

§ See papers by A. Blondel, *Comptes Rendus*, vol. cxvi. pp. 502 and 748, 1893; "Oscillographs," W. Duddell, *Brit. Ass. (Toronto)*, 1897; and *Electrician*, vol. xxxix. p. 636.

|| For an interesting account of such instruments see *The Physics of Earthquake Phenomena*, by C. G. Knott, D.Sc., chaps. iv. and v.

air velocities, and like those just mentioned be so designed, because of the lightness of its moving parts, efficient damping, etc., as to have no error of lag; but what is required is not a record of the velocity of the wind at any moment, but the time integral of that velocity. In theory, of course, this might be obtained by the subsequent use of a planimeter; but owing to the gustiness of the wind, either the record strip would have to be of prodigious length, or else the fluctuations would be crowded together upon it so closely as to put the use of a planimeter out of the question. For this reason it is necessary to give up the idea of using an instrument to record the velocity from moment to moment, and to turn rather to the consideration of totalling instruments such as the Robinson cup anemometer already spoken of. In considering the effects of lag upon it, and how they may be overcome, but little help can be got from the analogy of other totalling instruments. A novel form of ballistic galvanometer has recently been introduced possessed of the property that its indications are determined by the total quantity of electricity passed through it, and not at all by the duration or mode of fluctuation of the current.* The principle relied on here, however, has no analogy in air currents. Nor again, of course, has the principle so much used in photometry, according to which the eye and the photographic plate are both regarded as instruments capable, within a considerable range, of accurately integrating the total effect of a number of successive flashes of light.† As none of those cases, nor any others that could be thought of, seemed to be at all similar to that of the anemometer, and as the latter is of some consequence, it was decided to carry out a series of experiments with a view to ascertaining the extent of the error, and possibly of learning how it might be minimised.

APPARATUS AND EXPERIMENTS.

A special air meter of the usual fan type was constructed, the velocity of the fan being recorded at every instant on a moving strip of paper. Difficulties, however, in connection with the production of wind currents possessing accurately known fluctuations caused this method to be temporarily abandoned; and it was decided to employ currents of water, which are susceptible of much easier control and measurement, and which would probably give rise to analogous results. A small impact water turbine was

* The mathematical theory of the Grassot fluxmeter is to be found in the catalogue of the Cambridge Scientific Instrument Co.

† As regards cases in which this no longer holds true, see, e.g., E. F. Ferry, *Phys. Rev.*, vol. i., No. 5, p. 338; and for the contrary see, e.g., *Investigations on the Theory of the Photographic Process*, Sheppard and Mees, p. 18, 1907.

constructed, and, in order that the effects of "lag" might be rendered more apparent, the moment of inertia of the wheel was made relatively large, while the friction at the bearings was kept as small as possible. The turbine is shown diagrammatically in fig. 1.

Each end of the axle is supported on two anti-friction wheels, and, as the counting mechanism is very light, the whole runs very easily. A special cistern in which the water level could be kept constant was arranged in the room above vertically over the turbine, connection between the two being made by a straight-down 2-in. iron pipe. The pipe

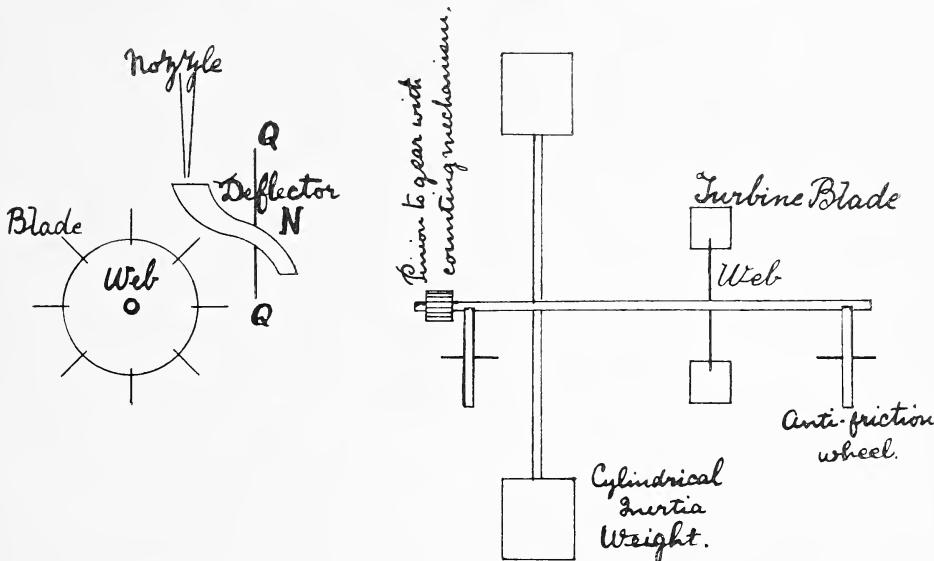


FIG. 1.

terminates in a key tap and tapering nozzle, with aperture $\frac{1}{8}$ in. in diameter. The key of the tap is moved by a long lever, the far end of which travels over a finely graduated scale, and a series of experiments was made to ascertain the rate of discharge from the nozzle in terms of the graduations. The constancy of the running of the turbine under the same conditions being of prime importance, experiments were undertaken to test this. The tap being opened to a predetermined extent, and the wheel having attained its full speed for that rate of flow, the time required for 500 revolutions was noted on successive days. It soon became apparent that changes were liable to take place in the running, and this effect was traced to the fact that the friction varied with the amount of oil present in the bearings. Accordingly, an oil-tank was built up round each end of the axle, so that the bearings of the anti-friction wheels could be kept

106 Proceedings of the Royal Society of Edinburgh. [Sess.
continually immersed in the oil, and since this was done the calibration has remained constant. The following table gives the results:—

Steady speed in revs./sec. = V_1 .	Velocity of the water per sec. = V . (In arbitrary units.)	Steady speed in revs./sec. = V_1 .	Velocity of the water per sec. = V . (In arbitrary units.)
13·9	·0276	11·4	·0228
13·8	·0275	10·6	·0213
13·7	·0274	9·5	·0195
13·5	·0270	8·5	·0174
13·4	·0268	7·4	·0151
13·2	·0262	5·9	·0125
12·9	·0256	4·6	·0100
12·4	·0248	3·6	·0074
11·9	·0238	2·4	·0049

On plotting these results (fig. 2) it appears that the steady speed is proportional to the velocity of discharge. If V denote the velocity of the water, and V_1 that of the paddle-wheel, then $V_1 = aV$. It may be noted that with this form of turbine the efficiency, defined as the fraction of the energy of the water abstracted by the wheel—and then lost in friction—varies inversely as the velocity of the nozzle discharge. If a be the cross section of the nozzle, then the mass of water discharged per second is ρaV , and its kinetic energy $\frac{1}{2}\rho aV^3$. The retarding frictional forces at the bearings, etc., being roughly kV_1 , as will be shown later, the work done against friction per second is kV_1^2 , and the efficiency is given by

$$\frac{kV_1^2}{\frac{1}{2}\rho aV^3} = \frac{2a^2k}{\rho aV}.$$

As a first step in the discussion of the problem of the effects of lag it is evidently necessary to determine how the speed of the turbine varies as the speed of the water is made to fluctuate in any given way. A continuous change in the velocity of the water can be considered as made up of a great number of small sudden changes, between each of which the flow is momentarily steady. This reduces the problem to that of ascertaining how the speed of the wheel running at its terminal velocity for any given rate of discharge varies to the terminal velocity for any other given rate of discharge, when the water flow is abruptly changed from the one rate to the other. It was sufficient for our immediate purpose, however, as will be apparent later, to find (1) how the speed falls to zero as a function of the time, when the water, previously full on, is abruptly shut completely off; and (2) to find how the speed increases from zero to the

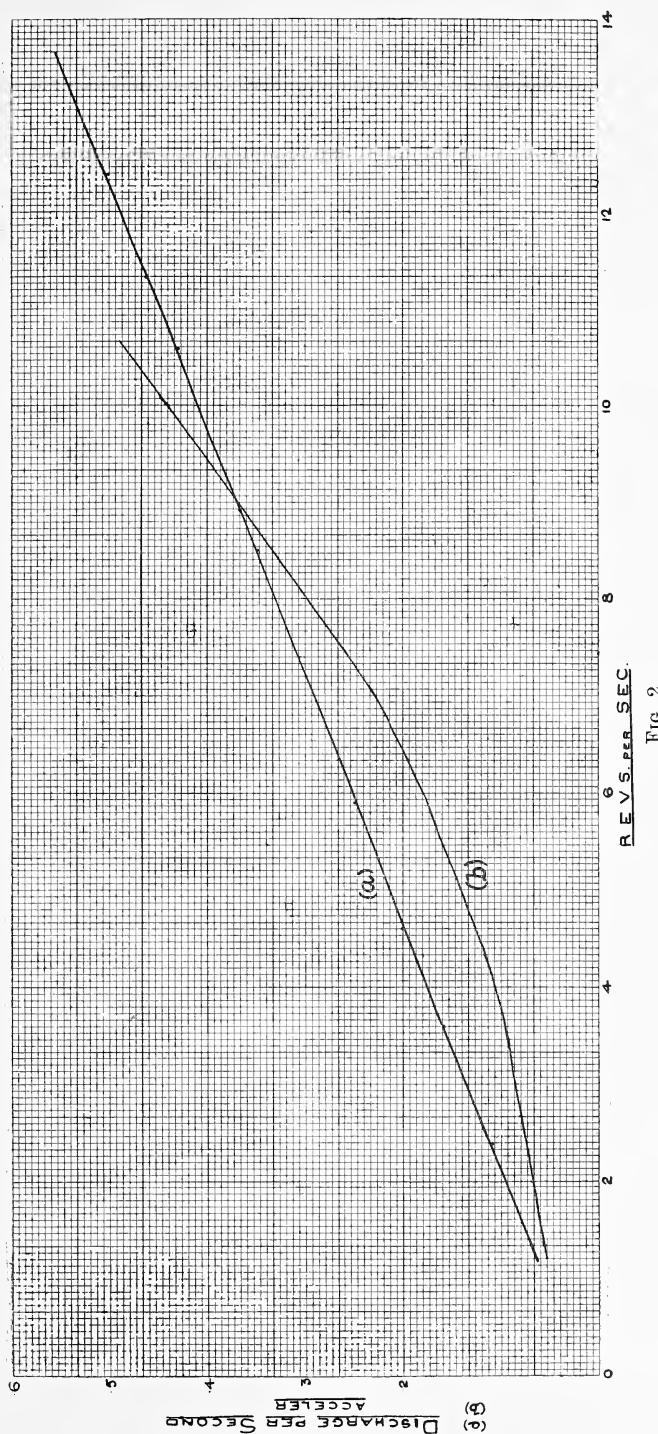


FIG. 2

terminal velocity, when the water previously shut off, with the wheel at rest, is suddenly turned on to any given rate of discharge.

Accordingly, a series of experiments was undertaken in which the tap was turned full on, and the turbine wheel allowed to attain its maximum velocity, which takes place in about two minutes; then suddenly the water was turned off, and by means of a chronograph the intervals of time between successive ten revolutions were recorded until the wheel gradually came to rest on account of its own friction. A curve was then constructed by plotting a number of revolutions against time. When the experiment was repeated, it was found that the new points lay exactly on the same curve; and that the latter was so smooth that it was quite possible to obtain the first and second gradient curves with accuracy, viz. speed against time, and acceleration (*i.e.* the frictional retardation) against time. The curve giving speed against time is shown in fig. 3, and may for convenience be termed the "stopping" curve.

To deduce the law connecting the retardation between the friction and the velocity it was only necessary to plot "acceleration" against "speed" as obtained from the two gradient curves. The result is represented in fig. 1, where for speeds less than about six revolutions per second "acceleration" is approximately proportional to "speed," while for greater values it is also linear, but of the form $a+bs$, a and b being constants.

A suitable formula representing this would take the shape

$$\ddot{s} = a(1 - e^{-ks}) + bs,$$

where for small values of s ,

$$\ddot{s} = (b + ak)s,$$

while for larger values

$$\ddot{s} = a + bs.$$

To obtain information concerning the increase of speed of the paddle-wheel from rest under a steady flow of water a set of experiments was carried out similar to that conducted to determine the stopping curve. The wheel being at rest, the tap was suddenly turned on to some definite angle, and the intervals of time that elapsed between successive ten revolutions as the speed of the wheel increased up to its terminal velocity were recorded by the chronograph. Once more a smooth and regular graph was obtained for "revolutions" against "time," so that the speed-time curve was easily derived. A separate curve had to be obtained, of course, for each angle of the tap, and therefore in fig. 3 a sheaf of curves appears, spreading out from the origin and finally running asymptotically to lines parallel to the time axis.

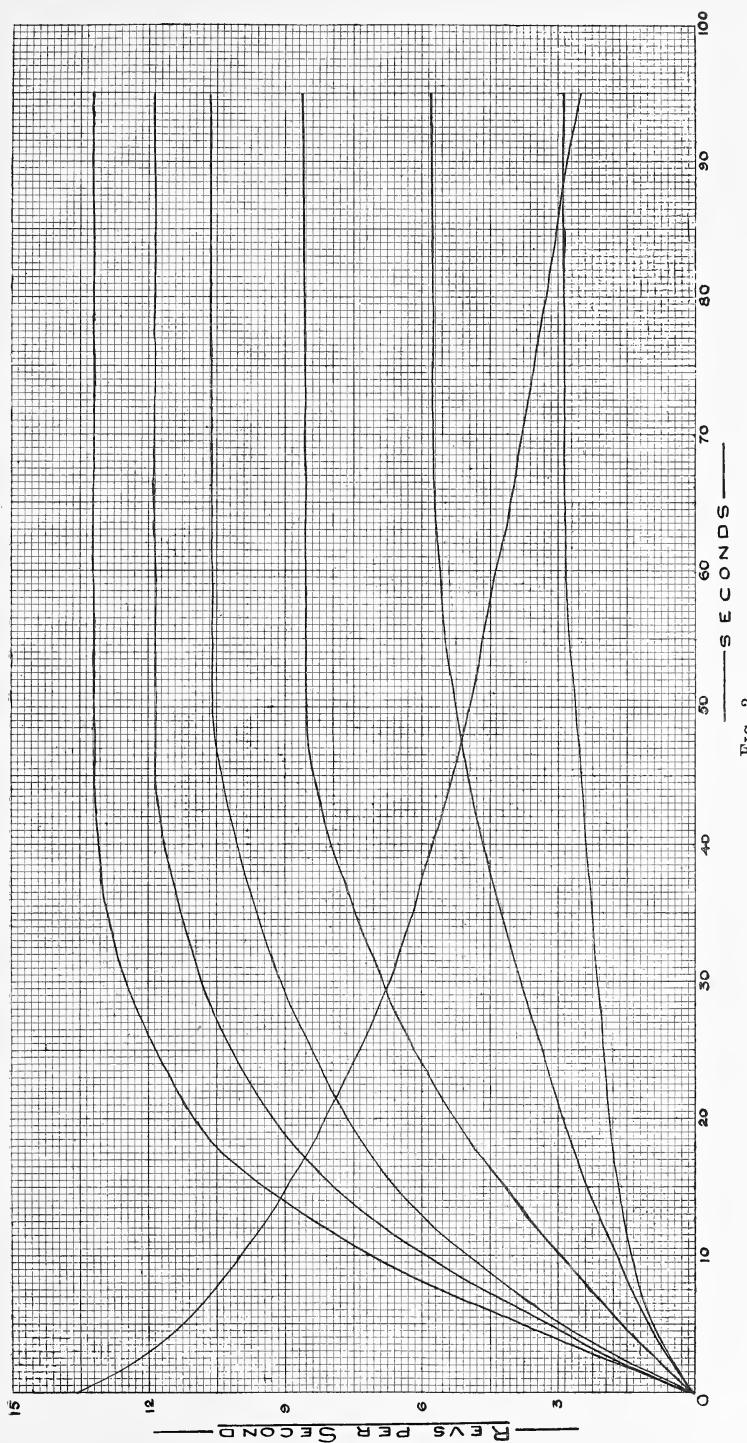


FIG. 3.

We have seen that approximately we may represent the retarding frictional force as proportional to the speed. On this assumption we may with comparative ease deduce the equations to the curves of fig. 3.

For the stopping curve

$$\begin{aligned}\ddot{s} &= -ks \\ \therefore \log \dot{s} &= -kt + A.\end{aligned}$$

If γ = maximum speed, and ω = speed at time t ,

$$\therefore \omega = \gamma e^{-kt}. \quad \quad (1)$$

which evidently represents a curve of the form of the stopping curve in fig. 3.

If V be the velocity of the water entering through the orifice of area a , then the volume that enters per second is aV , so that the momentum of the water before impact is $\rho a V^2$, say σV^2 . Let s be the resolved velocity of the turbine blade in the direction of the impinging jet of water, then the total momentum transferred per second is

$$\sigma V^2 - \sigma V \dot{s} = \sigma V(V - \dot{s}).$$

It is otherwise evident that this term must be merely of the first degree in s from the fact that when the steady state has been attained, the driving force equals the frictional retardation, which has already been shown to be proportional to $a + bs$ for large values of s . The equation of motion is therefore

$$\ddot{s} = \sigma V(V - \dot{s}) - ks \quad \quad (2)$$

We may write it

$$\begin{aligned}\ddot{s} &= \sigma V^2 - \dot{s}(k + \sigma V) \\ \therefore \dot{s} e^{(k+\sigma V)t} &= \sigma V^2 \int dt e^{(k+\sigma V)t} = \frac{\sigma V^2}{k + \sigma V} e^{(k+\sigma V)t} + A.\end{aligned}$$

When

$$\begin{aligned}t = 0 \quad \dot{s} = 0 \quad \therefore A &= -\frac{\sigma V^2}{k + \sigma V} \\ \therefore \dot{s} &= \left(\frac{\sigma V^2}{k + \sigma V} \right) [1 - e^{-(k+\sigma V)t}],\end{aligned}$$

which for different values of V gives a system of curves of the type shown in fig. 3.

The laws governing the rise and fall of speed of the turbine in cases of steady flow having thus been ascertained, it ought to be possible to predict the behaviour of the apparatus under any given condition of flow. In particular, if the flow fluctuates periodically with the time, the definite question presents itself, Do the ensuing imperfectly sympathetic fluctuations

in the running of the turbine, regarded as a water meter, cause it to register the total quantity of water in excess or in defect? We decided first of all to take the simplest possible case, namely, that in which the water is alternately turned on and shut off for predetermined intervals. To carry out the work on its experimental side it is necessary to have an auxiliary apparatus which will not only start and stop the delivery of water to the

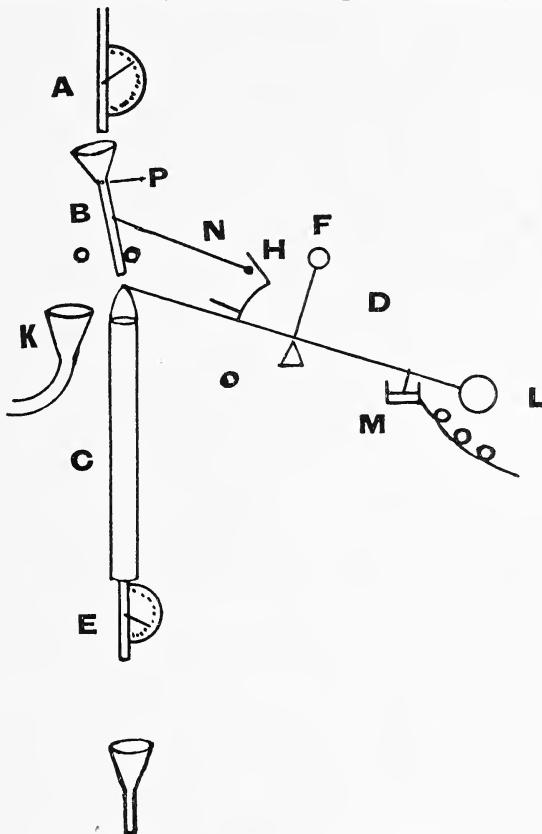


FIG. 4.

turbine; but which will restart it at full speed. To fully achieve this it is requisite, not to stop the water at all, but merely to divert its flow from the wheel at stated times. Accordingly, the apparatus shown diagrammatically in fig. 4 was constructed. It may be described as a sort of water-clock.

The water from the tap A flows into the funnel B, and falls from the nozzle of B into the glass tube C, hanging from one arm of the balance D. Despite the outflow from the tap E, C gradually fills up, and the balance beam tilts over. Owing to the presence of the weight F the action is sudden and definite, and in the course of the movement H comes in contact with N, the projecting arm of B, and pushes the latter, which is pivoted at

P, across to the other side. B is top-heavy so as to lie against a stop, to whichever side it is put, H not being required to hold it in place, and only coming momentarily in contact with it. The water now runs down the waste-pipe K, C gradually empties until at a certain stage the balance tilts back owing to the presence of the counter-weight L, and the whole cycle of operations begins again. The respective times of filling and emptying can be *independently* adjusted by means of the finely graduated taps at A and E. During the filling an electric contact is closed by the dipping of a wire into mercury at M, and during the emptying the contact is open. The circuit includes a cell; and an electro-magnet, which working against a spring actuates the deflecting tube N (fig. 1), which turning slightly about the vertical axle Q, can be made to swing under the nozzle of the turbine, and so to carry off the water before the latter reaches the wheel. Although a little tedious to describe, the whole arrangement is quite simple, and works admirably, the small movement of the deflecting tube taking place with great suddenness, so that there is no perceptible splashing of the water, or loss of time between the "off" and "on" positions. As the balance beam turns on a knife edge, and as it is quite free, H not coming in contact with B except during a swing, and even then not until the motion is well established, it follows that the timing is very accurate.

The experimental results will be given after the theory has been discussed.

THEORY AND RESULTS.

In fig. 5, if OABC represent one of the starting curves, and DEF the stopping curve, then, in order to choose the intervals of time during which the water is "off" and "on" so that the motion may be periodic, we must move up the starting curve from A to B, and down the stopping curve from E to F, so that BE and AF are parallel to the time axis. The intervals of "on" and "off" are then GH and KL respectively, while the number of revolutions described during a complete period is represented by the sum of the areas GABH and KEFL. The number that ought to be described is the terminal velocity multiplied by the interval during which the water is "on." Assuming the equations already determined to give the general shape of the curves of fig. 3, it is easy to show that there is always "overrunning" for such a periodic motion, and to determine an expression to represent it.

Let

$$OG = t_1, OH = t_2, OK = t_3, OL = t_4 \quad (\text{in fig. 5});$$

then, given t_1 and t_2 , we must determine t_3 and t_4 to give a periodic motion.

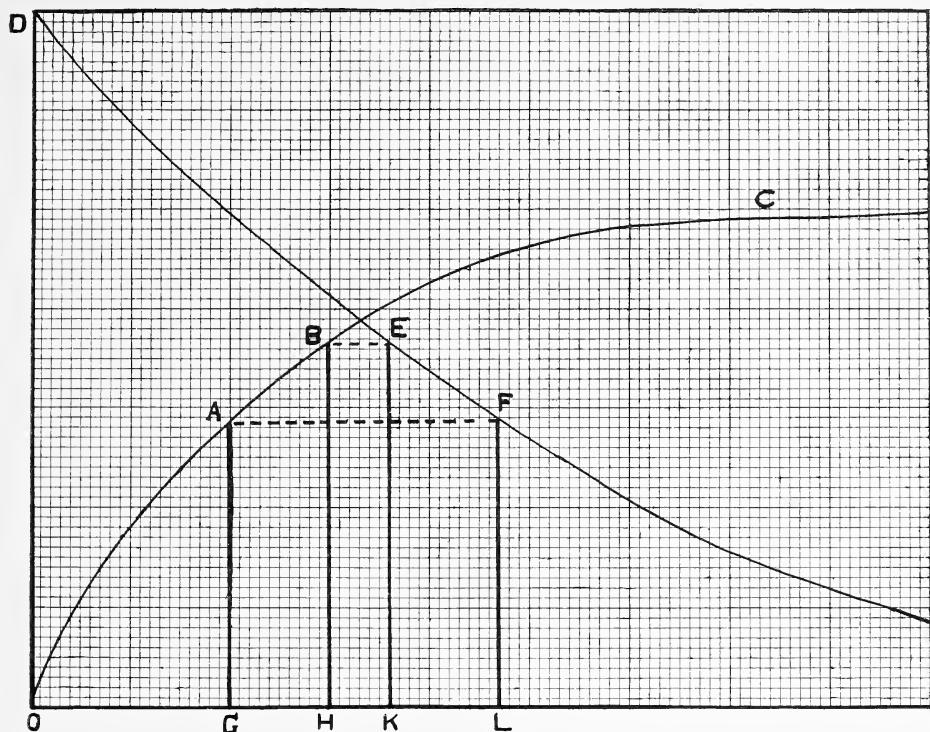


FIG. 5.

As already found, let the equations to the two curves be

$$w = \gamma e^{-kt}$$

$$v = \frac{\sigma V^2}{k + \sigma V} [1 - e^{-(k + \sigma V)t}]$$

then

$$v_{t_1} = w_{t_4} \quad \text{and} \quad v_{t_2} = w_{t_3}$$

$$\therefore \gamma e^{-kt_4} = \frac{\sigma V^2}{k + \sigma V} [1 - e^{-(k + \sigma V)t_1}]$$

and

$$\gamma e^{-kt_3} = \frac{\sigma V^2}{k + \sigma V} [1 - e^{-(k + \sigma V)t_2}]$$

Let R = number of revolutions actually described

$$\begin{aligned} \therefore R &= \int_{t_1}^{t_2} v dt + \int_{t_3}^{t_4} w dt \\ &= \frac{\sigma V^2}{k + \sigma V} (t_2 - t_1) + \frac{\sigma V^2}{(k + \sigma V)^2} (e^{-(k + \sigma V)t_2} - e^{-(k + \sigma V)t_1}) - \frac{\gamma}{k} (e^{-kt_4} - e^{-kt_3}) \\ &= \frac{\sigma V^2}{k + \sigma V} (t_2 - t_1) - \frac{\sigma^2 V^3}{(k + \sigma V)^2 k} (e^{-(k + \sigma V)t_2} - e^{-(k + \sigma V)t_1}) \end{aligned}$$

Let R' = number of revolutions that ought to be described

$$\therefore R' = \frac{\sigma V^2}{k + \sigma V} (t_2 - t_1),$$

since $\frac{\sigma V^2}{k + \sigma V}$ is the terminal velocity.

Hence the fractional overrunning is

$$\begin{aligned} \frac{R - R'}{R'} &= \frac{\sigma V [e^{-(k+\sigma V)t_1} - e^{-(k+\sigma V)t_2}]}{k(k + \sigma V)(t_2 - t_1)} = \frac{\gamma(e^{-kt_4} - e^{-kt_3})}{V k(t_2 - t_1)} \\ &= \frac{\gamma}{V} \frac{(t_3 - t_4)}{(t_2 - t_1)} \text{ approximately} = \frac{\gamma F}{VN}, \end{aligned}$$

where F = interval during which water is "off,"

N = interval during which water is "on,"

V = velocity of water.

This would indicate that there is always *overrunning*, which increases directly with the time during which the tap is off, and inversely with the time during which it is on, and with the velocity of the water.

Experiments were performed by means of the apparatus to give interrupted flow already described. The times t_1 and t_2 were determined from the graphs in fig. 3, and the apparatus adjusted to give this period. The number of revolutions which the paddle-wheel ought to describe if the starting and stopping curves obtained were accurate, was measured by means of a planimeter, and the number of revolutions actually described during a complete period was noted, and compared with them. From these data the percentage overrunning was calculated and tabulated as follows :—

R. (Observed.)	R. (Calculated.)	R'.	N. secs.	F. secs.	Terminal velocity.	Percentage overrunning.
179	178	132	10	5	13·2	36
183	183	119	10	7·2	11·9	54
165	164	106	10	8·2	10·6	56
167	167	85	10	17·5	8·5	96
282	284	132	10	21·2	13·2	113
260	259	119	10	24·2	11·9	119
235	234	106	10	27·2	10·6	122
208	207	85	10	41·5	8·5	145
220	220	82	6·2	22·5	13·2	169
235	236	105	8·8	22·5	11·9	124
297	298	173	16·3	22·5	10·6	72
223	225	63	4·8	33	13·2	252
234	236	77	6·5	33	11·9	203
250	257	108	10·2	33	10·6	131

If we compare the values of R in the first two columns it is evident that except in one case they never differ by more than 1 per cent. The

exceptionally large discrepancy in the last case may be accounted for by the fact that here we are working at that end of the stopping curve at which greatest doubt exists as to its exact shape, since being a derived curve the slope of the integral curve is most difficult to determine at its ends.

To find an empirical formula to represent the overrunning we may assume it to be of the form

$$KF^a V_1^b N^c,$$

where V_1 is the terminal velocity. It is useless to determine the constants K, a, b, c accurately, since their values will apply only to the set of experiments conducted above; but we desire to know at least their sign and their order, so that the laws governing the overrunning in this case may roughly be determined.

In the table quoted above, for the first eight sets, the values of N are constant, and these in pairs have V_1 constant also. Hence, if the formula we have assumed is at all justifiable we must have

$$\begin{aligned}\frac{36}{113} &= \left(\frac{5}{21.2}\right)^a \quad \therefore a = .78 \\ \frac{54}{119} &= \left(\frac{7.2}{24.2}\right)^a \quad \therefore a = .65 \\ \frac{56}{122} &= \left(\frac{8.2}{27.2}\right)^a \quad \therefore a = .65 \\ \frac{96}{145} &= \left(\frac{17.5}{41.5}\right)^a \quad \therefore a = .48.\end{aligned}$$

This gives us a rough average value $a = .6$.

Again, using all the data, we obtain roughly $a = .7, b = -.5, c = -1$.

The values of the constants being thus determined approximately, the similarity between this formula and that already derived, viz. $\frac{F}{VN}$, is at once seen. The overrunning in the experiments described varies from 36 per cent. to 252 per cent.; but as the apparatus used was specially designed to exaggerate this, no great stress can be laid on the magnitude of the result. It is doubtless significant, however, that the error is always found to be of the same sign.

SUMMARY.

All instruments designed to give a record of some fluctuating quantity as a function of the time, both those employed to give "instantaneous" values of the quantity measured and also those which totalise or integrate it during a given time, are liable to suffer from error due to the inertia

of their moving parts and consequent "lag." In each class instances are given of instruments which are free from the error, but the Robinson cup anemometer—in which the authors are more particularly interested—is not one of those. As nothing was known definitely as to the effects of lag in this case, an investigation was undertaken, using a specially designed apparatus analogous to the cup anemometer, and, for greater accuracy of measurement, currents of water instead of air.

A description of this apparatus is given; and the remainder of the paper is occupied with the mathematical theory and the results of experiment. The most striking of these is the fact that the instrumental error is always positive; that is to say, that when the current fluctuates the instrument registers too great a flow.

The authors desire to record their thanks to Professor MacGregor for the facilities he has provided, and for the kind interest he has taken in their work.

Part of the special apparatus described was obtained by a grant from the Tait Memorial Fund.

PHYSICS DEPARTMENT,
EDINBURGH UNIVERSITY.

(Issued separately April 11, 1913.)

X.—A New Method of Starting Mercury Vapour Apparatus.

By John S. Anderson, M.A., B.Sc., 1851 Exhibition Science Scholar (Glasgow); and George B. Burnside, Mechanical Assistant to the Professor of Natural Philosophy in the University of Glasgow. *Communicated by Prof. A. GRAY, F.R.S.* (With One Plate.)

(MS. received January 8, 1913. Read February 3, 1913.)

THE mercury vapour lamp,* although very economical in its use of electrical energy,† is not employed very extensively at the present day for illuminating purposes, chiefly because the light it emits is not white, but of a greenish hue. Many attempts have been made to produce the deficient red radiation, but these have so far not met with any great success.‡ The lamp, however, is useful as an illuminant where the colour of the light is of no great importance, such as in public works, etc. Further, it is very much used, in the form of rectifiers, for changing alternating to direct electrical current. During the past few years there has been an increasing demand for lamps which are rich in ultra-violet radiation, which is useful for photographic and medical purposes, as, for example, the sterilisation of water, milk, etc. This requirement is met by the mercury vapour lamp, for the ultra-violet spectrum of mercury is particularly intense; lamps used for this purpose must be made of quartz, since ordinary glass absorbs ultra-violet radiation. Then, again, the mercury spectrum is extremely useful in spectroscopic work as a reference spectrum.

In the best types of mercury vapour lamps and rectifiers at present on the market, the arc is started by tilting the lamp or rectifier, either by hand or automatically. Now, this tilting arrangement is cumbrous, and is often very inconvenient. This is found to be especially the case when one is dealing with lamps used for scientific purposes. For example, in making measurements on the Zeeman effect, an ordinary mercury high-tension vacuum tube in most cases does not form a sufficiently strong source of

* For history, see von Recklinghausen, "Ueber die Quecksilberdampflampe von P. C. Hewitt," *Elektrot. Zs.*, p. 492, 1902.

† Maurice Leblanc, "Expériences de M. Cooper Hewitt sur les tubes à vide," *Journ. de Phys.* (4), 4, p. 416, 1905.

‡ Leo Arons, "Eine Amalgamlampe mit reichem Linienspektrum," *Ann. d. Phys.* (4), 23, pp. 176-178; O. Bussmann, *Elektrot. Zs.*, vol. xxxviii., 1907; H. E. Ives, "White Light from the Mercury Arc and its Complementary," *Bull. Bureau of Standards*, vi. p. 265, 1909.

light, whereas a mercury vapour arc lamp is extremely suitable. But the difficulty arises from the fact that the lamp must be placed between the poles of an electro-magnet, the distance between the poles being usually so small that any tilting apparatus that may be employed interferes with the proper mounting of the lamp.

We have recently overcome this difficulty by constructing a lamp which may be fixed in position between the poles of an electro-magnet, or in any other suitable position, and then started without having to be tilted. This is brought about by the employment of a heating arrangement near one of the electrodes, preferably the negative electrode. The lamp tube is provided with a small vessel near this electrode, the vessel having a re-entrant portion or recess in which a heating element is placed. The part of the tube immediately above this small vessel and its recess is constricted. The heating element may conveniently consist of a small coil of platinum wire wound round a suitable support; it may be placed in the recess of the small vessel or removed at will, without interfering with the vacuum of the lamp. The heating coil of wire is connected in series or in shunt, in the latter case being provided with an automatic cut-out. An external resistance is placed in series with the lamp. Before starting, the small vessel is full of mercury, which forms a continuous connection inside the tube between the positive and negative electrodes. When the electrical current is switched on, the heating coil becomes incandescent, and the heat given off by the wire goes to raising the temperature of the vessel and its contained mercury, there being no appreciable loss by radiation into the surrounding air. Very little heat is required, because the first bubble of mercury vapour formed rises to the constricted portion of the lamp tube and is there caught, thus breaking the continuity of the mercury inside the lamp and starting the arc. Owing to the resistance of the mercury vapour, which is formed once the arc is started, the current is cut down to the value required for running the lamp. The platinum wire of the heating element can be made of such a thickness, and the external resistance can be so adjusted, that the wire does not emit heat when the lamp is working, but becomes incandescent when the lamp is started, the action being quite automatic.

The construction and method of working the lamp may be more particularly described with the help of the accompanying diagrams. Fig. 1 represents a vertical section of the small vessel A, with the heating element G, shown in elevation, in position. This figure shows one method of fitting in the heating element. B is the recess in the vessel A, and C represents the constricted portion of the lamp tube. Through a part of the

electrode D is sealed the leading-in wire E, the outer end of which is connected to the metallic cap F. The heating element G consists of a hollow quartz tube H, round the tapering end of which is wound a coil of thin platinum wire I, one end of which is connected to the metallic sleeve K. The other end of the wire passes down through the tube and is connected to the flexible conductor L, which is held in position by a packing of plaster-of-Paris M, or other suitable cement. A metallic clip N, having a binding screw O, serves to connect the sleeve K and the cap F, and also holds the heating element G in position. Figs. 2 and 3 are cross-sections taken on the lines XX, YY (fig. 1) respectively. The heating element G may be removed, whenever desired, by loosening the screw O. It is advisable to wind a thin piece of asbestos thread round the outside of the tapering portion of the heating element, or to adopt some similar precaution, in order to prevent the platinum wire, when hot, from coming into direct contact with the glass wall of the recess B of the small vessel A.

Fig. 4 shows, in elevation, the end of the lamp tube, and illustrates a second method of mounting the heating element, a vertical section of which is shown in fig. 5. This alternative arrangement is shown, partially in vertical section, in fig. 6, while fig. 7 is a horizontal section taken on the line ZZ (fig. 6). Referring to figs. 4, 5, 6, and 7: A represents the small vessel with the recess B and the constricted portion C of the lamp tube. D is the electrode, and the leading-in wire E is connected to a bayonet-holder P, which is attached to the vessel A by means of cement R. The heating element consists of a hollow porcelain tube H with fluted end, the coil of platinum wire I being attached as in the previous case. The metallic sleeve K is provided with two pins SS, which fit into two slots in the bayonet-holder P. The heating element may therefore be placed in position or removed with the greatest of ease.

In fig. 8 (Plate) is shown a convenient form of the complete lamp as used for spectroscopic purposes. For use in connection with work on the Zeeman effect, the lamp tube may be of the form shown, when it is to be placed in a vertical position between the poles of the electro-magnet, or it may be provided with a very short horizontal portion, so that it may be fixed up in such a position that the light passes in the direction of the lines of force. This does away with the necessity of boring the pole pieces, except where the light is observed in a direction parallel to the lines of force.

The method of working the lamp is as follows:—The flexible conductor L and the other electrode of the lamp than that shown in figs. 1-7 are connected through a suitable resistance to the poles of the battery. When the lamp is not burning, there is a continuous column of mercury between

the two electrodes of the lamp, thus forming a continuous electrical connection through the flexible conductor L, the heating coil I, the sleeve K, the clip N (figs. 1 and 3), the cap F (figs. 1 and 3), or the bayonet-holder P (figs. 4, 6, and 7), and thence through the leading-in wire E and the mercury of the lamp to the other electrode. When the current is switched on, the heating coil of platinum wire I becomes incandescent; the first bubble of mercury formed in the vessel A rises to the constriction C, where it is caught, thus breaking the continuous column of mercury in the lamp, and starting the arc.

The lamp, as we have described it, may be made of Jena glass, uviol glass, or quartz, according to the purpose for which it is required. For example, when the lamp is to be made use of in work on the visible spectrum, it may very conveniently be made of Jena-glass combustion tubing. Our first experimental lamps were made of soda glass, because of the great ease with which it can be worked. The method of starting the lamps proved to be entirely satisfactory, the arc being struck about 15 seconds after the current was switched on.* Of course, if the lamp was allowed to burn, it soon collapsed, owing to the fact that soda glass cannot withstand the temperature of the incandescent mercury vapour. The lamps, however, served the purpose of testing the method of starting. Once the method was fully developed, our attention was turned to the making of lamps from Jena glass. Here we met with a considerable amount of difficulty, because the existing methods of sealing electrical conductors, such as platinum wires, through Jena glass were found to be very unsatisfactory. In the course of our experiments an entirely new method of sealing electrical conductors through glass was developed by one of us, and it has been used by us since with the most satisfactory results. This method is described in a paper at present in the hands of the Institution of Electrical Engineers.† Briefly stated, it consists of fusing the glass round the electrical conductor and then cooling the seal thus formed, when at about a dull red heat, by successive immersions in a suitable cooling medium, such as sperm or other oil.

The Jena-glass lamps which we constructed for spectroscopic work were designed for a pressure of 100 volts; they were made from Jena-glass combustion tubing of 4 mms. internal and 6 mms. external diameter, the

* The time that elapses between the switching on of the current and the striking of the arc depends, for a given heating element and a given current strength, on two factors, namely, the pressure in the small vessel due to the head of mercury above the constricted portion, and the quantity of mercury contained in the small vessel. The lamp, as shown in fig. 8, was designed to cut down to a minimum the time of starting.

† "A New Method of Sealing Electrical Conductors through Glass," G. B. Burnside.

Fig. 1.

Fig. 2.

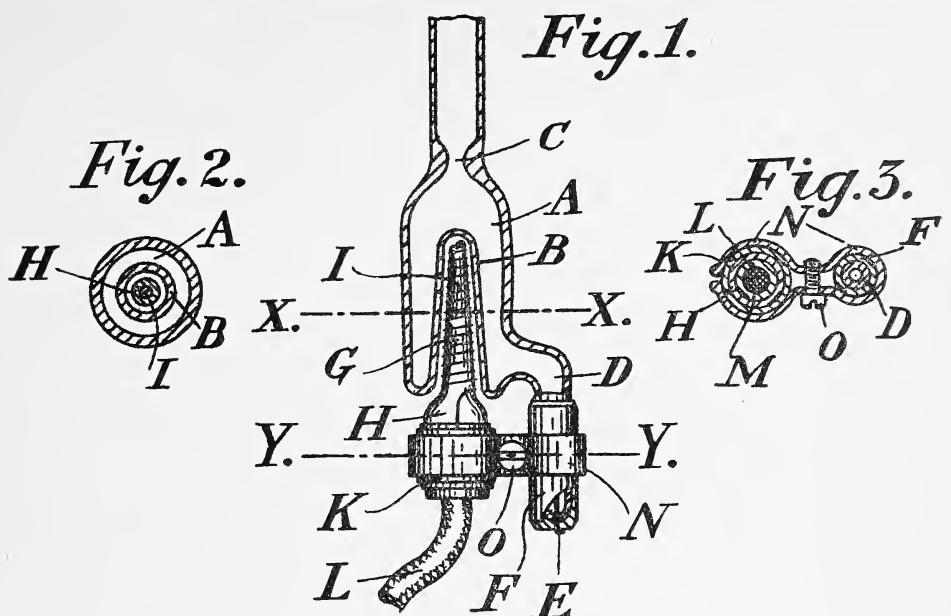


Fig. 4.

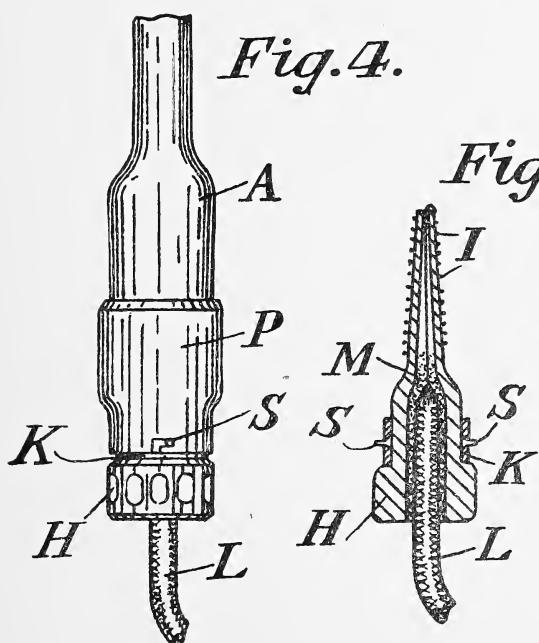
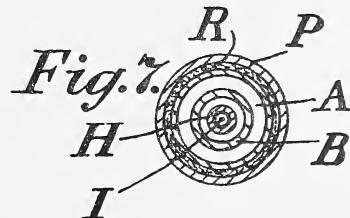
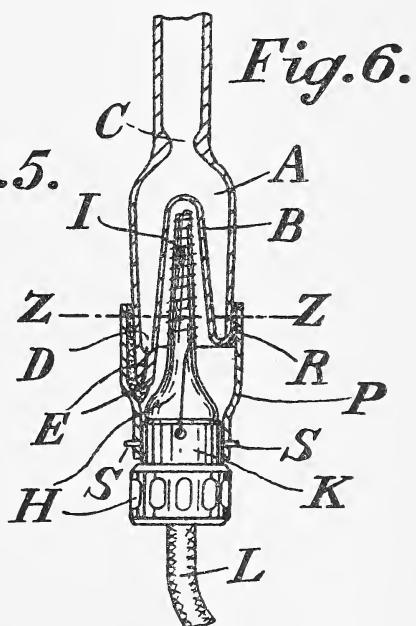


Fig. 5.



length of the illuminating portion of the lamp tube being about 11 cms. As may easily be seen, the lamp tube may be made in any convenient form, the essential parts of the lamp being the small vessel with its recess and the heating element situated in the recess. Thus a lamp suitable for illuminating purposes was constructed of the same kind of tubing as before, the tube being bent into the form of a spiral. This lamp was designed for 250 volts pressure, the effective length of the tube being about 22 cms.

It may be advisable to give here a short account of some of the more important points to be observed in the preparation of mercury vapour lamps. One of the chief precautions to be taken is to see that the lamp tube and the mercury to be put into it are perfectly clean and dry, for the presence of impurities or moisture interferes with the proper running of the lamp. A good method of cleaning the tube is as follows:—It is first of all thoroughly rinsed out two or three times with a fairly strong solution of chromic acid. Then it is washed several times with distilled water. Next, it is rinsed once or twice with absolute alcohol, and finally with ether. It is then fixed to a pump and exhausted, being slightly warmed after a time, in order to drive off the last traces of ether vapour.

The mercury, which should be of the purest quality, may be introduced into the lamp through a side tube which serves to connect the lamp to the pump. A very good plan to adopt is to constrict this side tube at two places, one next the lamp and the other a short distance from it. The object of this will be seen immediately. The lamp, after being connected to the pump, should be evacuated to a fairly high degree of exhaustion,* and should be strongly heated during the process, in order to facilitate the removal of any volatile gases that may be present. One or more tubes of concentrated sulphuric acid, calcium chloride, or phosphorus pentoxide should be introduced between the lamp and the pump, in order to absorb any moisture that may be given off. When the lamp is sufficiently exhausted, it should be sealed off at the constriction of the side tube nearer the pump. The lamp should then be connected to a suitable battery and run for some time, after which any gases that may still have been given off may be worked up into the side tube, which should then be sealed off at the other constriction.

The experimental lamps made of Jena glass naturally proved to be more satisfactory than the previous ones made of soda glass, because, not only could the method of starting the arc be observed, but also the lamps could be run for considerable periods, because Jena glass can withstand the heat of the lamp when burning. As in the other case, the arc was struck about

* A Sprengel three-fall pump was used by the authors.

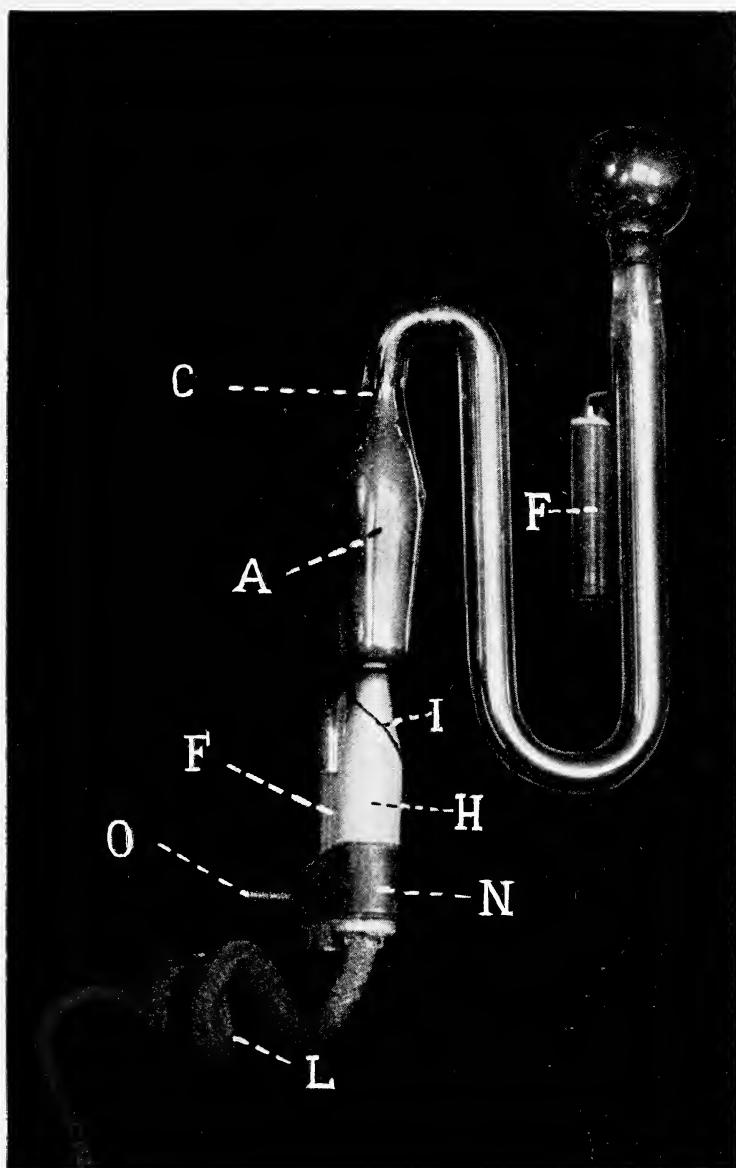


FIG. 8.

15 seconds after the current was switched on. As an example of durability, one lamp was found to work perfectly after a lapse of more than six months, although it had been in use from time to time during that period; no deterioration of the vacuum had taken place. This showed, amongst other things, that the previously mentioned new method of sealing in the platinum wires is entirely satisfactory.

It may easily be seen that this new method of starting mercury vapour apparatus does not depend on the form of the illuminating portion of the lamp, but may be applied without any difficulty to all forms of lamps, rectifiers, etc. We have found that the form of lamp specially described in this paper is extremely suitable for all kinds of spectroscopic work. One of the experimental Jena-glass lamps was successfully employed as a source of light in connection with a large concave diffraction grating spectrograph in use in the Natural Philosophy Department of the University of Glasgow.

We desire to express our thanks to Professor A. Gray for the kind interest he has taken in the work connected with this paper.

(Issued separately April 11, 1913.)

XI.—On the Application of Manley's Differential Densimeter to the Study of Sea-waters on board Ship. By Norman Phillips Campbell, B.A. Oxon. *Communicated by Sir JOHN MURRAY, K.C.B.*

(MS. received January 9, 1913. Read February 3, 1913.)

In a paper * communicated to the Royal Society of Edinburgh, Mr Manley describes a differential densimeter and its successful application in the laboratory to the rapid determination of the relative densities of sea-waters. The paper also contains the suggestion that the instrument would, if suspended in gimbals, be rendered available and convenient for use at sea: thus making it possible for the oceanographer to carry out observations upon the density of sea-water with greater frequency than the usual methods conveniently permit.

During a voyage of the *Oruba* from London to Colombo in October 1908, the Orient Steamship Company courteously afforded the present author the opportunity of testing the efficacy of such a differential densimeter under the varying conditions which obtain on board ship.

The instrument, which was built in Oxford under Mr Manley's supervision, is shown in fig. 1.

The densimeter tubes AA are mounted on a mahogany board 7 feet long and 6 inches wide; this board is suspended on gimbals GG. The gimbals are hung from the ceiling of the cabin by three copper wires. The lead weights WW, attached to the lower end of the board, greatly reduce the swinging. The lower ends of the tubes dip into two exactly similar flasks: these stand on a shelf C, which is covered with a piece of plate-glass. The average length of each column of liquid is 1·52 metre.

Fig. 2 shows the travelling microscope M (by Elliott Bros.) for comparing the heights of the columns. The drum-head D reads to .001 mm., but .01 mm. affords sufficient accuracy for the purpose. The microscope is mounted on a plate-glass base E, which slides on another piece of plate-glass F; the microscope can thus be brought opposite either tube and adjusted by the drum-head and screw S. If the plate-glass shelves C (fig. 1), F (fig. 2), were strictly horizontal, the difference in the microscope readings would at once give the difference between the heights of the two columns; it was found that C, F deviated from strict horizontal parallelism by 0·66 mm.

* *Proc. Roy. Soc. Edin.*, vol. xxvii., Part III., pp. 210-232.

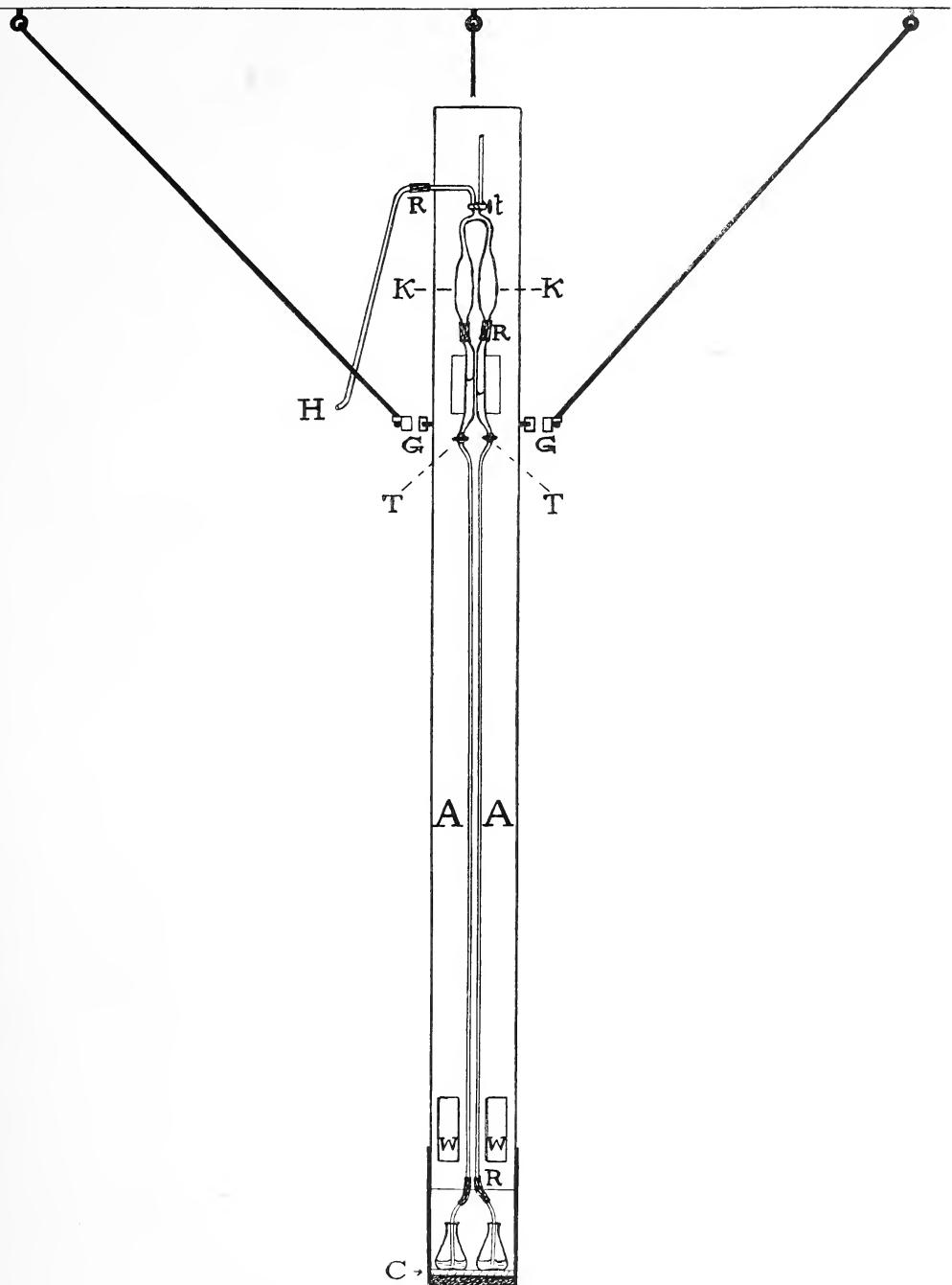


FIG. 1.

In fig. 3 the densimeter tubes AA are seen in section, and the shelf upon which the microscope stands is shown within the gimbals.

During the voyage the tubes were washed out with nitric and chromic acids once a week; and the rubber tubing RRR was frequently changed and kept in acidified bleaching-powder solution. The tubing was never allowed to remain dry for more than a few hours. The taps were lubricated with phosphoric acid or sea-water only. A close observance of these several precautions is, as Mr Manley has shown, absolutely necessary for ensuring success.

Before the voyage was commenced, a large jar of sea-water was obtained and its density in terms of air-free distilled water at 18° C. determined. The density measurements were carried out with the aid of thin-walled pyknometers somewhat similar to those devised by Sprengel: these determinations were repeated at Colombo. As the means of the first and second series were in very close agreement, it was concluded that the standard sea-water used throughout the voyage had remained practically unchanged.

In order to measure the density of any sea-water on board ship the following procedure was adopted:—100 c.c. of the standard sea-water were placed in one flask of the densimeter and an equal volume of the water to be examined introduced into the other; the waters were then drawn up into the tubes by applying suction at H, and the upper tap *t* closed. Sufficient time was then allowed for the two columns to attain not only equilibrium, but also a common temperature. The two taps TT were then simultaneously closed, and the difference between the lengths of the two columns measured with the microscope: the necessary correction (0·66 mm.), mentioned above, was then applied and the density calculated. Here it may be stated that the correcting factor remained, so far as could be discovered, constant during the whole of the voyage.

When the ship is in motion and the taps TT open, the water always oscillates within the tubes, owing to a slight swinging of the instrument; but with a little practice no difficulty is experienced in closing the taps during a moment when the tubes are vertical. Towards the end of the voyage it was found that the oscillations of the columns could be very considerably lessened by removing the bulbs KK: this modification of the original apparatus was accordingly introduced, and a distinct advantage gained.

In his paper* Mr Manley has shown that the ratio of the densities of two samples of sea-water measured at any common temperature within

* *Proc. Roy. Soc. Edin.*, vol. xxvii., Part III., p. 224.

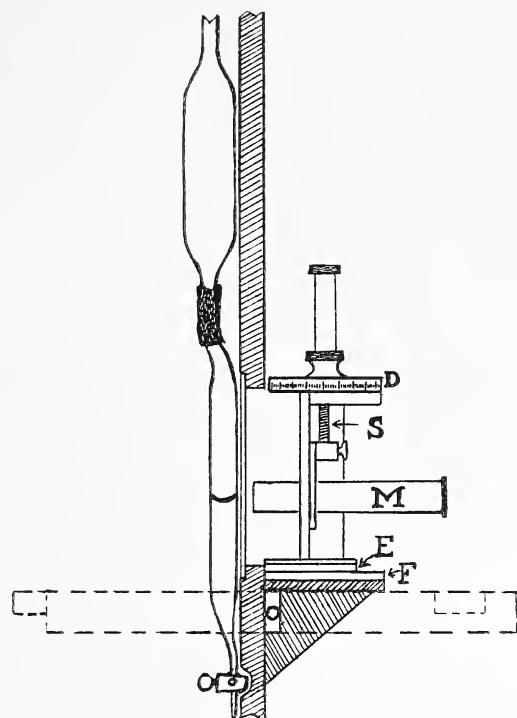


FIG. 2.

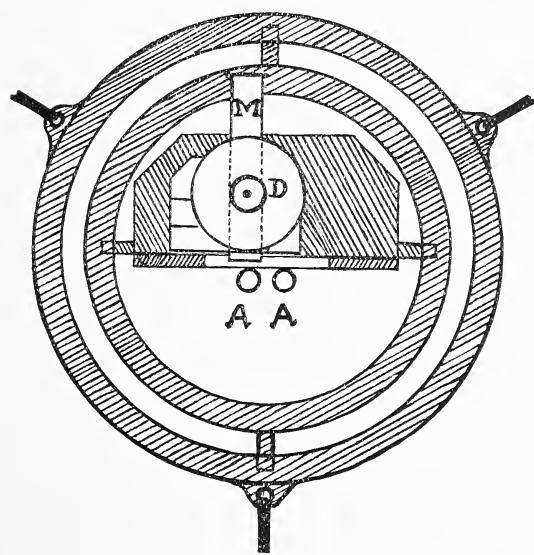


FIG. 3.

ordinary limits, may be taken as independent of the temperature at which the measurements are effected. It is largely due to this fact that the differential densimeter offers such decided advantages over the usual and, it may be added, tedious pyknometer methods. In the same paper it is also shown that the relative densities of sea-water and distilled water vary appreciably with variations in the common temperature of the two waters. Hence the necessity for first determining the true relative density of the sea-water, which is subsequently to be used as a standard, at some fixed temperature. This necessary determination may obviously be effected either with a pyknometer or, if desired, equally well in a room maintained at the selected standard temperature, with the aid of the densimeter itself. In this present instance the former method was, as we have already stated, adopted, and that chiefly because it was desired as far as possible to ascertain how nearly the ordinary pyknometer and the newly proposed densimeter methods might agree.

Some few of the samples of water obtained in the Bay of Biscay, the east coast of Spain, the Straits of Messina, and in the Red Sea were, after their densities had been measured by the densimeter, bottled and taken on to Colombo, where their densities were re-determined with a pyknometer at the temperature 29° C. In carrying out these pyknometer determinations the densities were found by a direct comparison, not with distilled water, but with the standard sea-water which had been used throughout. In this way the difficulty which arises in the former case owing to dissimilarities in the coefficients of expansion of distilled and sea-water is avoided. The following example will serve to illustrate our mode of checking the values obtained at sea with the densimeter, by means of pyknometer measurements carried out in the laboratory at Colombo:—

Example:

Water No. 57. (See Table II.)

Water required to fill pyknometer at 29° C. = 7·9825 grms.

Standard water " " " " = 7·9724 "

Density of standard water at 18° C. = 1·02767 "

$$\therefore \text{at } 18^\circ \text{ C. the density of water No. 57} = \frac{7\cdot9825}{7\cdot9724} \times 1\cdot02767 = 1\cdot02898.$$

The densimeter on board ship gave the density of this water as 1·02904; the difference between the two independent values is therefore .00006 only: and this may, for most purposes, be regarded as negligible. The points which have been dealt with in the immediately preceding paragraphs will, it is hoped, be rendered sufficiently clear by the data and results set forth in Table I.

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OF THE
ROYAL SOCIETY OF EDINBURGH.
SESSION 1912-13.

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TABLE I.

Water.	Place where determination was made.	Pyknometer measurement.			Densimeter measurement.	
		Temp.	Pyknometer used.	Weight of water corrected for weight of air displaced.	Specific gravity.	Place where determination was made.
A { Distilled water .	Oxford	18° C.	A	7.8975	{ 1.02765	...
A { Standard sea-water .	"	18°	A	8.1159	...}	...
B { Distilled water .	"	18°	B	7.7811	{ 1.02769	...
B { Standard sea-water .	"	18°	B	7.9966	...}	...
C { Distilled water .	Colombo	18°	B	7.7798	{ 1.02767	...
C { Standard sea-water .	"	18°	B	7.9951	...}	...
D { Distilled water .	"	29°	A	7.8776	{ 1.02709	...
D { Standard sea-water .	"	29°	A	8.0910	...}	...
E { Distilled water .	"	29°	B	7.7613	{ 1.02720	...
E { Standard sea-water .	"	29°	B	7.9724	...}	...
Sea-water No. 4 .	"	29°	A	8.0843	{ 1.02707	Bay of Biscay.
" No. 83 .	"	29°	A	8.1265	{ 1.02699	Bay of Biscay.
" No. 57 .	"	29°	B	7.9825	{ 1.02898	Red Sea.
" No. 26 .	"	29°	A	8.0923	{ 1.02784	Coast of Italy.
" No. 42 .	"	29°	B	7.9783	{ 1.02904	Red Sea.
" No. 87 .	"	29°	B	7.9867	{ 1.02932	East Coast of Spain.
" No. 98 .	"	29°	A	8.1077	{ 1.02980	Straits of Messina.
						Red Sea.
						Red Sea.
						-0.0004
						-0.00012
						+0.00115 (?)
						-0.0038

From the above table it may be seen that the average difference between the densimeter and pyknometer values (neglecting No. 87, which was probably misplaced in the case of bottles), is ± 0.0019 . This may be taken as ± 0.7 per cent. of the total salt content; whence it appears that the extreme variation between the densest water (No. 83), and the lightest (No. 3), is equal to ± 4 per cent. From the point of view adopted by the exacting physicist, it is therefore evident that unless the density determinations are refined to an accuracy of 5 decimal places, which is difficult even on land, the information which such determinations afford will not be of the highest order. We are, however, under the impression that the degree of accuracy attained by us with the densimeter at sea just fulfils the actual requirements of those who engage in the study of ocean currents and matters allied therewith.

As a result of the whole of this present investigation, the conclusion was reached that the following distinct and desirable advantages may be obtained by the use of the differential densimeter:—

1. The apparatus is simple, easily managed, and can be set up in quite a small cabin; in fine weather a reading can be taken in four minutes or less.
2. The use of many cumbersome cases of bottles is avoided, as well as the uncertainty attending the storing of samples for a more or less lengthy period.
3. The density values obtained are independent of the temperature at which they are measured; and in order that the densities sought may be stated for any given temperature, it is only necessary to determine once for all the true relative density of the standard sea-water at that temperature.
4. In fine weather observations upon the perfectly fresh and natural sea-water may be carried out as often as may be desired. When rough weather prevails the samples of water may be temporarily stored, their densities measured as soon as opportunity arises, and the bottles again set free for future occasions.

That the measured densities were practically independent of the temperature at which the observations were made, may be seen by referring to Table II. In the column headed "Specific gravity," two slightly different and bracketed values are give for certain waters (*e.g.* Nos. 24, 30, and 44); the first of these values was obtained when the temperature of the surrounding air was greatly different from that which prevailed during the second measurement; between the two determinations the water was suitably stored. As the differences are sometimes in one direction and sometimes in the other, and are, on the average, equal to ± 7 in the fifth decimal place, such discrepancies are to be assigned to un-

TABLE II.

	Place where the sample was taken.	Latitude.	Longitude.	Date when sample was taken, 1908.	Temp. in °C.	Specific gravity.	Place where the specific gravity was measured.
1	English Channel	50° 15' N.	4° 8' W.	Oct. 17.	16.0	1.02707	Bay of Biscay.
2	"	49 43	4 40	7.49 p.m.	15.0	1.02680	"
3	"	49 21	4 57	9.5	14.6	1.02677	"
4	Bay of Biscay	49 0	5 12	Oct. 18.	14.2	{ 1.02699 1.02707	{ Off the West Coast of Spain.
5	"	47 40	6 12	7.33 a.m.	16.4	1.02703	"
6	"	47 25	6 26	8.51	16.3	1.02712	"
7	"	47 15	6 32	10.41	17.8	1.02721	"
8	"	46 52	6 47	12.33 p.m.	18.1	1.02721	"
9	"	46 47	7 7	2.49	17.8	{ 1.02721 1.02720	{ East Coast of Spain. Gibraltar Harbour.
10	"	46 0	7 30	5.48	17.5	1.02722	"
11	"	45 20	8 0	9.15	17.4	{ 1.02720 1.02726	{ Marseilles. Gibraltar Harbour.
12	Off the West Coast of Spain	43 22	9 25	Oct. 19.	17.3	1.02727	"
13	"	42 42	9 30	10.0	16.9	1.02735	"
14	"	42 28	9 32	11.40	18.1	1.02737	E. Coast of Spain.
15	"	42 2	9 32	1.57 p.m.	17.3	{ 1.02737 1.02726	"
16	"	41 30	9 32	4.27	18.8	{ 1.02734 1.02739	Marseilles Harbour.
17	"	41 7	9 33	6.23	18.8	{ 1.02739 1.02730	E. Coast of Spain.
18	"	40 42	9 33	8.30	18.1	1.02726	"
19	"	40 13	9 35	10.44	18.4	1.02739	"
20	"	38 34	9 42	Oct. 20.	7.30 a.m.	1.02762	"
21	"	38 10	9 30	9.14	...	1.02764	Between Marseilles and Naples.
22	"	37 50	9 22	10.58	...	1.02760	"
23	"	37 13	9 11	12.58 p.m.	...	{ 1.02777 1.02775	{ Red Sea. Between Marseilles and Naples.
24	"	36 52	8 55	2.52	...	{ 1.02786 1.02780	{ Red Sea. Gulf of Suez.
25	"	36 50	8 40	3.51	...	1.02780	"

TABLE II.—*continued.*

	Place where the sample was taken.	Latitude.	Longitude.	Date when sample was taken, 1908.	Temp. in °C.	Specific gravity.	Place where the specific gravity was measured.
26	Off the West Coast of Spain	36° 38' N.	8° 5' W.	Oct. 20.	6.2	1.02780	East Coast of Spain.
27	"	36 28	7 30	8.27	...	1.02779	Gulf of Suez.
28	"	36 20	7 4	10.10	...	1.02780	"
29	"	36 12	6 35	12 midnight	...	1.02777	East Coast of Spain.
30	"	36 3	6 6	Oct. 21.	2.0 a.m.	1.02771	"
31	Off Gibraltar	35 56	5 35	4.0	...	1.02812	Gulf of Suez.
32	"	36 6	5 23	6.28	...	1.02808	"
33	Off the East Coast of Spain	36 19	4 12	2.27 p.m.	...	1.02783	"
34	"	36 27	3 42	4.41	21.1	1.02789	"
35	"	36 28	3 12	6.24	..."	1.02793	"
36	"	36 33	2 40	7.56	19.7	1.02789	"
37	"	36 43	2 2	9.58	19.5	1.02780	"
38	"	37 1	1 30	12 midnight	...	1.02803	"
39	"	37 20	1 0	Oct. 22.	2.0 a.m.	{ 1.02797	Opposite Aden.
40	"	37 40	0 30	4.0	..."	{ 1.02810	Gulf of Suez.
41	"	38 36	0 17 E.	8.45	..."	{ 1.02820	Opposite Aden.
42	"	38 53	0 35	10.43	..."	{ 1.02835	Near Messina.
43	"	39 4	0 52	12.11	20.6	{ 1.02832	"
44	"	39 22	1 12	1.53 p.m.	19.9	{ 1.02840	Off Aden.
45	"	39 47	1 38	4.1	19.5	{ 1.02851	Near Messina.
46	"	40 9	2 0	6.3	19.3	{ 1.02862	"
47	"	40 30	2 20	7.57	19.4	{ 1.02880	"
48	"	40 52	2 44	10.4	17.8	{ 1.02881	Between Naples and Messina.
49	"	41 12	3 8	12 midnight	..."	{ 1.02882	"
50	"	41 40	3 34	8.1	17.3	{ 1.02895	Between Naples and Messina.
51	"	42 50	4 48	...	1.02891	{ 1.02898	Off Naples.
52	Off Marseilles	43 14	5 12	10.15	17.2	{ 1.02882	Between Naples and Messina.
						{ 1.02900	Suez Canal.
						{ 1.02890	Near Messina.

53	Off Marseilles	43	3 N.	5	34 E.	Oct. 23.	7.45 p.m.	9.35		... 1.02898
54	Between Marseilles and Naples	42	46	5	50					1.02879
55	"	42	40	6	20	Oct. 23.	12 midnight			1.02898
56	"	42	29	6	44	Oct. 24.	2. 0 a.m.			1.02890
57	"	42	16	7	10			4. 0		1.02886
58	"	42	5	7	24					1.02904
59	"	41	44	8	20	10.30		7.49	19.9	1.02904
60	"	41	24	8	55	12.10 p.m.		10.30	20	1.02884
61	"	41	17	9	12			2. 9	19.8	1.02879
62	"	41	12	10	0			4. 0	19.3	1.02881
63	"	41	6	10	34			6. 2	19.0	1.02882
64	Off Naples	41	3	11	6			7.38	20.0	1.02865
65	Between Naples and Port Said . . .	40	57	11	52			10.3	20.5	1.02866
66	Between Naples and Port Said . . .	39	48	14	44	Oct. 26.	8. 1 a.m.	9.59	20.7	1.02863
67	"	38	50	15	20	12.12 p.m.		10.3	19.7	1.02914
68	"	37	58	15	38			4.19	20.6	1.02910
69	"	37	24	17	10			10.3	21.6	1.02898
70	"	37	0	18	16	Oct. 27.	2. 0 a.m.	22	22	1.02899
71	"	36	36	19	18			6. 0	21.6	1.02893
72	"	36	22	19	48			8. 1	20.0	1.02908
73	"	36	12	20	18			10.3	20.4	1.02908
74	"	36	0	20	48			12.13 p.m.	21.1	1.02945
75	"	35	50	21	13			1.58	21.1	1.02959
76	"	35	32	22	2			5.48	20.8	1.02898
77	"	35	19	22	48			8.23	20.5	1.02969
78	"	34	52	23	36			12 midnight	21.3	1.02949
79	"	34	40	24	8	Oct. 28.	2. 0 a.m.	... 1.02988		
80	"	34	16	25	12			8. 2	20.5	1.02988
81	"	33	43	26	44			12.36 p.m.	21.3	1.02979
82	Off Port Said	32	10	30	46	Oct. 29.	4. 0 a.m.	... 1.02940		
83	Between Port Said and Aden	29	35	32	20	Oct. 30.	1.33 p.m.	7. 6	23.4	1.03191
84	"	28	20	33	10			12 midnight	... 1.03061	
85	"	27	30	34	8				1.03072	1.03072

Between Naples and Messina.
Off Naples.

Suez Canal
Between Naples and Messina.
Off Naples.

Near Messina.
Suez Canal.
Suez Canal.

Off Naples.
Naples Harbour.

"
Suez Canal.

"
Suez Canal.

"
Indian Ocean.

"
Indian Ocean.

"
Near Port Said.

"
Gulf of Suez.

"
Suez Canal.

"
Gulf of Suez.

"
Suez Canal.

"
Indian Ocean.

"
Red Sea.

TABLE II.—*continued.*

Place where the sample was taken.	Latitude.	Longitude.	Date when sample was taken, 1908.	Temp. in °C.	Specific gravity.	Place where the specific gravity was measured.
Between Port Said and Aden .	26° 38' N. 25° 36'	34° 38' E. 35° 15'	Oct. 31. 4. 0 a.m. 8.52	25.6	1.03068 1.03067	Red Sea. "
86 " .	25° 36'	35° 45'	11.40	...	1.03020	"
87 " .	25° 0	35° 10'	2.55 p.m. 6.55	27.8 29.0	1.03039 1.03000	"
88 " .	24° 23'	36° 45'	10.32	29.2	1.03008	"
89 " .	23° 30'	36° 10'	12 midnight	...	1.02983	"
90 " .	23° 30'	36° 14'	Nov. 1. 2. 0 a.m. 4. 0	...	1.02993 1.02974	"
91 " .	22° 45'	37° 10'	6. 0	...	1.02981	"
92 " .	22° 25'	37° 25'	9.20	30.0	1.02978	"
93 " .	22° 5	37° 37'	12.20 p.m. 3.55	30.5 31.0	1.02954 1.02942	"
94 " .	21° 40'	37° 55'	9.52	31.0	1.02911	"
95 " .	21° 10'	38° 14'	12 midnight	...	1.02931	"
96 " .	20° 25'	38° 43'	Nov. 2. 2. 0 a.m. 4. 0	...	1.02940	"
97 " .	19° 48'	38° 52'	6. 0	...	1.02929	"
98 " .	19° 5	39° 35'	9. 8	29.0	1.02911 1.02885	"
99 " .	17° 55'	40° 20'	12. 0 a.m. 4. 0	...	1.02892	Indian Ocean. Red Sea.
100 " .	17° 30'	40° 30'	11.35	29.8	1.02864	"
101 " .	17° 13'	40° 42'	3.15 p.m. 7.25	29.3 28.2	1.02817 1.02820	"
102 " .	16° 45'	41° 0	10.12	28.4	1.02856	"
103 " .	16° 18'	41° 18'	12 midnight	...	1.02865	"
104 " .	15° 28'	41° 45'	Nov. 3. 4. 0 a.m. 9.51	...	1.02835	Indian Ocean.
105 " .	15° 2	42° 5	4. 5	...	1.02841	"
106 " .	14° 18'	42° 40'	7.51	...	1.02841	"
107 " .	13° 22'	43° 0	1.18 p.m. 4. 5	...	1.02841 1.02851	"
108 " .	12° 45'	43° 14'	7.51	...	1.02859	"
109 " .	12° 36'	43° 30'	12 midnight	...	1.02865	"
110 Off Aden .	12° 23'	44° 45'	Nov. 3. 4. 0 a.m. 12 midnight	...	1.02835	Indian Ocean.
111 Gulf of Aden .	12° 18'	45° 57'	12. 0 a.m. 4. 0	...	1.02841	"
112 " .	12° 16'	46° 46'	1.18 p.m. 4. 5	...	1.02841 1.02851	"
113 " .	12° 16'	47° 20'	7.51	...	1.02859	"
114 " .	12° 16'	48° 20'	12 midnight	...	1.02865	"
115 " .	12° 16'	49° 20'	12 midnight	...	1.02865	"

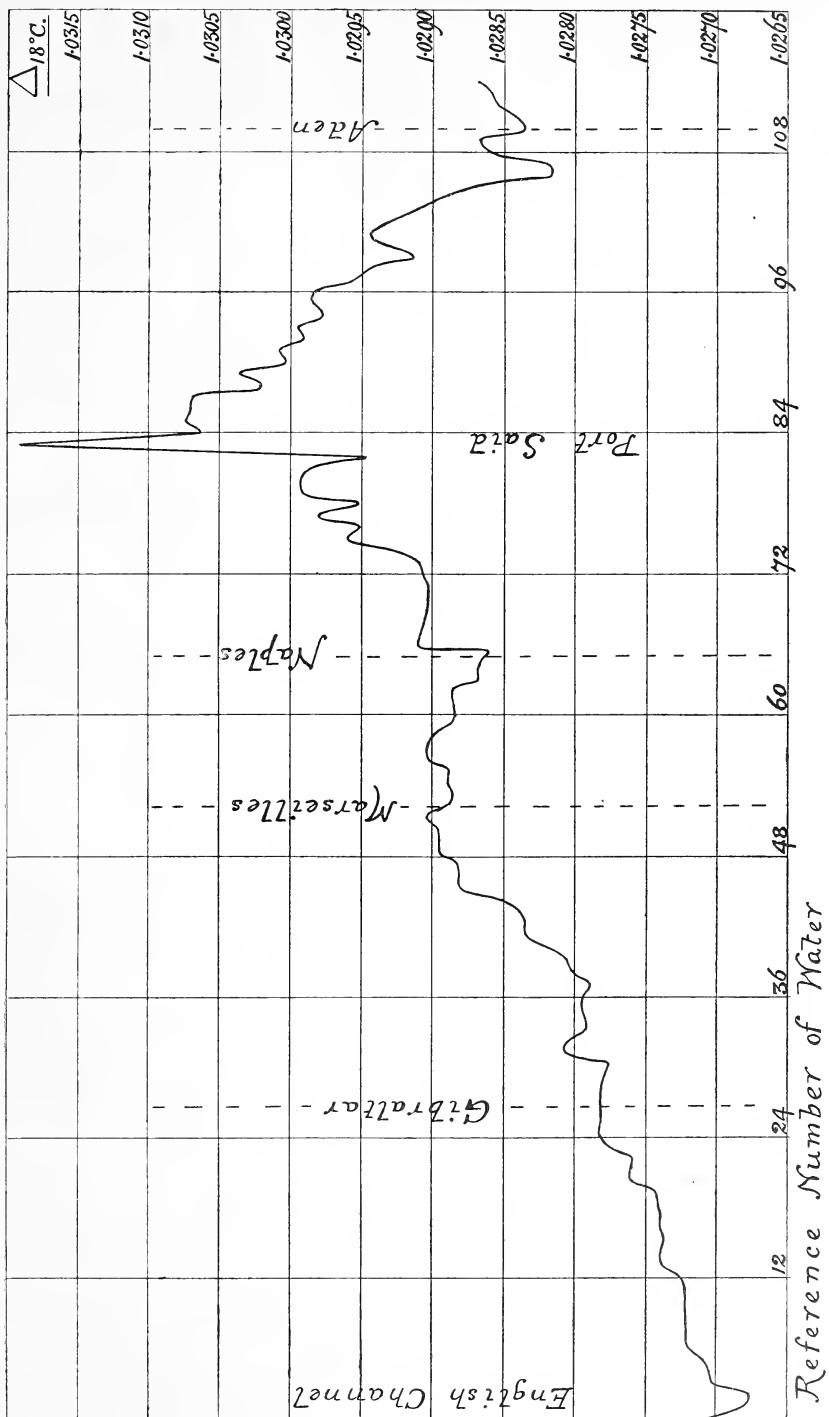


FIG. 4.

avoidable experimental error rather than to the fact that the densities were determined at very different temperatures. During this present investigation the temperature ranged in value from 16° C. in the English Channel to 30° C. in the Red Sea.

All the samples of water examined were taken from the surface at the bow of the ship; and a sample was collected every four hours during the voyage. In Table II., above, are given the latitude and longitude of the spots whence the waters were obtained, the date and hour of the day, and, in most cases, the temperature of the water at the time of collecting. The specific gravities of the 115 sea-water samples taken aboard, are set forth in column 7 in the table; and in the last column are indicated the localities in which the actual specific gravity measurements were made.

Throughout Table II. the "specific gravity" may be taken to mean *the ratio between the densities of the sea-water and air-free distilled water, both at 18° C.*

The variations in the density of the sea-water at the surface and along the ship's track are most conveniently shown according to the plan pursued in the "*Challenger*" Reports. They are therefore plotted graphically in fig. 4.

In conclusion, I wish to thank Mr J. J. Manley for much practical assistance given in Oxford; Dr H. B. Baker; the Government Grant Committee of the Royal Society for a grant to cover the cost of the reading microscope; and Mr M. J. Sarson, second officer of the *Oruba*, for his kindness in furnishing me from time to time with the ship's bearings.

TRINITY COLLEGE,
KANDY, CEYLON.

(Issued separately April 19, 1913.)

XII.—The Absorption of Light by Inorganic Salts. No. IX.: Solutions of Copper, Nickel, and Cobalt Salts in Alcohol and in Acetone. By R. A. Houstoun, M.A., Ph.D., D.Sc., and A. H. Gray, M.A., B.Sc., Research Student in the University of Glasgow.

(MS. received February 10, 1913. Read March 3, 1913.)

THE absorption spectra of the copper, nickel, and cobalt salts investigated in this series do not seem to depend so much upon the acid radical as on the water of solution. The latter evidently plays a very important role, and possibly complicates matters. It was therefore resolved to determine the molecular extinction coefficients in alcohol and in acetone to see whether conditions were any simpler there. It may be stated in advance that they are not.

As these solvents mix with water and with each other, their use seemed to promise an application of the law of mass action similar to that discussed in the sixth article of this series. This was an additional reason for studying them.

Some determinations of molecular extinction coefficients in the ultra-violet for solutions in alcohol have already been made in the eighth paper of the series; the work described in this article is wholly in the visible spectrum.

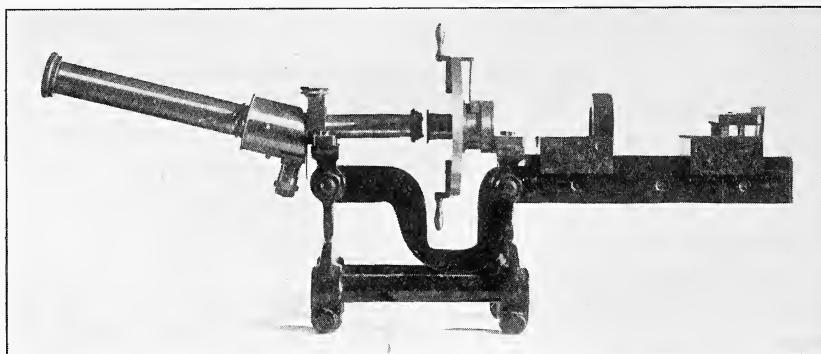
As the spectrophotometer used in previous work was in constant use, it was necessary to get a new instrument, and a convenient and inexpensive one was made to the design of one of us by Messrs R. and J. Beck, at the expense of the Carnegie Trust for the Universities of Scotland. It was exhibited at the recent Optical Convention in London, and is shown in the photograph on p. 138.

The light from the right falls first upon a prism of glass and Iceland spar of the design described in a previous paper,* and is divided into two beams polarised in planes at right angles to one another. These beams are focussed by a lens on the slit, and pass through a polarising prism mounted with a divided circle, before reaching the latter. The sharp edge of the glass and spar prism is vertical and the slit horizontal. The spectroscope was a Beck-Rafferty grating one. The wave-length could be read

* *Phil. Mag.* (6), 15 (1908), p. 282.

off on a milled head. To an eye looking into the telescope two spectra divided by a sharp vertical line were visible, and the intensities of these spectra were matched for any wave-length by rotating the divided circle and polarising prism in front of the slit. The reading of the circle could be seen by the observer reflected in a mirror inclined above it without his having to rise from his seat. This mirror is not shown.

The glass troughs containing the solution and the water against which it was balanced were placed side by side upon the ledge to the extreme right in front of the glass and spar prism, a vertical sheet of zinc with two apertures or windows punched in it intervening to cut off extraneous light. The source of light was a screened incandescent mantle.



The instrument therefore resembles the older one except that the horizontal and vertical planes are interchanged, and the rotating prism is placed in front of the slit instead of in the eyepiece. This necessitates the introduction of the lens to get the vertical edge of the prism in focus at the same time as the slit.

There was more difficulty in making the preliminary adjustments than in the case of the former instrument; and owing to the absence of an arrangement for screening off the sides of the field of view, determinations at the ends of the spectrum were not so accurate. Owing to a grating being used instead of a prism, the field was not so bright as in the former instrument, and this also made work at the ends of the spectrum difficult. On the other hand, the instrument was compact and very rapid to work. The method of taking readings and reducing results was the same as with the older instrument. It was found convenient to keep to a definite set of wave-lengths, corrected for small errors of the scale and for the width of the slit; the range was usually from 432 to 667 $\mu\mu$. In accordance with the design of the prism, the source of light had to be placed about eight inches away from it.

It had been intended originally to study the absorption spectra in alcohol and in acetone of the sulphates, nitrates, chlorides, and bromides of copper, nickel, and cobalt. The three sulphates when reduced to the anhydrous state were found insoluble. The nitrates, which were in the form of deliquescent crystals having the respective compositions $\text{Cu}(\text{NO}_3)_2 \cdot 3\text{H}_2\text{O}$, $\text{Ni}(\text{NO}_3)_2 \cdot 6\text{H}_2\text{O}$, and $\text{Co}(\text{NO}_3)_2 \cdot 6\text{H}_2\text{O}$, could not be obtained in the pure anhydrous state; they fused readily on heating and decomposed before all the water of crystallisation could be driven off. The nickel chloride $\text{NiCl}_2 \cdot 6\text{H}_2\text{O}$ was, like the sulphates, found to be insoluble when dehydrated. The curves in the case of these four substances therefore give A for the salts in solution in a hydrated form; but they were not dissolved in the form given above, the attempt being made to reduce them to lower states of hydration, and the values of A in these cases are put forward more from the qualitative than the quantitative point of view. In the case of these four substances the curves have much the same shape as for aqueous solutions, but the values of A are as a rule much higher.

The other salts were dissolved in the anhydrous state, and the values of A are in this case more characteristic. The pale blue crystals of the dihydrate cupric chloride, $\text{CuCl}_2 \cdot 2\text{H}_2\text{O}$, passed when heated through the green monohydrated phase to the rusty brown anhydrous salt, giving in alcohol an olive-green solution, and in acetone a yellowish-brown solution with a distinct new band in the violet. This solution, like most of those in acetone, did not keep well, turning rapidly to pale yellow and depositing a whitish crust. The cupric bromide, a black anhydrous powder, dissolved with great readiness in both solvents, producing in alcohol a black solution quite opaque except in very thin layers, when it appeared reddish brown. The acetone solution was lighter in colour and more transparent.

The nickel bromide $\text{NiBr}_2 \cdot 3\text{H}_2\text{O}$ * was a light brown powder with traces of green. On heating, it darkened slightly and then turned yellowish, giving a turbid green solution in both solvents, with an insoluble yellowish residue. As with all the other alcoholic solutions of nickel, there was a rapid rise of absorption in the violet.

The cobalt chloride consisted of hexahydrate crystals; the anhydrous salt was a light blue powder giving in both solvents an intense blue solution. The bromide was in the form of dark crimson crystals of the hexahydrate, so deliquescent that they became liquid on exposure to the air. The anhydrous salt was bluish green, giving deep blue solutions,

* We assumed this formula as it gave the percentage of water right. There is a trihydrate mentioned in Watt's *Dictionary of Chemistry*. Hartley, however, describes only two hydrates, the dihydrate and the hexahydrate.

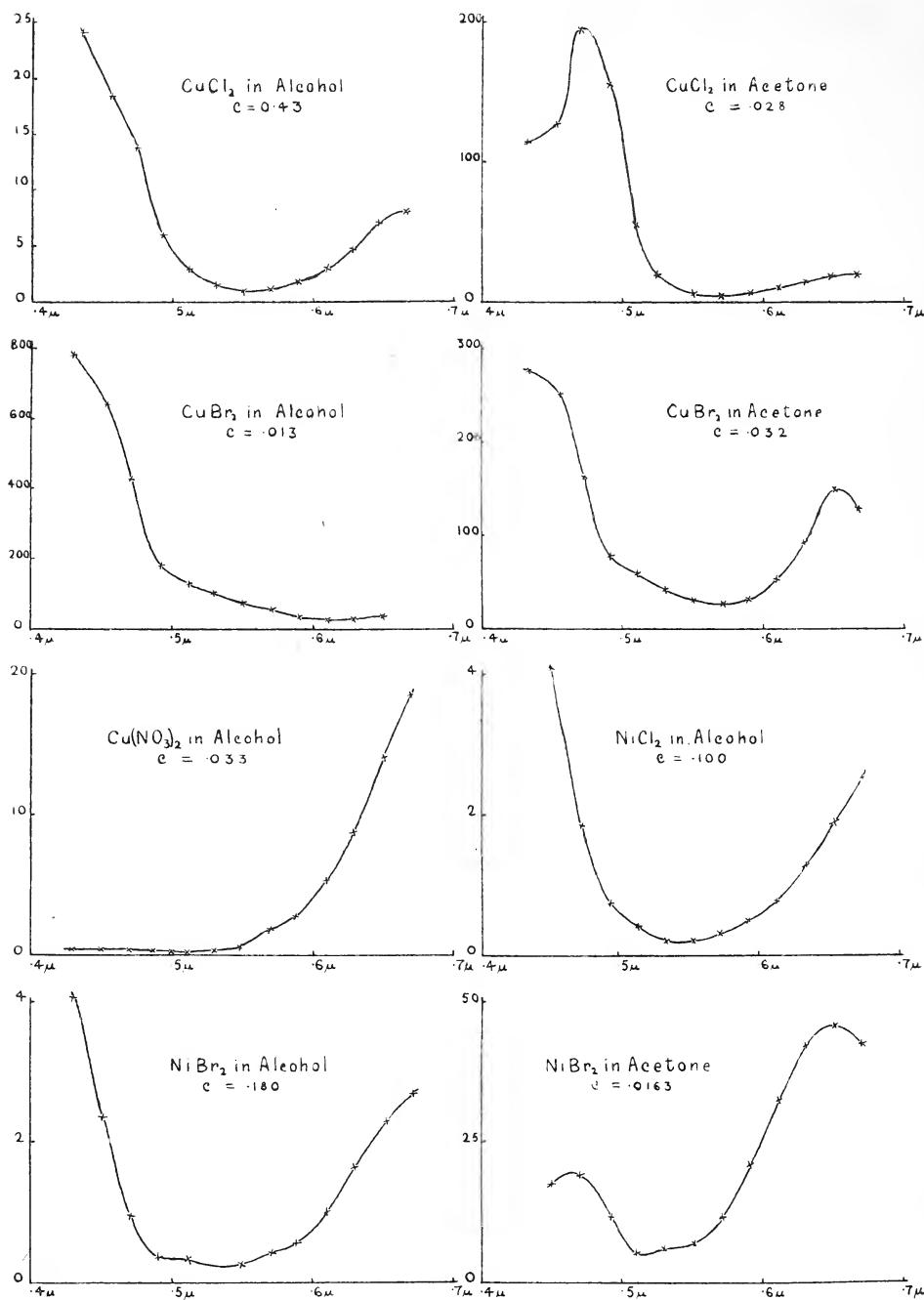


FIG. 1.

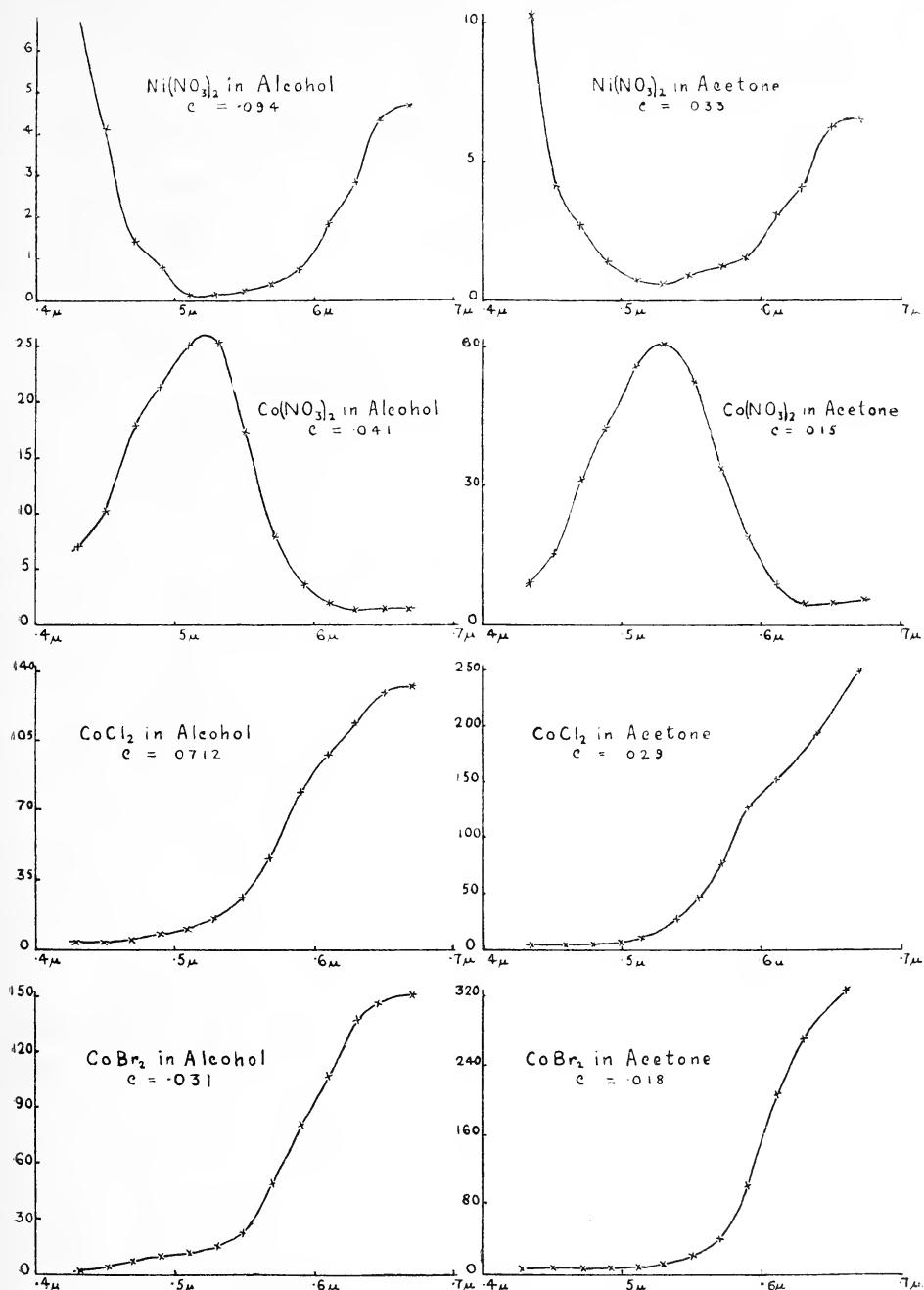


FIG. 2.

purplish in thick layers. Both solutions were extremely hygroscopic. The curves in the case of the last two salts are very similar for both solvents.

The acetone, except that used in the case of nickel bromide and copper bromide, was obtained from Kahlbaum as derived from the sodium bisulphite compound.

The results are given below and the curves in figs. 1 and 2. It will be noticed that the vertical scale is different in almost every case:—

$\lambda.$	CuCl ₂ , Alc. $c=43$ last two $\frac{1}{3}$ strength.	CuCl ₂ , Acet. $c=0284$.	CuBr ₂ , Alc. $c=013$ last $\frac{1}{2}$ strength.	CuBr ₂ , Acet. $c=032$.	Cu(NO ₃) ₂ , Alc. $c=033$.	NiCl ₂ , Alc. $c=100$.	NiBr ₂ , Alc. $c=18$.	NiBr ₂ , Acet. $c=0163$.
667 $\mu\mu$	7.90	19.2	23.2	132	18.6	2.49	2.75	42.4
647	7.22	16.7	22.8	146	14.1	1.82	2.29	46.6
628	4.84	11.8	21.3	95.2	8.90	1.24	1.61	42.7
609	2.93	8.4	28.3	50.5	5.08	.76	1.04	32.9
590	1.61	5.6	37.0	28.4	2.72	.41	.57	20.4
570	.98	5.3	51.6	25.9	1.66	.29	.33	10.1
550	.81	7.0	69.3	31.7	.50	.17	.18	6.16
530	1.40	19.7	91.9	39.7	.12	.19	...	5.55
510	2.89	57.2	125	59.4	.07	.40	.29	4.84
490	5.79	155	170	72.6	.27	.77	.30	11.9
471	13.7	196	431	169	.53	1.82	.95	19.5
452	18.3	126	647	257	.25	4.08	2.31	18.1
432	24.0	116	794	280	.43	...	4.04	...

$\lambda.$	Ni(NO ₃) ₂ , Alc. $c=094$.	Ni(NO ₃) ₂ , Acet. $c=033$.	Co(NO ₃) ₂ , Alc. $c=041$.	Co(NO ₃) ₂ , Acet. $c=029$.	CoCl ₂ , Alc. $c=0712$.	CoCl ₂ , Acet. $c=029$.	CoBr ₂ , Alc. $c=031$.	CoBr ₂ , Acet. $c=0179$.
667 $\mu\mu$	4.61	6.49	1.18	4.31	131	254	151	...
647	4.22	6.44	1.22	3.84	129	226	146	325
628	2.74	3.92	1.13	3.77	112	190	142	274
609	1.09	3.12	1.88	8.50	95.2	146	106	205
590	.57	1.25	3.56	18.4	76.3	116	80.8	99.2
570	.33	1.21	7.60	33.0	46.0	70.6	55.5	42.4
550	.12	.80	16.93	52.4	26.9	40.3	19.3	17.2
530	.002	.53	25.3	61.4	14.3	18.2	13.9	11.5
510	.001	.58	25.2	54.8	9.83	7.5	11.0	8.9
490	.72	1.37	21.0	42.3	7.01	4.10	7.89	7.3
471	1.42	2.52	17.8	30.7	4.92	3.40	5.53	7.5
452	4.04	6.88	10.3	15.5	3.58	2.13	2.99	5.9
432	6.77	11.4	7.39	9.41	3.36	2.11	2.57	6.1

The values for cobalt chloride in acetone were taken by Mr Cochrane with the other spectrophotometer.

The curves bring out the important facts that when copper, nickel, and cobalt salts are dissolved in alcohol or acetone their absorption is much greater, and at the same time is much more characteristic of the molecule

than when they are dissolved in water. The absorption in alcohol and acetone varies very greatly from one salt to another of the same base, and seems to depend more on the bonds connecting the base to the acid radical.

An attempt was next made to apply the law of mass action to some of the colour changes. The case of cobalt chloride in alcohol has already been dealt with in the sixth article. Only three of the other salts used here seemed suitable for investigation in this respect, namely, copper chloride, copper bromide, and cobalt bromide.

Three solutions of copper chloride in alcohol were prepared of concentrations .600, .300, and .150 respectively, and the change in A at the point $\lambda = 470 \mu\mu$ caused by the addition of water was examined. This point

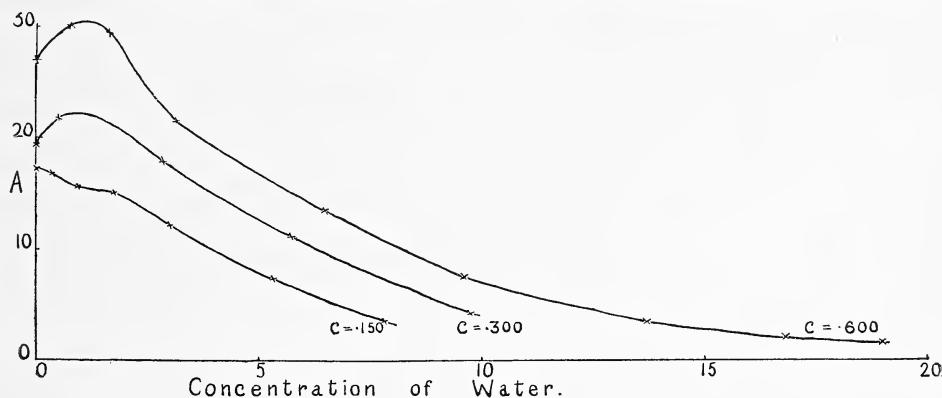


FIG. 3.

was selected as being the farthest into the violet for comfortable observation. The changes in A are shown by the curves in fig. 3. The abscissæ give the concentration of the water present in the solution in gm.-mols. per litre.

The solution gradually changed from olive-green to pale green; and when a certain concentration of water was reached, it became blue and began to throw down a colloidal precipitate which could not be removed by filtration. Moreover, the curves show a peculiar behaviour on the first addition of water, the absorption first rising and then falling asymptotically to the aqueous value. Conditions thus seemed too complex for mathematical investigation.

Copper bromide also gave trouble owing to the absorption varying much too rapidly on the addition of water, a single drop of water added to 8 c.cs. alcoholic solution bringing A down almost to its value for water. The solutions of copper chloride and copper bromide in acetone decomposed slightly on the addition of water.

There remained, therefore, only cobalt bromide. Here again the solution in acetone gave trouble owing to decomposition. We also inferred from the fact that cobalt bromide dissolved in acetone with great evolution of heat that some chemical change took place on solution.

We expected the results in the case of the alcoholic solution of cobalt bromide to be analogous with the case of the chloride. After making up three solutions of anhydrous cobalt bromide in alcohol, of concentrations .218, .109, and .054, and selecting the point $\lambda = 628 \mu\mu$ for observation of the changes in A, water was gradually added to each solution until the

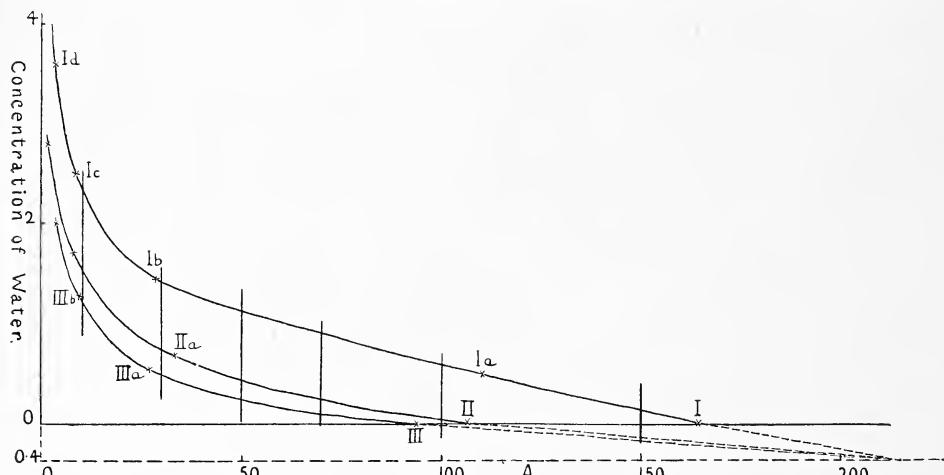


FIG. 4.

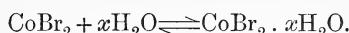
colour had changed through the various stages from deep blue to the pink colour existing in dilute aqueous solution. Thus solutions I_a, I_b, etc., were obtained from I, and solutions II_a, II_b, etc., from II. The different values of A, together with c_1 , the concentration of the salt, and c_2 , the concentration of the added water, both expressed in gm.-mols. per litre, are given below:—

Solution.	A.	c_1 .	c_2 .	Solution.	A.	c_1 .	c_2 .	Solution.	A.	c_1 .	c_2 .
I.	164	.218	.4	II.	106	.109	.4	III.	94	.054	.4
I. _a	109	.217	.86	II. _a	34.6	.108	1.09	III. _a	26.2	.054	.98
I. _b	28.4	.216	1.80	II. _b	7.13	.107	2.02	III. _b	9.83	.053	1.68
I. _c	9.39	.212	2.89	II. _c	2.01	.104	3.18	III. _c	3.83	.052	2.40
I. _d	2.87	.210	4.04	II. _d	.73	.095	9.40	III. _d	1.05	.052	4.07
I. _e	.89	.182	11.77					III. _e	.61	.047	10.27

A is plotted as a function of c_2 in the curves in fig. 4, the higher concentrations of water being omitted to save space.

The curves when produced back met at first at the point $c = -4$, $A = 215$. From this we inferred, as was done in the sixth article, that the alcohol contained $\cdot 4$ gm.-mol. of water per litre, or $\cdot 95$ per cent. by weight, and that the value of A for a solution in pure alcohol would be 215. The values of c_2 given in the preceding table are the corrected values, *i.e.* they include this percentage of water present originally in the absolute alcohol.

Assuming that the change in colour is due to the formation of a hydrate of the form $\text{CoBr}_2 \cdot x\text{H}_2\text{O}$ at the expense of the anhydrous salt, and that the alcohol takes no active part in the change, we shall have a balanced action between the anhydrous salt, uncombined water, and hydrate of the form



Assume that $A=0$ for the hydrated phase at the wave-length in question. Then the active mass of the anhydrous phase is $\frac{A}{215}c_1$, and therefore that of the hydrated phase $(1 - \frac{A}{215})c_1$. The active mass of the free water is $c_2 - x(1 - \frac{A}{215})c_1$.

The law of mass action gives

$$\frac{\frac{A}{215}c_1 \left\{ c_2 - x\left(1 - \frac{A}{215}\right)c_1 \right\}^x}{\left(1 - \frac{A}{215}\right)c_1} = k = \frac{\frac{A'}{215}c'_1 \left\{ c'_2 - x\left(1 - \frac{A'}{215}\right)c'_1 \right\}^x}{\left(1 - \frac{A'}{215}\right)c'_1},$$

where k is a constant and the dashes denote another set of possible values of A , c_1 , c_2 .

Put $A = A'$ and multiply up. Then

$$x = \frac{c_2 - c'_2}{\left(1 - \frac{A}{215}\right)(c_1 - c'_1)}.$$

To calculate x , ordinates were drawn in fig. 4 at $A = 10, 30, 50, 70, 100$, and 150, and the length of the ordinate intercepted between curves I. and III. or between the curves I. and II. measured. This gave $c_2 - c'_2$ for the value of A in question. c_1 varies very slowly along each curve, and the value of $c_1 - c'_1$ could be obtained from the preceding table. The table on p. 146 gives the results.

Of course, the results depend on the way the curves are drawn through the points in fig. 4. They could easily be drawn to secure more consistent

values of x ; but when we consider the difficulty of measuring A , c_1 , and c_2 —for, to economise material, small quantities were used,—the results, such

A.	x .	
	From I. and III.	From I. and II.
150	7·5	9·2
100	6·6	8·9
70	6·2	7·4
50	6·4	7·3
30	6·1	6·3
10	7·3	9·1

as they are, make it very probable that the colour change is due to the anhydrous bromide changing into the hexahydrate.

(Issued separately April 22, 1913.)

XIII.—The Absorption of Light by Inorganic Salts. No. X. By
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Thomson Experimental Scholar in the University of Glasgow.

(MS. received February 10, 1913. Read March 3, 1913.)

THIS article describes work done with the spectrophotometer with the view of clearing up four different points arising out of the earlier articles. At the time it was commenced, the hope of finding a mathematical connection between absorption spectra and molecular structure had been given up, at least so far as aqueous solutions were concerned, and attention was being concentrated on some chemical questions arising out of the work. The spectrophotometer and methods were the same as had been used in Nos. II., III., VI., VII., and VIII. of the series.

The first point investigated was in connection with the acetates of cobalt, nickel, and copper. Ewan * had stated that the molecular extinction coefficient of copper acetate varied with the wave-length in the same way as the molecular extinction coefficients of the other salts of copper, but that its value was always roughly two and a half times as great as the values of the latter. He worked with solutions the concentrations of which were of the order .004 gm.-mol. per litre.

Our results are tabulated on p. 148: c is the concentration in gm.-mols. per litre, A the molecular extinction coefficient, and λ the wave-length. In the graphs the heavy curves are for the acetates, the dotted ones are for the corresponding sulphates. The values for the cobalt and nickel sulphates are taken from Nos. II. and III. respectively of this series, those for copper sulphate from a paper by Grünbaum.†

The difference in the behaviour of copper acetate from the other acetates is possibly to be found in a different chemical constitution. It has recently been shown that ferric acetate is not so simple as was formerly thought.‡

* *Proc. Roy. Soc.*, 56, p. 286 (1894); *ibid.*, 57, p. 117 (1895).

† *Ann. d. Phys.* (4), 12, p. 1004 (1903).

‡ R. F. Weinland and E. Gussmann, *Zs. f. an. Chem.*, 66, p. 157 (1910), 67, p. 250 (1910).

The next point investigated was in connection with the so-called "colour of the ions." Various physical chemists have ascribed the colour

VALUES OF A FOR THE ACETATES.

$\lambda.$	Cobalt. $c = .288.$	Nickel. $c = .264.$	Copper. $c = .068.$
703 $\mu\mu$.22	2.35	20.4
670	.27	2.26	15.5
640	.33	1.80	10.6
614	.44	1.07	6.21
592	.68	.54	3.83
573	1.22	.31	2.56
556	2.47	.22	1.70
540	4.37	.17	.98
525	5.94	.14	
512	6.30	.13	
502	5.84	.09	
492	5.48	.12	
482	4.99	.21	
473	4.29	.32	
465	3.67	.42	
457	2.79	.51	
450	2.15	.62	
443	1.63	.88	
438	1.10	1.31	

changes of cobalt, nickel, and copper salts to ionisation. In Ostwald's *Solutions*, translated by M. M. Pattison Muir, 1891, pp. 269-270, we find stated :—

"The colours of salt solutions are essentially the colours of the parts of molecules, or ions, contained therein, and all salt solutions which contain a certain ion must exhibit the characteristic colour of that ion. Should the expected colour not appear, we may conclude that the corresponding ion is absent.

"The red colour of dilute solutions of cobalt salts, for instance, indicates the presence of cobalt ions. The sulphate, nitrate, chloride, etc., have the same colour in solution; the colour is independent of the nature of the negative ions. If one of these solutions is boiled with an excess of potassium cyanide, it is decolorised, and the colourless solution no longer shows the reaction of cobalt. The compound potassium cobalticyanide, $K_3Co(CN)_6$, has been formed; free cobalt ions are no longer present. The green colour of a solution of a nickel salt is changed, under the same conditions, to yellow, which shows that the nickel ions have entered into combination.

"If the foregoing statement is correct, the intensity of the coloration of those salts whose ions are coloured must be proportional to the quantity

of these ions, supposing that the part of the salt which is not separated into ions is itself colourless. . . .

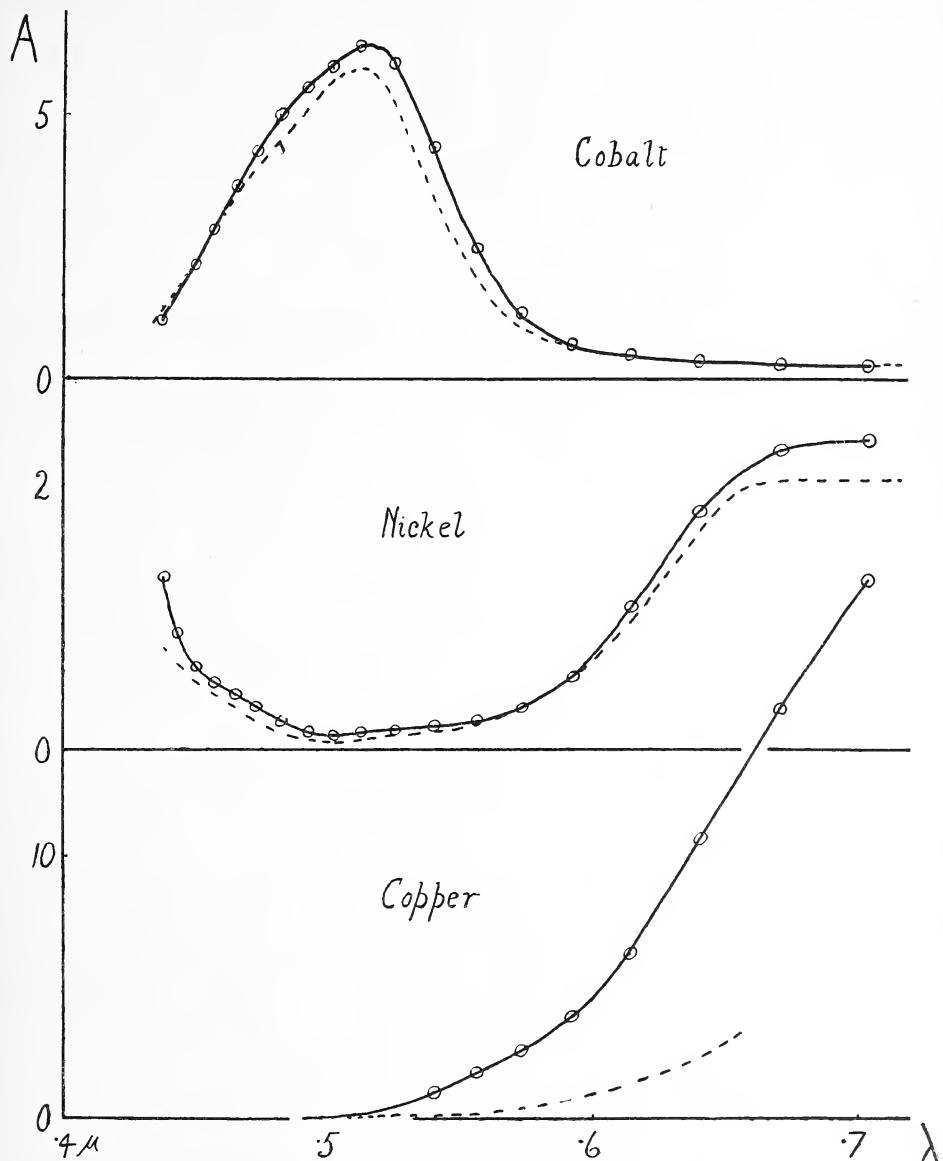


FIG. 1.

"Special relations are observed when the non-dissociated salt is itself coloured. For instance, anhydrous cupric chloride, or a solution of this salt in indifferent solvents (wherein it is not separated into ions), is coloured intensely yellowish brown. Therefore, a solution of this salt, which

contains many free ions of Cu and Cl, but also a certain amount of undissociated molecules of CuCl_2 , will show the colour which results from mixing the blue of the copper ions with the yellow of copper chloride. Agreeably with this conclusion, it is found that very dilute solutions of copper chloride—which contain but little undissociated salt—are blue, like solutions of other copper salts; but the more concentrated the solutions become, and therefore the greater becomes the number of the non-dissociated molecules, the more does the colour incline towards green. Addition of hydrochloric acid, or raising the temperature, acts in the same way as increasing the concentration; in both cases, the number of copper ions decreases, and the number of the undecomposed molecules of copper chloride increases."

Whetham also (*Theory of Solution*, 1902, p. 334), referring to the additive properties of electrolytic solutions, says:—

"Similar phenomena appear when we study the colour of a salt solution, which is found to be produced by the superposition of the colours of the ions and the colour of the undissociated salt. If the absorption spectra of a series of coloured salt solutions containing a common ion are examined, the additive character of the colour is well seen, the absorption bands due to the common constituent being unaffected by the presence of the other part of the salt. The light transmitted through a solution is composed of all these rays which have been absorbed by neither constituent. Anhydrous cobalt chloride is blue, while in cold aqueous solution all cobalt salts are red. Red, then, is the colour of the cobalt ion, and only appears when the salt is more or less dissociated. When cobalt chloride is dissolved in alcohol, the conductivity is very low, showing very incomplete ionisation. The colour is, accordingly, the blue of the undissociated salt. If we slowly add water to this solution, the ionisation gradually increases, and the colour changes to purple and then red. An aqueous solution, boiled with potassium cyanide, is decolorised, for a cobaltcyanide, $\text{K}_3\text{Co}(\text{CN})_6$, has been formed; the ions of this compound are 3K and $\text{Co}(\text{CN})_6$; the free cobalt ions no longer exist, and the solution ceases to respond to the usual tests for cobalt. That the red colour is really due to the ionisation, and not to a hydrate formed between the cobalt salt and the solvent, is indicated by the additive nature of the phenomena; for, like many other properties, the colour of non-electrolytes depends on the constitution and is not additive."

The results given in this series—which are really the first *systematic measurements* on the subject—had shown that ionisation had really nothing to do with these changes, but the question of ionisation was only treated

as a side issue in the earlier papers. It was thought advisable to collect here the data already obtained and to obtain additional data.

Our previous work in the case of cobalt and nickel, and our work together with that of Müller * in the case of copper, showed that the molecular extinction coefficient did not vary much with the concentration except at special points in the spectrum. Such points we might term "sensitive points." At these points the change was much more marked in the case of the chlorides and bromides than in the case of the sulphates and nitrates. The data for these points are collected in the table on p. 152, and illustrated in the following curves. The values for nickel chloride are taken from No. III. of this series, for ferric chloride and bromide from No. VII., and those for copper bromide from Müller's observations. The other values are new.

For the sake of comparison, the variation with the concentration of μ_0 , the molecular electrical conductivity at 0° C., as determined by Jones,† is shown.

Our results show that the molecular extinction coefficient remains constant at low concentrations—the single exception in the case of ferric chloride is due to the formation of colloid hydroxide—and increases asymptotically as saturation is reached. Its behaviour is of a totally different nature from that of the molecular conductivity, as may be seen from the curves. There is thus no connection between the two quantities. Ionisation has thus nothing whatever to do with the colour changes of cobalt, nickel, copper, and iron salts. What is more, supporters of the theory of ionic dissociation are forced to assume that the ions have exactly the same colour as the neutral molecules.

The third point investigated was the absorption of anhydrous cobalt chloride dissolved in acetone, and the effect on the absorption of adding water to this solution. In the sixth article, it will be remembered, the effect of adding water to an alcoholic solution was studied very fully, and the results were explained by the law of mass action on the assumption that the alcoholic solution gave the spectrum of the anhydrous salt, and that when water was added a polyhydrate was formed.

When anhydrous cobalt chloride is dissolved in acetone the solution does not keep well. On standing it deposits a black residue; also there is trouble due to evaporation. Consequently the values of A could not be determined very exactly. However, they agree roughly with the values obtained with ethyl alcohol as solvent.

* *Ann. d. Phys.* (4), 12, p. 767 (1903).

† "Hydrates in Aqueous Solutions," Jones, *Publication No. 60 of the Carnegie Institution of Washington.*

<i>c.</i>	A.	<i>c.</i>	A.
Cobalt Chloride. $\lambda = 683 \mu\mu.$		Cobalt Bromide. $\lambda = 703 \mu\mu.$	
4·0	2·83	3·73	.76
3·11	.88	3·50	.56
2·74	.51	3·19	.37
2·47	.39	2·76	.21
2·17	.32	1·87	.17
1·65	.29	.93	.13
.547	.21	.46	.25
.321	.30		
Nickel Chloride. $\lambda = 434 \mu\mu.$		Nickel Bromide. $\lambda = 434 \mu\mu.$	
4·09	2·65	4·40	3·9
3·07	1·64	3·98	3·0
2·27	1·27	3·23	2·30
2·05	1·17	2·04	1·55
.76	1·03	1·56	1·41
.34	.96	.79	1·24
.17	.94	.38	1·21
.085	1·04		
Copper Chloride. $\lambda = 466 \mu\mu.$		Copper Bromide. $\lambda = 541 \mu\mu.$	
4·03	2·38	1·390	1·81
3·62	1·69	.737	.322
2·92	1·02	.545	.230
2·66	.75	.059	.165
2·02	.32		
1·61	.15		
.93	.07		
.54	.03		
Ferric Chloride. $\lambda = 529 \mu\mu.$		Ferric Bromide. $\lambda = 653 \mu\mu.$	
2·0	6·85	1·0	11·8
1·5	3·53	.7	2·52
1·0	1·72	.4	.66
.7	1·26	.1	.37
.4	1·26		
.1	1·68		
.01	3·49		

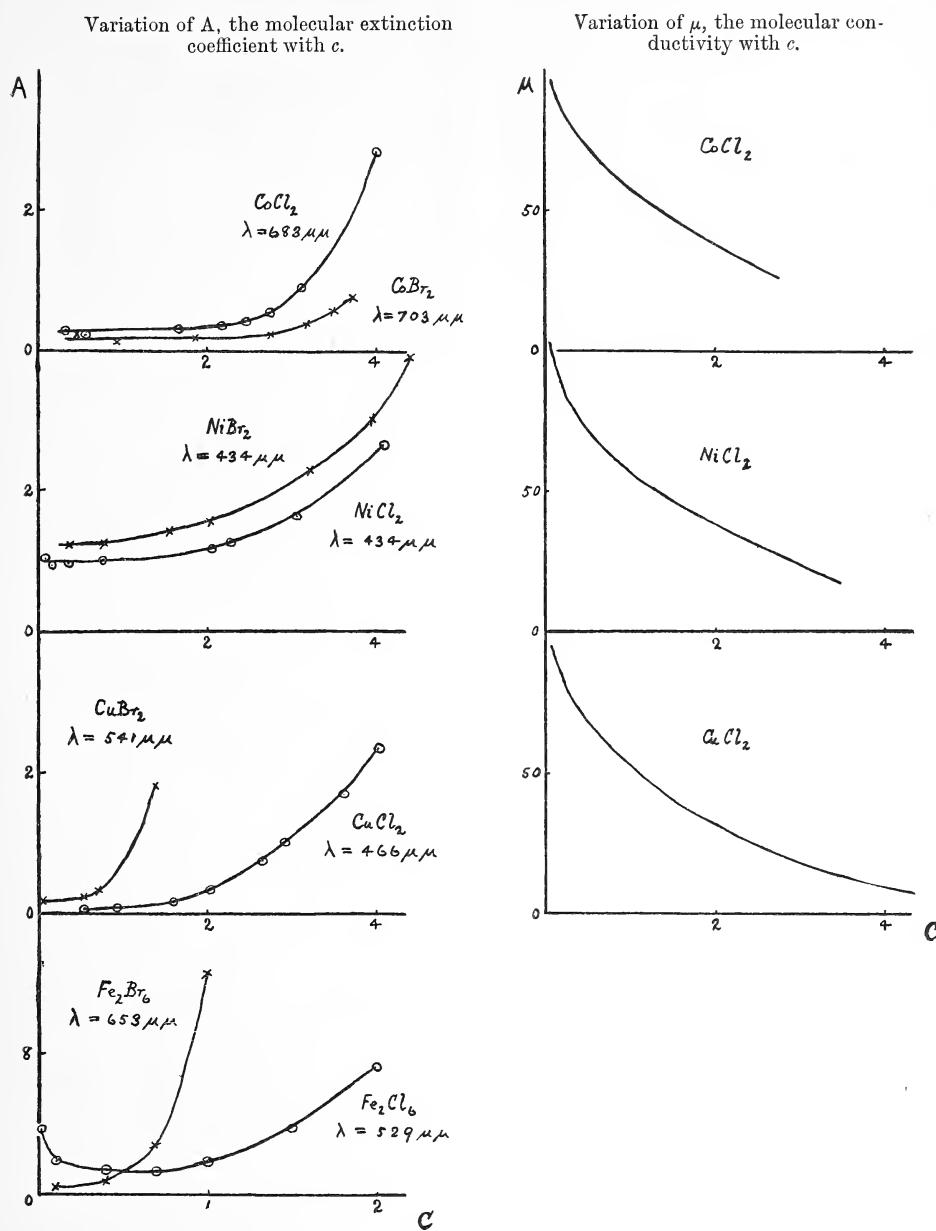


FIG. 2.

But the effect of adding water is quite different, as may be seen from the following table and curve. The concentration of the cobalt chloride in gm.-mols. per litre is denoted by c_1 , and that of the water in the same units by c_2 . The increase of c_1 in the table is due to loss of acetone by evaporation.

EFFECT OF ADDING WATER TO A SOLUTION OF ANHYDROUS COBALT CHLORIDE IN ACETONE.

c_1 .	c_2 .	A.
.0508	.00	253
.0508	.63	257
.0523	1.08	257
.0508	1.68	252
.0508	2.51	240
.0496	4.20	210
.0472	6.46	77

It is probable from the shape of the curve that the anhydrous cobalt chloride combines with the acetone. We did not expect this result. It did not seem worth while to attempt to apply the mathematics of mass action to our data.

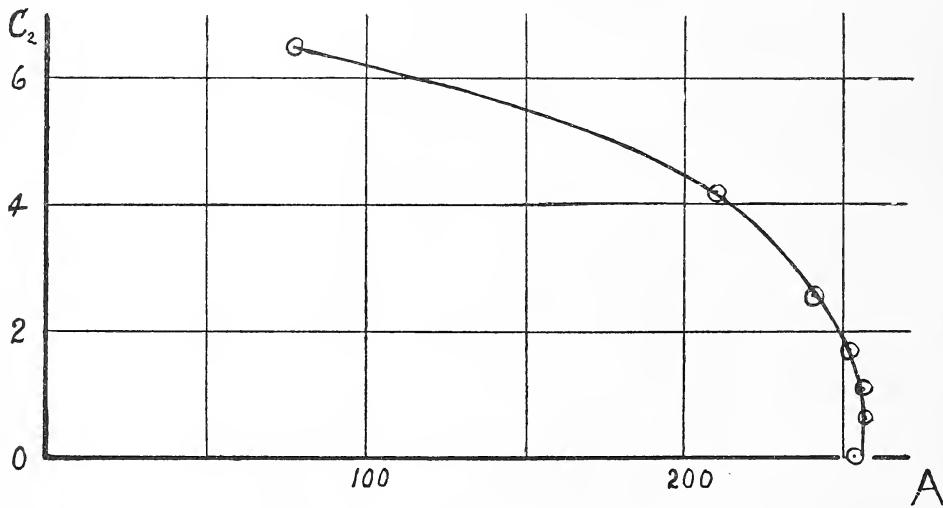


FIG. 3.

Finally, the case of the cobalt and nickel iodides was studied afresh. It will be remembered that according to the fourth article these salts gave an enormous absorption in the ultra-violet similar to that of an aqueous solution of iodine, A for nickel iodide agreeing roughly with its

value for iodine, and A for cobalt iodide varying in the same way with the wave-length, but having only a value about one-fifth as large. Also the absorption of a cobalt iodide solution two months old was very much less than this.

Our intention here was to study the effect on A of varying c for a wave-length sufficiently far in the violet to get the beginning of the characteristic absorption, but still one suitable for use with the spectrophotometer. The wave-length $\lambda=434 \mu\mu$ was selected. It was found, however, that different solutions gave contradictory results, the solution prepared from iodide that had been standing a long time in the bottle giving a much greater absorption. Investigation showed that this was due to the formation of the oxyiodide and the consequent liberation of free iodine. When the solution was made up, the oxyiodide being insoluble was removed by filtration and the iodine went into solution.

Nickel oxyiodide has been described by Erdmann,* and the formula $\text{NiI}_2\text{NiO}15\text{H}_2\text{O}$ has been fitted to the results of his analysis in Morley and Muir's *Dictionary of Chemistry*. Cobalt oxyiodide seems, according to Hartley,† to have the formula $\text{Co}_2\text{I}_3\text{O}$.

Our results in the second, third, and fourth articles thus require correction. Apparently the absorption of the pure iodides should not differ from that of the sulphates. Iodine itself absorbs so strongly, and it seems almost impossible to avoid a trace of iodine in the solution, so no attempt was made to determine again the absorption of the pure iodides. Indeed, the formation of the oxyiodide shows itself by the colour of the solution long before the precipitate has become appreciable.

It should be noticed that in the third article we used $\text{NiI}_26\text{H}_2\text{O}$, the commonly accepted formula for the usual hydrate of nickel iodide. Hartley ‡ has shown it to be $\text{NiI}_27\text{H}_2\text{O}$.

* *Journ. f. prak. Chem.*, 7, p. 254 (1836).

† *Journ. Chem. Soc.*, 12 (1874).

‡ *Dublin Trans.* (2), 7, p. 264 (1900).

XIV.—The Absorption of Light by Inorganic Salts. No. XI.:
Conclusion. By R. A. Houstoun, M.A., Ph.D., D.Sc.

(MS. received February 10, 1913. Read March 3, 1913.)

In this paper a short account will be given of the present state of the theory of the absorption of light, with special reference to the results gained in this series of investigations.

Theories of the dispersion of light may be divided into two classes: (1) those in which the body is regarded as consisting of particles which vibrate under the influence of the light wave; and (2) those in which the body is regarded as consisting of obstacles which diffract the light wave. According to (2), light is scattered, not absorbed; a wave going through the body diminishes in intensity, but the energy lost is radiated out laterally without change of wave-length. Theories of the second type can account for a genuine absorption only by supposing light to be absorbed inside the obstacles according to theories of the first type.

If we consider the theories of the first type we find that the particles may do either of two things. They may move about against a frictional resistance. This is the case of a metal. Or they may vibrate about their positions of equilibrium. In this case we get an absorption spectrum with well-defined bands, and this case alone will be considered in the present paper.

Since the time of Sellmeier there have been various theories of dispersion founded on the principle of sympathetic vibrations.* The best-known modern theories are those of H. A. Lorentz, Drude, and Planck. They assume that in the body various classes of electrons are in vibration, and from the equations of motion of these electrons they obtain the dielectric constant of the medium, and hence its index of refraction and coefficient of absorption. Though these theories agree in the main, they differ in some minor points. According to Drude, it is the electric intensity of the incident wave that acts on the electrons; according to Lorentz and Planck, it is the latter modified by the presence of the surrounding electrons. Drude makes no attempt to explain the friction term in the equation of motion of the electrons. Lorentz explains it by the transfer-

* For a clear account of them all, cf. Professor Pflüger's monograph in the fourth volume of Kayser's *Spectroscopie*.

ence of energy into the free vibrations by impact;* while Planck puts it down to loss of energy by electromagnetic radiation.

When we examine these theories and ask what they have done, we find that they have explained:

- (1) anomalous dispersion;
- (2) the Faraday rotation of the plane of polarisation in a magnetic field and its connection with the absorption bands;
- (3) the Zeeman effect observed on the absorption lines of certain crystals of the rare earths at low temperatures.

Of course, to explain these facts it is not absolutely necessary to introduce electrons and ions, although they are much more satisfactory to the mind and give it something concrete to take hold of; more cautious theorists like Voigt, who discovered (2), have merely supposed that part of the electric displacement satisfied the differential equation for forced vibrations. But the numerical results obtained by the assumption of electrons and ions are very suggestive, and the more general standpoint has found relatively few supporters.

The question naturally arises as to the relation which the electrons concerned in the theory of dispersion bear to the atoms and the molecules. This question has been tackled by Drude.† From the constants of the dispersion formulæ for hydrogen, fluorite, quartz, carbon disulphide, etc., he sought to show that in every molecule there was a number of negative electrons equal to the sum of the valencies of the atoms in the molecule, and also an ion or ions with a total positive charge equal to the sum of the negative charges. The periods of the electrons were in the ultra-violet, and the periods of the ions were in the infra-red.

Drude's results are certainly very interesting. He cannot, however, be said to prove them, but only to make them plausible. Indeed, the general opinion in this country seems to be that it is asking too much of any theory of dispersion to demand information on such points; that while it may give the general features right enough, it must not be pushed into too much detail.

Drude's views about the absorption bands in the infra-red being caused by the vibrations of ions or charged atoms have recently received an interesting confirmation from another side. Einstein ‡ assumed that the

* It is explained in the same way in Schuster's *Optics*.

† "Optische Eigenschaften und Electronentheorie," Drude's *Ann.*, 14, pp. 677-726, pp. 936-961 (1904).

‡ A. Einstein, "Eine Beziehung zwischen dem elastischen Verhalten und der spezifischen Wärme bei festen Körpern mit einatomigen Molekülen," *Ann. d. Phys.* (4), 34, p. 170, 1911.

forces under which the atoms of a body oscillate are merely the ordinary elastic forces of the body, and hence calculated the free periods of the atoms from the compressibility of the body. The wave-lengths obtained varied from 45μ to 168μ . He found that these oscillations were very rapidly damped.* Also, from his theory of the variation of atomic heat with temperature and Nernst's observations on the latter for silver, he was able to calculate the wave-length of the radiation emitted by silver in the infra-red, and found it to be 90μ , while the value given by elastic considerations was 73μ . He was not able to make the test for other substances owing to the want of the necessary data.

A class of electrons executing harmonic vibrations causes an absorption band, and influences the index of refraction in the neighbourhood of the absorption band. Drude's results were all derived from this influence on the refractive index. It occurred to me that the results would be more certain if derived from the variation of the coefficient of absorption throughout the band itself, and in 1909† I derived the following formula for this purpose:

$$\frac{pe}{m} = 1.300 \frac{\nu \bar{\kappa}}{c} \frac{(\lambda_1 - \lambda_0)}{\lambda_0^3}.$$

This formula applies to any well-defined absorption band. $\bar{\kappa}$ is the maximum value of the coefficient of absorption in the band, κ being defined by

$$I = I_0 e^{-\frac{4\pi\kappa d}{\lambda}},$$

where d is the thickness of a layer and I_0, I the intensities of the wave before and after traversing the layer. λ_0 is the wave-length of the maximum; ‡ λ_1 the wave-length for which κ has half its maximum value; c the concentration of the colouring matter in gm.-mols. per litre; ν the refractive index of the substance; e the unit charge of electricity; m the mass of its carrier; and p the number of such vibrators per molecule of colouring matter. The formula is applicable only if the class of electrons or ions concerned is uninfluenced by any others in the field. I nevertheless

* A. Einstein, "Elementare Betrachtungen über die thermische Molekularbewegung in festen Körpern," *Ann. d. Phys.* (4), 35, p. 679, 1911.

† "On the Mechanism of the Absorption Spectra of Solutions," *Proc. Roy. Soc., A*, 82, p. 607, 1909.

‡ I think that, in view of the difficulties connected with the friction term and the irregularities in the shape of the bands, it is an over-refinement to distinguish between the maximum of absorption and the natural free period of the electrons. Cf. Havelock, "Optical Dispersion: a Comparison of the Maxima of Absorption and Selective Reflection for Certain Substances," *Proc. Roy. Soc., A*, 86, p. 1, 1911.

applied it to all the well-defined absorption bands on which I could get measurements, and found :—

(1) For certain aqueous solutions of anilin colouring matters pe/m was of the order $1\cdot8 \times 10^7$. These bands are therefore caused by the vibrations of electrons, and in their case the assumptions of the theory are justified.

(2) For the same colouring matters in certain other solvents pe/m was much smaller.

(3) The values for aqueous solutions of cobalt chloride and uranyl nitrate suggested ions.

It is somewhat arbitrary to assume, as is usually done, that in the medium there are only electric doublets or single electrons and ions, and not more complicated systems. I felt that progress could not be made unless more complicated systems were constructed. Drude's results for fluorite, etc., seemed to supply a clue. Simply by taking the chemical formula and assuming that the chemical bonds were electric forces and acted something like spiral springs, I thought it might be possible to get systems giving the spectra of some simple molecules. But it was no use constructing such systems until there were observations available to test them with. My purpose in starting the experiments recorded in this series of articles was to get such observations. The salts experimented on were chosen so as to make the working out of a dynamical constitution for them as easy as possible. For example, if the fluorides, chlorides, bromides, and iodides of cobalt and nickel had dynamical constitutions of the nature I supposed, there would be some sort of progression in the spectrum as the weight of the acid radical increased. I hoped before starting the observations that water of hydration and of solution would not have much influence on the spectra—that in the case of cobalt chloride, for example, the chief absorption bands would depend only on the bonds connecting the cobalt and chlorine atoms.

The results obtained in the first four articles of the series and represented in the curves given on pp. 540 and 541 of the third article completely shattered this view. These curves showed that the effects of the acid radical and base were roughly additive. In the six nickel salts we had three bands occurring throughout at the same places in the spectrum. In the six cobalt salts we had two bands occurring at the same places in the spectrum. The concentration for which these curves were taken was much too great to explain the similarity by ionic dissociation.

When the concentration was increased almost to saturation, however, changes occurred, particularly in the case of the chlorides and bromides. Also, on heating the crystallised salts so as to drive off the water of

crystallisation, striking colour changes were passed through. I therefore came to the conclusion that the three bands common to the six nickel salts were due not to the nickel atom but to its watery atmosphere, to the bonds connecting it to the water molecules round about it. Similarly the two bands common to the six cobalt salts were characteristic, not of the cobalt atom, but of the bonds connecting it to its watery atmosphere. My view is that when any of these twelve salts is dissolved in water, one watery atmosphere forms round the base and another round the acid radical. The bonds connecting the acid radical and base to one another are much weaker than those connecting each to its own watery atmosphere, and do not affect the spectrum much, even at great concentrations. Only when there is not sufficient water left for the base to form its proper watery atmosphere is the absorption spectrum much affected.

These watery atmospheres I believe to be the same as surround the ions in electrolysis. My view thus explains why the colour of cobalt, nickel, copper salts, permanganates, etc., is not altered when they are ionised. The colour is characteristic mainly of the system *ion + watery atmosphere*, and in the neutral molecule the anion and cation systems are but slightly bound together.

The application of the formula for pe/m to the cobalt and nickel salts (fourth article) did not give any definite result. It was hopeless to try and construct a dynamical system representing a watery atmosphere. Having failed to make anything mathematically of the cobalt and nickel salts, I turned my attention to the salts of the alkali metals (fifth article). As the alkali metals are monovalent, conditions might have been simpler there. But in their case the absorption was almost all off the range of the apparatus. Also in the case of the iron and copper salts it proved impossible to connect their chemical constitution with their absorption spectra, owing, in this case also I think, to the occurrence of watery atmospheres.

Conditions in aqueous solutions having proved so complicated, I turned my attention to other solvents. The eighth and ninth articles give some results for ethyl alcohol and acetone. The absorption in these solvents is more intense and the additive character of the results breaks down; they become more characteristic and irregular. Nothing could be done with them from the standpoint of the theory of dispersion, although the law of mass action was applied successfully to the colour change which occurs when water is added to an alcoholic solution of cobalt chloride or bromide.

As a result of my own experimental work I have not been able to construct any dynamical models giving absorption spectra. The only case

in which I have had any success is that of water.* It has two great bands in the infra-red, and by assuming the system represented in the diagram below, where M is an oxygen atom with two negative charges, and m, m hydrogen atoms, one with a positive charge and the other with two positive charges and a negative charge, I obtained 2·32 for the ratio of the wave-lengths instead of 2·00, and 7110 or 1550 for the value of e/m instead of 9660, according as I used the first or second band. The result is certainly significant.

I have carefully examined the radiometric results of W. W. Coblenz† in the hope of interpreting some of his infra-red spectra in the same way. Some of his spectra, notably sulphur dioxide, carbon dioxide, carbon monoxide, and carbon disulphide, looked very promising, giving only one or two bands, but the values of pe/m obtained from them were much too low. That is, if we suppose the atoms to have each one elementary charge

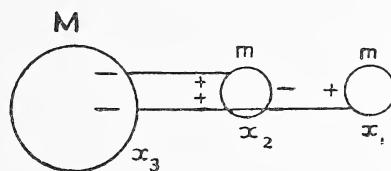


FIG. 1.

of electricity and to be in a state of vibration, the absorption bands should be very much more intense than they really are.

I do not, therefore, regard the prospects of extending further the mathematical theory of dispersion and absorption as very bright at present.

The only other worker who has been approaching the subject from the same standpoint as myself is Prof. J. Koenigsberger‡ of Freiburg, Baden. He has derived methods for obtaining pe/m from the values of the coefficient of absorption throughout an absorption band, but they are not so convenient for calculation as my formula. I did not become aware of his methods until within the past year. He finds that Planck's theory of

* "The Mechanics of the Water Molecule," *Proc. Roy. Soc., A*, 86, p. 102, 1911.

† *Publications of the Carnegie Institution, Washington*, Nos. 35, 64-67, 97-99.

‡ "Über das Verhalten gebundener und 'freier' Electronen gegen elektromagnetische Strahlung," J. Koenigsberger u. K. Kilching, *Ann. d. Phys.* (4), 28, p. 889, 1909; (4), 32, p. 843, 1910.

"Über die Bestimmung der Zahl schwingender Teile in Dämpfen, Lösungen, leuchtenden Gasen," J. Koenigsberger, *Ph. Zs.*, 12, p. 1, 1911.

"Über die Farbe anorganischer Salze und die Berechnung der schwingenden Teile," Eva von Bahr u. J. Koenigsberger, *Heidelberg. Sitzungsber.*, 1911.

"On the Origin of Selective Continuous Absorption of Band and Series Spectra," J. Koenigsberger, *Astroph. Jr.*, 35, p. 139, 1912.

the damping of the vibrators—namely, that it is due to electromagnetic radiation—is inapplicable to the bands of the anilin colouring matters or other broad bands, because it gives much too small a rate of damping. This is a view with which I am in thorough agreement. He divides absorption bands into two kinds: the first kind characterised by great intensity and breadth, and due to an ion or electron in every molecule; and the second kind including bands, such as those of didymium, which are narrower and much less intense, and are supposed to be caused only by exceptional molecules in the body. Thus by making p a small fraction he explains the very small values of pe/m obtained. He believes that absorption measurements are much more valuable than dispersion measurements from the point of view of calculating pe/m , and with this opinion I am in emphatic agreement. His views on the absorption spectra of gases are specially interesting. Continuously selective absorption is due to the normal state of gas molecules; the band spectrum absorption and emission are due only to an intermediary non-stationary condition, and are connected with the rupture of the bonds to which the continuously selective absorption is due.

Having stated my own views, I shall now criticise some ideas current in the field.

First of all, take the question of the colour of the ions. According to the theory of electrolytic dissociation, in an aqueous solution a certain proportion of the salt molecules splits into ions. The absorption spectrum of a salt solution should consequently be the sum of the separate spectra due to the neutral molecules, anions, and cations respectively. If the solution is very dilute it is entirely ionised, and the colour should be entirely due to the ions. This conclusion was first drawn by Ostwald, and he sought to prove it in one case by showing that the absorption spectra of thirteen permanganates were identical at great dilution. He did not investigate concentrated solutions, but it was shown later that the absorption spectra of concentrated permanganates were also identical, so they cannot be adduced as a proof of electrolytic dissociation. They are, nevertheless, still cited for that purpose in books on physical chemistry.

As a result of my studies, both in the literature and at first hand, my opinion can be put in the following sentence: There is no spectroscopic evidence in favour of the theory of electrolytic dissociation; there is some evidence against it, which can, however, be got over by the assumptions of hydrolysis, complex ions, hydration, etc. Few of the researches on the subject have been carried out with the necessary thoroughness. The photographic method is applicable only to the salts of the rare earths,

where the bands are sharp and the wave-lengths can be determined accurately. In other cases a spectrophotometer must be employed, but the spectrophotometry of solutions of the necessary dilution is very troublesome. The colour changes of the cobalt and copper salts have often been ascribed to ionic dissociation; but, as has been decidedly shown in the tenth article, these changes occur only at great concentrations, and must be put down to hydration.

There is one interesting point in connection with the colour of the ions which has not, I think, yet received attention. To explain an absorption band we must have vibrating charged particles. These particles must have centres to vibrate about. If, for example, we take the case of CoSO_4 , we can suppose that the Co has a positive charge, the SO_4 a negative charge, and that they vibrate towards each other under the influence of an electrical attraction. When the CoSO_4 is ionised, these vibrations cease. Consequently, the Co ion cannot have a spectrum with absorption bands unless we introduce vibrations inside the atom or provide it with a watery atmosphere.

In 1877 Kundt stated the law that if one colourless solvent had a greater dispersive and refractive power than another, and if a colouring matter producing an absorption band was dissolved in both, then in the case of the first medium the absorption band would be nearer the red than in the case of the second. Kundt's own measurements showed exceptions to the rule, and it is as often disobeyed as obeyed; yet it has persisted in the literature and has become a kind of definition of chemical change. When it is not obeyed, the solvent is supposed to enter into chemical combination with the colouring matter, and when it is obeyed there is no chemical combination. Kayser* thinks that, on the whole, there is something in the law, but that it is very often masked by disturbing factors.

My own belief is, that when the disturbing factors are taken away there is nothing left. The idea of molecules of the dissolved substance moving about in the solvent without essential change is a very tempting one, and suggests analogies with Hertzian vibrators in media of different dielectric constants. The period of the Hertzian vibrator should, of course, be proportional to the square root of the dielectric constant of the medium. Also, we can think of the electric doublets associated with the absorption bands as being affected by weak electric forces from the doublets in the ultra-violet [which cause the dispersion and refraction of the solvent, and I developed a theory on these lines seven years

* *Spectroscopie*, vol. iii. p. 85.

ago.* This line of reasoning has doubtless also occurred to others. But even with the best will in the world I have never found anything that could be interpreted in the sense of the above theories.† All shifts of absorption bands are accompanied by changes of shape and intensity which are usually much more important; for example, cf. a little-known paper by G. J. Katz,‡ which is quite the best on the subject. Kundt's law, I feel, has been a false clue, and should be forgotten.

Recently a considerable amount of work has been done by Jones and his colleagues § photographing various absorption spectra. They started their work in the belief that the dissolved substance usually formed loose compounds or "solvates" with the solvent, and that from the absorption spectra it would be possible both to adduce evidence in support of this view and at the same time assign the different bands to their sources, solvates, ions, or whatever these were. The solvate coincides to a considerable extent with my own view as to the watery atmosphere. I cannot help feeling, however, that they would have made more progress if they had restricted themselves to a narrower field, and used spectro-photometry instead of photography. Photographic results are liable to misinterpretation; e.g., cf. Jones and Strong, *Publications of the Carnegie Institution*, No. 130, p. 141, and elsewhere, where the gradual shift of an absorption band as one salt is transformed into another by replacement of the acid radical is said to indicate intermediate compounds. This shift is merely apparent, and Merton's criticism || on the point is fully justified.

With regard to work done from the side of organic chemistry, I think the theory of chromophores, chromogenes, etc., is more a question of terminology than anything else; also that the chemists have too high an opinion of Stark's theories.¶

The benzene ring has given rise to much speculation owing to its seven bands in the ultra-violet, and a connection has been sought between these bands and the ways in which the ring can oscillate. For this purpose the absorption bands in the infra-red are always ignored. I have had rough measurements made on the molecular extinction coefficient of

* "Untersuchungen über den Einfluss der Temperatur auf die Absorption des Lichtes in isotropen Körpern," *Ann. d. Phys.* (4), **21**, p. 572, 1906.

† This applies also to Dr Havelock's theory, *Proc. Roy. Soc.*, A, **86**, p. 15, 1911.

‡ "Verschiebung der Absorptionsstreifen in verschiedenen Lösungsmitteln," Inaug. Diss., Erlangen, 1898, *Arch. f. wissenschaftl. Photogr.*, **1**, p. 21, 1899.

§ *Publications of the Carnegie Institution, Washington*, Nos. 60, 110, 130, 160.

|| "The Changes in Certain Absorption Spectra in Different Solvents," Thos. Ralph Merton, *Proc. Roy. Soc.*, A, **87**, p. 138, 1912.

¶ Cf. *Die Beziehungen zwischen Farbe und Konstitution bei organischen Verbindungen*, H. Ley, Leipsic, 1911.

1912-13.] The Absorption of Light by Inorganic Salts, No. XI. 165
benzene in ethyl alcohol in the ultra-violet, and have examined the various views, but am afraid that none can be regarded as satisfactory.

In connection with the critical attitude of this paper it should be remembered that the difficulty in establishing laws for the behaviour of absorption spectra is due to their being so extraordinarily characteristic. In this series of papers we have obtained values of A , the molecular extinction coefficient, ranging all the way from 27,000 to 0·01, and A can be measured for an aqueous solution for all wave-lengths between $\cdot23\ \mu$ in the ultra-violet and $1\cdot3\ \mu$ in the infra-red. Molecular refraction and the other additive properties of physical chemistry are not very characteristic.

(Issued separately April 22, 1913.)

XV.—On the Occurrence of Functional Teeth in the Upper Jaw of the Sperm Whale. By James Ritchie, M.A., D.Sc., Royal Scottish Museum; and A. J. H. Edwards, National Museum of Antiquities. *Communicated by Sir WM. TURNER. (With One Plate.)*

(MS. received February 25, 1913. Read March 17, 1913.)

WRITERS on mammalian anatomy are united in stating that one of the characteristics of the Cetacean family Physeteridae is the presence of functional teeth in the lower jaw only. Typical statements are those of Flower and Lydekker (1891): "No functional teeth in the upper jaw";* Beddard (1900): "Teeth found in both jaws, but those of lower jaw alone functional; often very reduced in number";† Turner (1912): "Functional teeth in lower, but not in upper jaw."‡ A possible exception is the Pigmy Whale, *Kogia*, in which the upper jaw may altogether lack teeth or may possess an anterior rudimentary pair (as in *K. simus*, Owen) the utility of which is a matter of some doubt.

As regards the Cachalot or Sperm Whale (*Physeter macrocephalus*), it is well known that an indefinite number of rudimentary teeth occur in the upper jaw; but these have been held to be relatively small, embedded in the gum so that they do not reach the surface, and necessarily, therefore, altogether functionless. We have already published a short note § recording the result of an examination of six aged bull Sperm Whales caught by the Bunaveneader Whaling Company in the vicinity of Rockall, in one of which a row of small teeth was visible in the upper jaw. The teeth projected from the gum less than half an inch, and each lay close to one of the large indentations into which a tooth of the lower jaw had fitted. It was remarked that the exposed portions of the teeth did not resemble the sharp tips of the rudimentary maxillary teeth described and figured by Sir William Turner,|| where out of fifteen examples "in no specimen was the crown polished or worn"; that, instead, the crowns were distinctly flattened

* Flower and Lydekker, *An Introduction to the Study of Mammals, Living and Extinct*, London, 1891, p. 247.

† Beddard, *A Book of Whales*, London, 1900, p. 182.

‡ Turner, *The Marine Mammals in the Anatomical Museum of the University of Edinburgh*, London, 1912, p. 71.

§ *Scottish Naturalist*, 1912 (March), p. 65.

|| Turner, "The Occurrence of the Sperm Whale or Cachalot in the Shetland Seas," *Ann. Scot. Nat. Hist.*, 1904, p. 6, pl. i.; and previously in *Proc. Roy. Soc. Edinburgh*, 1903.

or polished, apparently by friction ; and the obvious conclusion was that, in this case, the maxillary teeth had been functional.

A second visit to the whaling station at Bunaveneader in Harris in 1912, for the purpose of obtaining a Sperm skull for the Royal Scottish Museum, has resulted in even more convincing demonstration of the utility of the maxillary teeth. A single male Cachalot was captured and brought in during this visit, and it, fortunately, also bore exposed teeth, about a score in number, in the upper jaw. With considerable difficulty, after the breaking of a strong knife-blade and the final use of axes, made necessary by the density of the flesh, one of the teeth was excised along with the surrounding portions of gum, and this forms the subject of fig. 1.

The maxillary teeth lay in a row along a well-defined groove running the length of the jaw on the inner side of the depressions caused by the mandibular teeth. Each tooth was situated near to, and on the inner and posterior side of, a mandibular pit, the inner margin of a pit being indicated by the curve behind the tooth in fig. 1. The exposed portion of the tooth presents in plan an oval section, but the sharp point common in maxillary teeth has been completely abraded, and on the flattened top the concentric arrangement of the dentine is clearly visible—a central translucent core surrounded by a ring of whiter and more opaque material. The surface is marked by fine grooves and scratches, additional evidence of its having been in use.

The removal of part of the surrounding gum showed that, as is so often the case, the tooth was much twisted (fig. 2). It was 11 cms. long ; and its curve approximated a quadrant of a circle, the fang, which was broad and jagged, being turned outwards so that it lay almost at the base of the mandibular pit, where it may have served as an opponent to the mandibular tooth. All but the tip of the tooth was simply embedded in the gum, far removed from the maxillary bone itself. The stability of the tooth, however, was not affected thereby, for the consistency of the gum was so dense and ligamentous that it afforded a perfectly firm setting, as indeed also occurs in the case of the mandibular teeth, which in the specimen prepared for the Royal Scottish Museum were held in place by the gum, the sockets in the lower jaw being far too wide and too shallow to grip the fangs of the teeth—a fact observed many years ago by Owen.*

That the occurrence of functional teeth in the upper jaw of the Sperm Whale is rare, the negative results of the observations of many naturalists make clear ; but that it is not an abnormal phenomenon is equally apparent from the evidence furnished by two out of seven specimens examined by

* Owen, *Odontography*, London, 1840-45, p. 354.

us, and from the statement of Mr Carl F. Herlofson, the director of the Bunaveneader Whaling Station, that on previous occasions he had noticed teeth protruding from the upper jaw of Sperm Whales landed at the station. Rare occurrences of projecting teeth have at other times been observed, but we are aware of no evidence of their utility, apart from a deduction formed by Pouchet and Beauregard from one out of several maxillary teeth received from Ponta Delgada in the Azores, and mentioned in a footnote: "L'une d'elles paraît avoir été saillante en dehors de la muqueuse, ce qui arriva, on le sait, quelquefois, et présente à son extrémité une légère usure."*

As there is still some indefiniteness regarding the shape of the head of the Sperm Whale, and especially regarding the distance of the mouth behind the extremity of the head, a photograph of the specimen bearing the functional maxillary teeth is reproduced, showing the head, from the under surface, with the lower jaw open (fig. 3). In this are clearly visible the mandibular pits in the upper jaw, the grooves in which the maxillary teeth lie (though these teeth themselves are, unfortunately, on too small a scale to be visible), and an enormous squarely truncated pre-oral rostrum. In this last respect the Bunaveneader specimen differs greatly from the Inniskea bull sperm figured by Lillie,† in which the tip of the mandible is close to the anterior of the head; and it differs as much from the smooth round-muzzled Azores specimens figured from photographs by Pouchet and Chaves, who stigmatise as inexact those figures "représentant l'animal avec l'extrémité de la tête aplatie et comme coupée carrément."‡ Those statements can be correlated with our own observations of square, truncated muzzles and receding mouth, only by supposing that there occurs a wide range of variation in the proportions and shape of the head parts, but it is possible that some differences, as perhaps those between our and the Azores specimens (the sexes of the latter are unrecorded), may indicate sexual dimorphism. Nevertheless, in each of the seven male Bunaveneader examples examined by us there was a distinct pre-oral rostrum similar in proportion to that here figured. It projected about a yard in front of the tip of the mandible, and was so tough and fibrous in nature that it could be cut only with great difficulty.

* Pouchet et Beauregard, "Sur un Cachalot échoué à l'Île de Ré," *Jour. de l'Anat. et Phys.*, vol. xxvii., 1891, p. 132, f.n.

† Lillie, "Observations on the Anatomy and General Biology of some Members of the Larger Cetacea," *Proc. Zool. Soc. London*, 1910, part 2, p. 788, fig. 78.

‡ Pouchet et Chaves, "Des Formes extérieures du Cachalot," *Jour. de l'Anat. et Phys.*, vol. xxvi., 1890, p. 270.



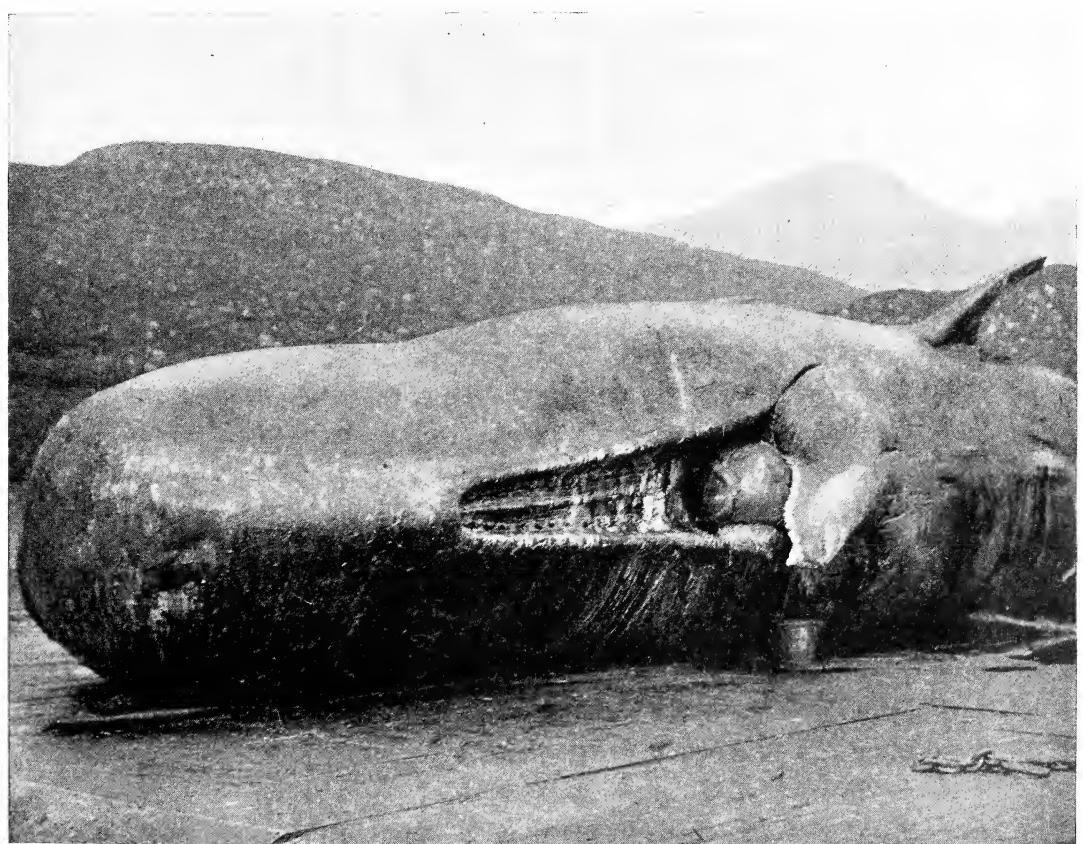
R. H. R. photo.

FIG. 1.—Functional Maxillary Tooth of Sperm Whale, with surrounding portions of gum ; nat. size.



R. H. R. photo.

FIG. 2.—Functional Maxillary Tooth of Sperm Whale, showing shape of embedded portion.



A. J. H. E. photo.

FIG. 3.—Head of Sperm Whale, under surface, with lower jaw hanging open.

XVI.—On the Electron Theory of Thermo-electricity. By John M'Whan, M.A., Ph.D., Assistant to the Professor of Mathematics in the University of Glasgow. *Communicated by* Professor A. GRAY, F.R.S.

(MS. received January 8, 1913. Read February 3, 1913.)

THE rôle played by the “free” (negative) electrons in metallic conductors as carriers of the electric current has formed the basis of theories developed by H. A. Lorentz,* Sir J. J. Thomson,† E. Riecke,‡ and others, regarding the thermo-electromotive force. In these theories the free electrons are considered collectively as an ideal gas permeating the metal and subject to the usual gas laws. The kinetic theory of gases is then applied to the problem of the motion of the atoms of the electronic “gas,” and expressions for the thermo-E.M.F.—differing in the various instances by a constant multiplier only—are deduced. The formulæ so derived are not open to test as against experimental result, containing as they do the ratio of the electronic densities in the two metals forming the couple: they have been shown, in addition, to be inconsistent with some of the known facts.

A new method of attack has been originated by Krüger,§ and, independently, by Baedeker,|| which admits of the application of thermodynamic processes in the electron theory. This method is based upon the researches of O. W. Richardson into the emission of electrons from incandescent bodies, and it is the object of the present paper, starting from this point, to derive an expression for the thermo-E.M.F. differing in important respects from those found by Krüger and Baedeker, and probably approximating more nearly to the actual physical processes at a thermo-junction. It should, however, at once be stated that—as will be seen later—even this expression cannot pretend to account for all the known facts.

The preliminary argument may be briefly summarised thus:—Incandescent bodies have been found—speaking generally—to emit negative electrons with great readiness, the saturation current of emission of electrons

* Lorentz, *Proc. Amsterdam*, vii. 585 (1905), vii. 684 (1905); *Arch. Néerl.* (2) xx. 336 (1905).

† Thomson, *Corpuscular Theory of Matter*, p. 73.

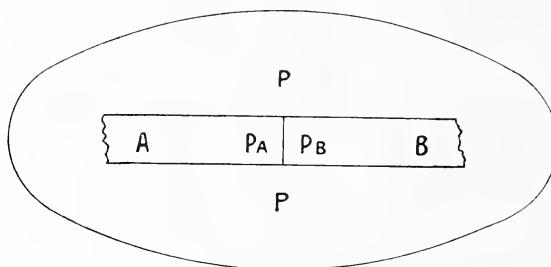
‡ Riecke, *Wied. Ann.*, lxvi. 353, 545, and 1199 (1898); *Ann. d. Phys.*, (4) ii. 835 (1900).

§ Krüger, *Phys. ZS.*, xi. 800 (1910).

|| Baedeker, *Phys. ZS.*, xi. 809 (1910); *Elektrische Erscheinungen in metallischen Leitern*, p. 91.

(H. A. Wilson's "thermionic current") being found to be a definite function of the absolute temperature, which we shall presently employ. H. A. Wilson has regarded this emission of electrons as an evaporation of negative electricity through the surface of the hot body, determined therefore by the nature and temperature of the body and by a definite "vapour pressure of electrons." Regarded so, the ordinary thermodynamic laws of evaporation are at once available, and Wilson has shown* that in this way Richardson's expression for the thermionic current may be derived. Now it is reasonable to suppose that the emission, though of observable magnitude at high temperatures only, takes place at *all* temperatures, varying in amount only according to the temperature and the pressure of the electron "vapour," which we may assume to obey the laws for an ideal gas.

Suppose then that two metals A, B are in contact and that the vapour-pressurest of electrons in the metals are p_A , p_B , at absolute tem-



perature θ . Let us place the arrangement inside a chamber whose walls are completely impervious to the electrons and, having completely exhausted this chamber, let us fill it with our electron-vapour, at a vapour-pressure p , at absolute temperature θ . Lastly, let the whole arrangement be kept at constant (absolute) temperature θ and let the contact potential-difference between A and B at this temperature be E. The thermodynamic process (assume $p < p_A < p_B$)† is then as follows:—

Allow a definite quantity of the electron-gas to evaporate out of metal A into the surrounding chamber. For convenience this quantity may be chosen as that carrying a charge of $1.772 \times 10^7 \left(\equiv \frac{e}{m} \right)$ electro-magnetic units: we have then unit mass of gas to deal with and, in the gas equation

* H. A. Wilson, *Phil. Trans.*, A, vol. ccii. 243 (1903).

† We shall use this term throughout, though more probably, Nernst's "pressure of solution" would convey the conception more clearly.

‡ $p_A > p > p_B$ would be a more convenient assumption, as indicating evaporation *out* of A into the chamber and *out* of the chamber into B. It has been thought better to avoid such a particular case; in fact any assumption will do.

$pV = R\theta$, R retains its usual significance. The choice is, however, immaterial, involving at most the introduction of a constant. In the evaporation (i) the gas expands isothermally, doing work of amount $R\theta \log \frac{p_A}{p}$, and (ii) it absorbs latent heat of amount q_A , say. Now cause it to condense back isothermally into metal B. In this process, (i) work has to be done on the gas, of amount $R\theta \log \frac{p_B}{p}$, and (ii) the gas liberates latent heat of amount q_B , say. Thus the nett effect of both changes has been an expenditure of work and heat on the gas of amount

$$R\theta \log \frac{p_B}{p} - R\theta \log \frac{p_A}{p} + q_A - q_B = R\theta \log \frac{p_B}{p_A} + (q_A - q_B).$$

But as the process considered has merely resulted in the transfer from one side of the interface between A and B to the other side, against a potential-difference E (e.m.u.), of a quantity $\frac{e}{m}$ (e.m.u.) of electricity, the above energy-expenditure must be balanced by the necessary electrical work $E \cdot \frac{e}{m} \equiv E\mu$, say; so that

$$E = \frac{R\theta}{\mu} \log \frac{p_B}{p_A} + \frac{1}{\mu} (q_A - q_B) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

gives the contact difference of potential at temperature θ between the metals A, B.

For the temperature-coefficient, since the electronic vapour-pressure and latent heats are functions of the temperature, we have

$$e \equiv \frac{dE}{d\theta} = \frac{R}{\mu} \log \frac{p_B}{p_A} + \frac{R\theta}{\mu} \frac{d}{d\theta} \log \frac{p_B}{p_A} + \frac{1}{\mu} \frac{d}{d\theta} (q_A - q_B).$$

Now the transfer of electrons took place in two distinct "evaporation" or "condensation" stages, to either of which accordingly Clapeyron's equation will apply. Thus, for the evaporation out of A,

$$q_A = v_A \theta \frac{\partial \mu_A}{\partial \theta},$$

assuming as usual that the volume of the vapour is large compared with that of the un-evaporated state, and writing v_A for the volume-change, which is thus practically the whole vapour-volume. Or,

$$\begin{aligned} \frac{q_A}{p_A v_A \theta} &= \frac{1}{p_A} \frac{\partial p_A}{\partial \theta}, \\ i.e. \quad \frac{\partial}{\partial \theta} \log p_A &= \frac{q_A}{R \theta^2}. \end{aligned}$$

A similar equation is found for $\frac{\partial}{\partial \theta} \log p_B$, and subtracting,

$$\frac{d}{d\theta} \log \frac{p_B}{p_A} = \frac{q_B - q_A}{R\theta^2} \quad \quad (2)$$

To evaluate $\frac{d}{d\theta}(q_A - q_B)$ we apply the equation

$$\frac{dL_\theta}{d\theta} + c - h = \frac{L_\theta}{\theta},$$

in which c, h are the specific heats in the first and second phases, and so obtain

$$\frac{dq_A}{d\theta} = \frac{q_A}{\theta} + h - c_A,$$

h referring to the external (*i.e.* chamber-) electron-vapour, and

$$\frac{dq_B}{d\theta} = \frac{q_B}{\theta} + h - c_B,$$

whence

$$\frac{d}{d\theta}(q_A - q_B) = \frac{q_A - q_B}{\theta} - (c_A - c_B) \quad \quad (3)$$

Our equation for the temperature-coefficient of the E.M.F. thus becomes, on insertion of (2) and (3),

$$\mu \frac{dE}{d\theta} = R \log \frac{p_B}{p_A} - (c_A - c_B);$$

and from (1),

$$R \log \frac{p_B}{p_A} = \frac{\mu E}{\theta} - \frac{q_A - q_B}{\theta},$$

$$\therefore \frac{dE}{d\theta} = \frac{E - \frac{q_A - q_B}{\mu} - \frac{\theta(c_A - c_B)}{\mu}}{\theta}.$$

Lastly, writing q_{AB} for $(q_A - q_B)$, the latent heat per unit mass for the passage from A to B, we see that (i) E is the purely electrical energy-change, per unit charge, involved in the transference from A to B; (ii) $\frac{q_{AB}}{\mu}$ is the latent heat of change per unit charge; and (iii) $\frac{\theta(c_A - c_B)}{\mu}$ is the heat absorbed or liberated per unit charge on account of the change in specific heat of the electronic vapour. Hence the numerator in

$$e \equiv \frac{dE}{d\theta} = \frac{E - \frac{1}{\mu} \{q_{AB} + \theta(c_A - c_B)\}}{\theta} \quad \quad (4)$$

accounts entirely for the heat absorbed or liberated at the junction by the

passage across it of unit charge of electricity, and must accordingly be the Peltier effect;

$$\text{i.e. } e \equiv \frac{dE}{d\theta} = \frac{\Pi}{\theta}, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

which is the first of the two equations derived from purely thermodynamical considerations by Lord Kelvin.* The theories thus appear so far in consonance, but it has to be remarked that whereas the Kelvin result assumes reversibility of the thermo-electric processes and consequently neglects the irreversible heat conduction in the thermo-element—assumes, in fact, the equation $\int \frac{dQ}{\theta} = 0$ —the present treatment, though assuming reversibility of the individual “evaporations,” does not *require* reversibility on the whole, but rests only upon the First Law. Further, “ dE ” has not the same significance in the two cases. As obtained from (4) and (5) in the form $dE = \frac{\Pi}{\theta} d\theta$, it is obviously, for the case of a thermo-couple, only the difference of the E.M.F.’s at the junctions: in Kelvin’s expression of $dE = \frac{\Pi}{\theta} d\theta$, dE is the *whole* E.M.F. over the whole circuit. Identity of the two expressions would therefore imply, as Krüger † has pointed out for the case of his own result, the non-existence of any potential-difference due to the unequal heating of the conductors forming the couple, an implication quite at variance with the electron theory of metallic conduction. The result, therefore, only serves to cast further doubt on the justification for assuming reversibility in the thermo-electric problem.

The variance in the meaning of E will prevent anything more than an approximation to the second Kelvin equation,

$$\frac{de}{d\theta} = \frac{\sigma_A - \sigma_B}{\theta},$$

being obtained. Thus, on differentiating (4) and again employing (3), we obtain

$$\frac{de}{d\theta} = -\frac{q_{AB}}{\mu\theta^2} - \frac{1}{\mu} \frac{d}{d\theta} (c_A - c_B),$$

or, identifying $\frac{c_A}{\mu}$, $\frac{c_B}{\mu}$, the “specific heats of unit charge of electricity” in the two metals, with the Thomson effects σ_A , σ_B ,

$$\frac{de}{d\theta} = -\frac{q_{AB}}{\mu\theta^2} - \frac{d}{d\theta} (\sigma_A - \sigma_B), \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6),$$

an equation from which the proportionality of the difference of the Thomson

* Kelvin, *Math. and Phys. Papers*, i., art. xlvi., part vi. p. 249.

† *Loc. cit.*, p. 805.

effects to the absolute temperature can only be derived on the assumption that $\frac{q_{AB}}{\mu\theta^2}$ is constant or negligibly small. For neither of these assumptions is there any sufficient justification. The equation of Krieger leads to no better a conclusion when the same process is employed.

There still remains the possibility of examination of equations (1) or (4) by comparison with Avenarius's empirical law for the thermo-electromotive force. This may be attempted in the following way.

If the conception of electronic-vapour-pressure is to have any physical significance whatever, it is certain that that pressure will determine the rate of evaporation of electrons out of the metal—*i.e.* the negative thermionic current,—and that, conversely, the vapour-pressure must be a one-valued function of the thermionic current.

If n be the number of electrons leaving the metal per second and per sq. cm. of surface, $ne=j$ gives the saturation current. But if θ be the absolute temperature, m the mass of an electron (molecule of electronic vapour), and v the root mean square of all the electronic velocities (in the usual significance of the kinetic gas theory), then for the vapour-pressure

$$p \propto mnv.$$

Again, from the kinetic theory, $v^2 \propto \theta$, so that, substituting for n

$$p = k_e^j \sqrt{\theta}, \quad \dots \dots \dots \dots \dots \dots \quad (7)$$

where k is a constant, gives the relation between p and j .

Thomson, Wilson, and Richardson have, however, arrived at the expression * for the thermionic current—

$$j = a\theta^{\frac{1}{2}} e^{-b/\theta}, \quad \dots \dots \dots \dots \dots \quad (8)$$

from different theories, a, b being constants, so that we may write for the electronic-vapour-pressure

$$p = a\theta e^{-\beta/\theta}, \quad \dots \dots \dots \dots \dots \quad (9)$$

an expression in which a, β will vary in general, if at all, only with the physical nature of the body from which the emission is taking place.

For uniform absolute temperature θ , this gives

$$\begin{aligned} \log \frac{p_B}{p_A} &= \log \left(\frac{a_B}{a_A} \cdot \frac{e^{-\beta_B/\theta}}{e^{-\beta_A/\theta}} \right) \\ &= \log \frac{a_B}{a_A} + \frac{\beta_A - \beta_B}{\theta}, \end{aligned}$$

* See, for example, J. J. Thomson, *Cond'n. of Elect. through Gases*, § 93 (1903); H. A. Wilson, *Elect. Props. of Flames and Incandescent Solids*, p. 14 (1912); O. W. Richardson *Phil. Trans., cci. 516* (1903), *Phys. ZS.*, v. 6 (1904).

and, differentiating (1) and inserting this value, we have

$$\begin{aligned}\frac{dE}{d\theta} &\equiv \frac{R}{\mu} \log \frac{p_B}{p_A} + \frac{R\theta}{\mu} \frac{d}{d\theta} \log \frac{p_B}{p_A} + \frac{1}{\mu} \frac{d}{d\theta} q_{AB} \\ &= \frac{R}{\mu} \left\{ \log \frac{a_B}{a_A} + \frac{\beta_A - \beta_B}{\theta} \right\} - \frac{R\theta}{\mu} \cdot \frac{\beta_A - \beta_B}{\theta^2} + \frac{1}{\mu} \frac{d}{d\theta} q_{AB}, \\ \text{i.e. } \frac{dE}{d\theta} &= (\text{a constant}) + \frac{1}{\mu} \frac{d}{d\theta} q_{AB}.\end{aligned}$$

Avenarius's law, however, gives—it is obviously impossible to attempt a deduction of the law from the above equation, in view of the presently indefinite character of q_{AB} as given by (3)—

$$\frac{dE}{d\theta} = A + B\theta + C\theta^2 + \dots$$

(or, to a first approximation, $\frac{dE}{d\theta} = A + B\theta$), whence, in the differential identity

$$A + B\theta + C\theta^2 + \dots \equiv (\text{a constant}) + \frac{d}{d\theta} \frac{q_{AB}}{\mu},$$

it is only necessary that $\frac{q_{AB}}{\mu}$ should be of the form $p\theta + q\theta^2 + r\theta^3 + \dots$.

(or, to a first approximation, $p\theta$),* and so from equation (3), written

$$\frac{d}{d\theta} \left(\frac{q_{AB}}{\mu} \right) = \frac{q_{AB}}{\mu\theta} - (\sigma_A - \sigma_B),$$

we require an equality of the form

$$\sigma_A - \sigma_B = A^1\theta + B^1\theta^2 + C^1\theta^3 + \dots \quad . \quad . \quad . \quad . \quad . \quad (10)$$

or, to a first approximation, $\sigma_A - \sigma_B = A^1\theta$, i.e., in view of the certainly extremely small coefficients B^1, C^1, \dots we may say that the difference of the Thomson effects is approximately simply proportional to the absolute temperature.

We conclude accordingly that the theory is essentially reconcilable with the standard Kelvin equations, with the exception of small divergences to be traced to the irreversible nature of the heat flow along the conductors, a consideration omitted in the Kelvin treatment. Whether the present theory will give more information as to the actual physical processes involved in the action of a thermo-couple is another question. It certainly seems to indicate the positive existence of potential-differences, however small, due solely to unequal heating of conductors. Experimental results cannot help us much so far; it has, for example, proved impossible, even

* Cf. Preston's *Heat*, p. 316 (1894 edition): "It has been suggested that . . . the molecular latent heat is proportional to the absolute temperature."

with the aid of the bolometer in measuring the Thomson effect, to prove beyond question the proportionality to the absolute temperature, so that no help may be looked for, experimentally, as to the superiority or inferiority of the relation $\sigma_A - \sigma_B = A^1\theta + B^1\theta^2 + C^1\theta^3 + \dots$ to that of simple proportionality.

I hope, however, in a further paper, to communicate some experimental results on the thermo-electrical properties of strained metals, experiments on which, with the aid of a grant from the Carnegie Trust, I have presently in progress, and to essay a test of the theory from that standpoint.

PHYSICAL INSTITUTE OF THE
UNIVERSITY OF GLASGOW,
November 20, 1912.

(Issued separately April 22, 1913.

XVII.—On a Fuller Test of the Law of Torsional Oscillation. By James B. Ritchie, B.Sc., Carnegie Research Scholar in Physics, University College, Dundee. *Communicated by Professor W. PEDDIE.*

(MS. received November 18, 1912. Read November 18, 1912.)

TEST OF A MODIFICATION OF THE FORMULA.

IT has been shown in a former paper * that an equation of the form

$$y^n(x+a) = b$$

can be applied to give close representation of results in the determination of the law of decrease of torsional oscillations of wires of different materials, when the range of oscillation is large in comparison with the palpable limits of elasticity. In the equation as written

y =the range of oscillation.

x =the number of oscillations since the commencement of observations.

n , a , and b =quantities, constants for any one experiment, their numerical values depending on the initial conditions of the wire in question, and its subsequent treatment.

This empirical equation was deduced by Dr Peddie† on the supposition that the loss of potential energy when the wire is twisted through an angle, θ say, is due to the rupture of molecular configurations, and is proportional to a power of the angle of torsion. Thus, the potential energy during the twist θ being represented in accordance with Hooke's Law, as

$$V = \frac{1}{2}k\theta^2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

we obtain on taking the loss into account,

$$V = \frac{1}{2}k\theta^2 - p\theta^m \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This loss of energy per swing was taken as being small in comparison with the total energy, so that it was practically equal to $k\theta d\theta$, and hence equation (2) took the form

$$-k\theta d\theta = p\theta^m dt \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where dt is the time of an outward swing.

The integral is

$$\theta^n(t + t_0) = b \quad . \quad . \quad . \quad . \quad . \quad (4)$$

* *Proc. Roy. Soc. Edin.*, vol. xxxi. p. 424.

† *Phil. Mag.*, July 1894.

where b and t_0 are constants, and $n=m-2$. This is identical with the equation used, and it has been shown that if $\log y$ be plotted against $\log(x+a)$, the points can in many cases, such as brass at the ordinary temperature, be made to lie along one straight line, when a suitable value of a has been chosen. The tangent of this line's inclination to the $\log y$ axis then gives the value of n . In other cases, however, such as copper in the soft condition, it was found (*Proc. Roy. Soc. Edin.*, p. 436, 1910-11) that no one value of a could be found to cause the points over the range taken to fall on one straight line, but that an *s*-shaped curve was in general got. This required two separate values of a for large and small oscillations respectively, and two straight lines, inclined at an angle concave to the origin, were obtained.

In order to test the applicability of the empirical law further, a fuller approximation to the true representation of results was taken as follows:

If θ_1 and θ_2 be the ranges of the first and second out swings respectively, the loss of energy is represented by

$$\frac{1}{2}k(\theta_1^2 - \theta_2^2),$$

and this loss will, taking the arithmetic mean of the two swings, be more approximately equal to

$$p\left(\frac{\theta_1^m + \theta_2^m}{2}\right)$$

instead of $p\theta^m$ as formerly.

Thus we can write

$$\frac{k}{2}(\theta_1^2 - \theta_2^2) = p\left(\frac{\theta_1^m + \theta_2^m}{2}\right),$$

the interval of time between the swings being taken as unity. Or, substituting the proportional scale readings,

$$\frac{k}{2}(y_1^2 - y_2^2) = p\left(\frac{y_1^m + y_2^m}{2}\right).$$

But if we suppose

$$y_2 = y_1 - dy_1,$$

then

$$\begin{aligned} k(2y_1 - dy_1)dy_1 &= p[y_1^m + (y_1 - dy_1)^m] \\ &= p\left[2y_1^m - my_1^{m-1}dy_1 + \frac{m(m-1)}{2}y_1^{m-3}dy_1^2 + \dots\right] \end{aligned}$$

Approximately, p being small,

$$k(2y - dy)dy = p[2y^m - my^{m-1}dy].$$

Hence

$$dy = y^{m-1} \frac{p}{k} \left[1 - \frac{m}{2} \frac{dy}{y}\right] \left[1 + \frac{dy}{2y}\right],$$

an equation which gives discrete values of y corresponding to successive integral values of x . This condition also holds in the expression

$$dy \left[1 + \frac{p}{2k} (m-1) y^{m-2} \right] = -\frac{p}{k} y^{m-1},$$

where we explicitly make dy negative to suit the actual case. Hence, if three such successive points (y, x) lie nearly enough on a straight line,

$$\frac{dy}{y} + \frac{2k}{p(m-1)} \frac{dy}{y^{m-1}} = \frac{-2}{m-1} dx,$$

i.e. in integration,

$$\log y - \frac{2k}{p} \frac{1}{(m-1)(m-2)} y^{-(m-2)} = \frac{-2}{m-1} (x+a),$$

i.e.

$$y^{m-2} \left[\log y + \frac{2}{(m-1)} (x+a) \right] = \frac{2k}{p(m-1)(m-2)}. \quad \cdot$$

And if, as before, we write $m=n+2$, this becomes

$$y^n \left[\log y + \frac{2}{n+1} (x+a) \right] = \frac{2k}{p^n(n+1)} = b,$$

or

$$y^n [\log y + k'(x+a)] = b \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This, then, is the new form of working equation, which differs from (4) simply in the introduction of the $\log y$ term.

As before

$$n \log y + \log [\log y + k'(x+a)] = \log b,$$

so that the curve now to be used to give n is got by plotting $\log y$ against $\log [\log y + k'(x+a)]$. But, as the value of k' is unknown, direct representation of results is impossible.

In diagram 8,* showing the values of $\log y$ plotted against $\log(x+a)$, an s-shaped curve is obtained, which could, by choosing the two values of $a=0$ and $a=16$, be changed to two straight lines, giving $n=3.8$ and $n=1.6$ respectively. From this diagram the curve obtained when $a=5$ (an intermediate value) is chosen, and suitable values of $y_1, y_2, y_3, x_1, x_2, x_3$ are taken, so that a series of equations can be written,

$$\begin{aligned} y_1^n [\log y_1 + k(x_1+a)] &= b \\ y_2^n [\log y_2 + k(x_2+a)] &= b \\ y_3^n [\log y_3 + k(x_3+a)] &= b. \end{aligned}$$

Assume, for convenience

$$y_2 = \lambda y_1$$

$$y_3 = \lambda y_2$$

* Proc. Roy. Soc. Edin., vol. xxxi. p. 436, 1910-11.

where $\lambda = 1.5$, this value of λ being chosen so as to cause the values of y taken to lie well over the range plotted.

The values taken were

$$\begin{array}{ll} x_1 = 62.0 & y_1 = 2.40 \\ x_2 = 23.0 & \log y_1 = .38. \\ x_3 = 6.0 & \end{array}$$

Now

$$\begin{aligned} y_1^n [\log y_1 + k'(x_1 + a)] &= b, \\ \lambda^n y_1^n [\log y_1 + \log \lambda + k'(x_2 + a)] &= b, \\ i.e., \quad \lambda^n [\log y_1 + \log \lambda + k'(x_2 + a)] &= \log y_1 + k'(x_1 + a), \end{aligned}$$

and

$$\lambda^n [2 \log \lambda + \log y_1 + k'(x_3 + a)] = \log y_1 + \log \lambda + k'(x_2 + a).$$

Using the contractions

$$\begin{aligned} 2 \log \lambda + \log y_1 &= A, \\ \log \lambda + \log y_1 &= B, \end{aligned}$$

we have

$$\frac{A + k'(x_3 + a)}{B + k'(x_2 + a)} = \frac{B + k'(x_2 + a)}{\log y_1 + k'(x_1 + a)}.$$

Now substitute various (likely) values of a , and find the corresponding values of k' from the resulting quadratic.

Thus if $a = 4$, since $A = .732$ and $B = .556$, we get $k' = .32$ or $.002$.

Proceeding in this way, we find the following:—

a	k'	a	k'	a	k'
0	.15	6	.88	10	-.35
1	.15	6.5	3.16	11	-.25
2	.15	7	7.45	12	-.20
3	.17	7.5	-2.50	13	-.15
4	.32	8	-1.15	14	-.15
5	.47	9	-.65	15	-.15

These values are seen plotted in diagram I.

Again,

$$\lambda^n [\log y_1 + \log \lambda + k'(x_2 + a)] = \log y + k'(x_1 + a) \quad . \quad . \quad . \quad (6)$$

When a was equal to 5 in the original curve for soft copper, this value seemed on the whole to give the closest approximation to one straight line. Let $a = 5$ be tried in the further approximation. In this case $k' = .47$.

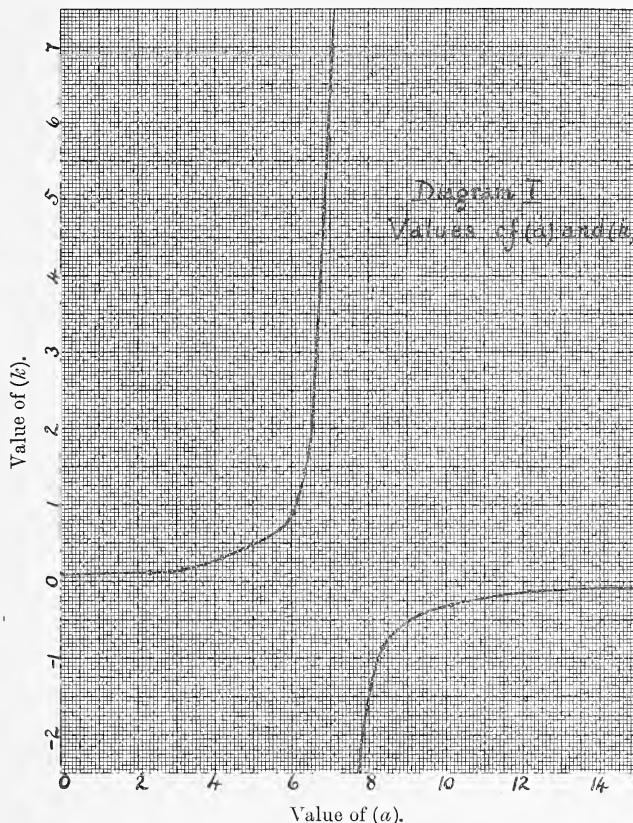
Substituting in equation (6), we find

$$n = 2.00.$$

But we know from the form of our equation that our value of n must be given by the equation

$$k' = \frac{2}{n+1}.$$

This value of a then is not suitable. With $a=6$, however, approximate agreement is obtained.



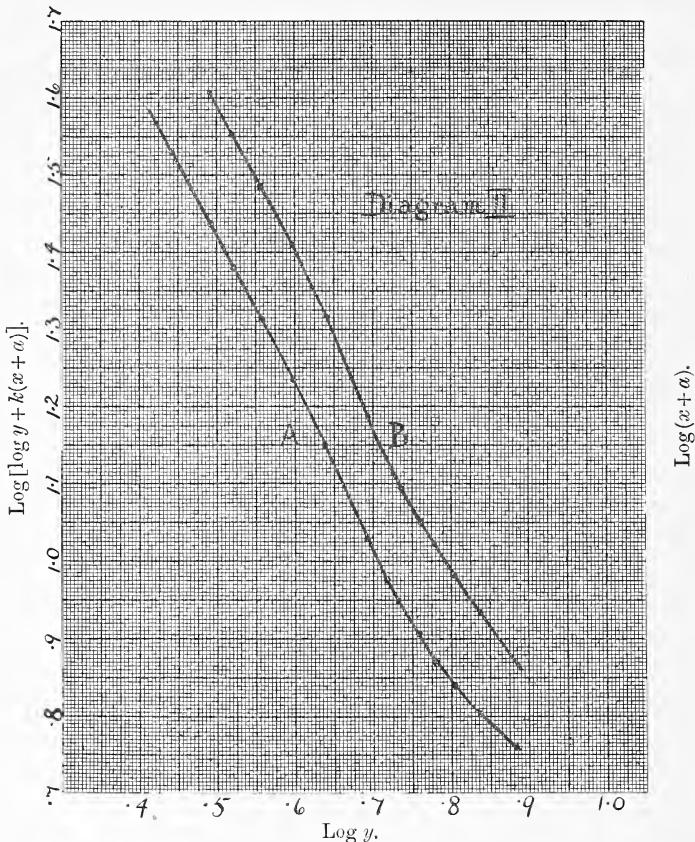
With the relation $n \log y + \log [\log y + k'(x+a)] = \log b$, plot $[\log y]$ against $\log [\log y + k'(x+a)]$, with $[\log y]$ as ordinate and $\log [\log y + k'(x+a)]$ as abscissa, using the values

$$a = 6, k' = .88,$$

and taking the corresponding values of $\log y$ and $(x+a)$ from the original curve for soft copper in its *s*-shape. Diagram II., curve B, shows the resulting curve, which is practically identical in form with curve A, the original $\log y$ against $\log(x+a)$ curve. It is still *s*-shaped, but shifted slightly further from the origin, and parallel to the old curve. The further

approximation, then, gives no better representation of results than that formerly used, and the approximation

$$y^n(x+a) = b$$



can be regarded as sufficiently good. This is only what might be expected from the fact that the $\log y, \log(x+a)$ curve consists of two very accurately linear parts meeting at a very sharply defined point.

(Issued separately April 23, 1913.)

XVIII.—On a Continued Investigation of the Application of the Law of Torsional Oscillation to Various Metals and Alloys.
By James B. Ritchie, B.Sc., Carnegie Research Scholar in Physics,
University College, Dundee. *Communicated by Professor W. PEDDIE.*

(MS. received November 18, 1912. Read November 18, 1912.)

THIS paper is a continuation of one already submitted to the Society, and published in their *Proceedings*.*

It showed that an equation of the form $y^n(x+a)=b$ could be applied to give close representation of results in the determination of the law of decrease of torsional oscillations of wires of different materials. This empirical equation, in which y represented the range of oscillation, x the number of oscillations since the commencement of observations, and n , a , and b constants for any one experiment, their numerical values depending upon the initial conditions of the wire and its subsequent treatment, was found to hold over a large range of oscillation for wires of many metals, and the present paper gives an account of further work on the subject.

CONTINUATION OF FORMER WORK.*

Brass Wire.

In the paper referred to there is given (p. 430) a diagram showing the relationship between the value of the constant n , and the temperature to which the wire had been initially subjected. After the temperature reached 370° C. two values of n were obtained for each temperature, the plotted curves ($\log y$ against $\log(x+a)$) giving two straight lines inclined at a gradually increasing angle, and cutting each other at a point corresponding to, roughly, twenty oscillations. It was found on comparing the separate curves for each temperature that the crossing point shifted fairly uniformly with increase of temperature, the change being directed upwards towards the points representing the smaller oscillations.

The actual results obtained are as follows. It will be noticed that the product of the temperature and the value of $\log y$ at the crossing-point remains fairly constant.

* *Proc. Roy. Soc. Edin.*, vol. xxxi. p. 424.

Temp.	Value of $\log y$ at break.
376° C.	·77
413° C.	·59
466° C.	·50
485° C.	·47
535° C.	·47
625° C.	·36
695° C.	·36

Since a specimen of brass wire heated above 370° C. invariably required two separate values of a to accommodate the points on two straight lines, and since the crossing-points of these lines occurred after twenty to thirty oscillations, several experiments were carried out with such a wire to discover if the average period of oscillation during the first fifty oscillations differed to any extent from that during the second fifty. Results showed that there was invariably a small decrease in the period of the latter, the values obtained for the given specimen being :

Unheated state : Average time of 1st fifty oscillations	= 4 mins. 35 secs.
, , 2nd , ,	= 4 , 35 ,
After heating to 477° C. : Average time of 1st fifty oscillations	= 6 , 35 ,
, , 2nd , ,	= 6 , 31 ,

Wires of Rarer Metals.

Wires of gold, platinum-iridium, platinum, and Assay silver were got in the pure state and tested for their conformity to the empirical law.

Gold.—Length taken = 29 cms. Diameter = 1 mm. The oscillations were found to die down fairly rapidly, and, although one line could not approximate to all the observed points, the same value of a sufficed. The values were :

A. $a = 20$	B. $a = 20.$
$n = .80$	$n = .90$
$b = 155.6$	$b = 168.4.$

A repeated experiment gave :

$a = 25$	$a = 25$
$n = .80$	$n = .90$
$b = 155.6$	$b = 174.7.$

In each of these cases the range taken was eighty oscillations, the break occurring after 50 had been observed.

A rather curious result was obtained on heating this wire and testing

when cold. It was raised to a temperature of 530° C., and, on plotting, it was found that, with $a=0$, an s-shaped curve was obtained; but, with $a=20$, all the points after the first five oscillations could be brought to lie on one straight line, the values being:

$$\begin{aligned}a &= 20 \\n &= 1.7 \\b &= 826.\end{aligned}$$

The slope of the line has thus been practically doubled. It was found that no value of a could be got to cause the first five points to lie on one line.

Platinum-Iridium.—Length = 30 cms. Diameter = 1 mm. Two lines were found in this case over a range of 90 oscillations, the values being:

A. $a=40$	B. $a=40$
$n= .90$	$n= .96$
$b=301$	$b=362.$

The doubling occurred at the value $.66$ for $\log y$. These results agree fairly closely with those got for commercial platinum as described formerly, the values for this being:

A. $a=28$	B. $a=30.$
$n= .95$	$n= 1.05$
$b=341$	$b=429,$

and the doubling points occurred at a value of $\log y= .80$.

Pure Platinum.—Length = 30 cms. Diameter = 1 mm. A range of 100 oscillations was observed in this case, and not only was one line insufficient to accommodate all the points, but even two lines could represent only up to 50 oscillations, and a third line was necessary. These are shown in Diagram I., and the values obtained were:

A. $a=25$	B. $a=75$	C. $a=75$
$n= 1.75$	$n= .60$	$n= .50$
$b=1136$	$b=275$	$b=243.$

Repeated experiments gave exactly similar values, the values of n being 1.85 , $.60$, and $.50$ respectively. This is the only wire yet experimented with which, over a range of a hundred oscillations, required three straight lines to cover the observed points.

Assay Silver.—Length = 30.5 cms. Diameter = 1 mm. A change was

experienced here again from the values got from the commercial metal. Two lines, with totally different values of (a) were required, the values being:

$$\text{A. } \alpha = 15$$

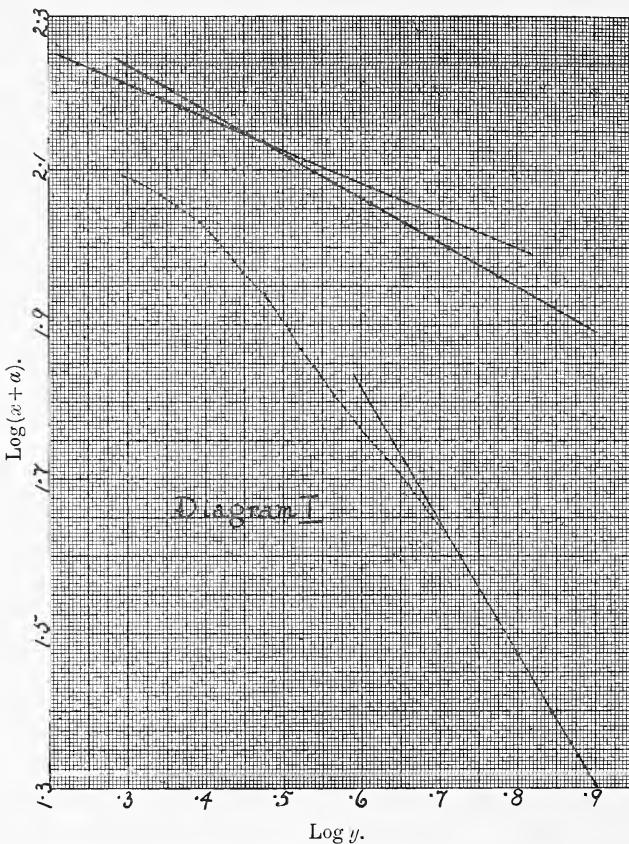
$$n = .42$$

$$b = .681$$

$$\text{B. } \alpha = 55$$

$$n = .33$$

$$b = 104.$$



In the commercial specimen one line was found sufficient, with $\alpha = 65$, $n = .45$, $b = .166$.

Aluminium Wire—Effect of Antecedent Heating or Cooling.

A series of experiments was carried out with wires of aluminium raised to increasing temperatures and cooled in the same manner as has been described in the former paper in the case of brass. The wires were heated in an electric furnace, and the range taken was between the ordinary temperature and 560° C. , the temperatures being recorded as before by a platinum thermometer. In each case two values of α were

required, giving two straight lines with different value of n . The actual values were :—

Temp.	1st Value of a .	2nd Value of a .	1st Value of n .	2nd Value of n .
15° C.	0	105	1·34	.20
150° C.	0	60	1·40	.45
200° C.	7	20	1·40	.47
243° C.	2	50	1·48	.47
280° C.	1	20	1·85	.67
315° C.	2	12	1·90	.92
325° C.	1	0	1·90	1·25
330° C.	4	0	1·90	1·70
350° C.	4	2	1·90	1·80
373° C.	1	2	1·90	1·90
380° C.	1	1	1·90	1·90
390° C.	3	6	2·20	1·80
420° C.	2	12	1·70	1·50
450° C.	0	12	2·80	2·05
475° C.	0	40	2·90	.95
565° C.	0	65	3·70	1·40

The first value of a remains in the vicinity of zero, whilst the second one, although fluctuating occasionally, tends on the whole to decrease to zero and increase thereafter. The first value of n tends to increase uniformly, whilst the second increases till a temperature in the neighbourhood of 370° C. is reached, and thereafter fluctuates. It is noticeable, however, that both values of a and both values of n tend to become identical at that temperature. That was also the temperature at which, in the case of brass, the one value of n , which till then had increased uniformly, split up into two values. Both metals, then, would seem to undergo a molecular change at that temperature, a change which is not noticeable in the larger oscillations, but whose effects come into play after the swing has died down to about half its original value.

Diagram II. shows the relationship between temperature and the value of n , and shows how the curves approach and finally touch at a temperature of about 370° C. Thereafter the values fluctuate so much that no conclusion can be drawn regarding the form of the curves, but they certainly tend to separate again after this point.

A length of 30 cms. was immersed in liquid air for 30 minutes, and subsequently tested, but no change in the constants was evident.

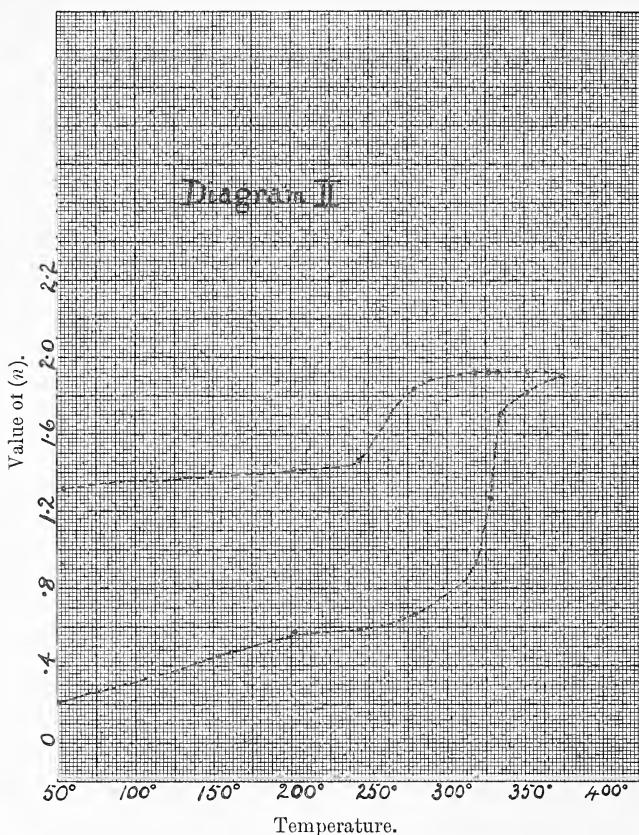
Aluminium Wire—Effect of Fatigue.

The same wire was then fatigued by means of a repeated rotational strain* for a period of two hours, but again, except for a slight lowering

* Proc. Roy. Soc. Edin., vol. xxxi. p. 440.

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of b , no change was obtained in the constants, although the period of oscillation after fatigue was found to have increased slightly. The average values gave :

$$\begin{array}{ll} \text{Before fatigue, period of 10 oscillations} = 178 \text{ secs.} \\ \text{After } , , , , = 182.5 \text{ secs.} \end{array}$$



Effects of Quenching.

A rather interesting result was obtained when copper* was raised to a bright red heat and cooled suddenly. The subsequent testing showed that three lines were necessary to cover all the points over the usual range, the values in the first experiment being :

$$\begin{array}{l} \text{A. } a=0 \\ n=2.45 \end{array}$$

$$\begin{array}{l} \text{B. } a=50 \\ n=.30 \end{array}$$

$$\begin{array}{l} \text{C. } a=50 \\ n=.50 \end{array}$$

* Cut from the same wire as was formerly tested.

Repeated experiments gave the values :

A. $a = 0$	B. $a = 50$	C. $a = 50$
$n = 2.30$	$n = .40$	$n = .60,$

and

A. $a = 1$	B. $a = 20$	C. $a = 25$
$n = 1.70$	$n = .40$	$n = .50.$

Although no exact record of the temperature obtained was got, the wire in the latter case was raised to a much brighter heat than in the former two cases.

German silver wire raised to a red heat and quenched gave values of n far higher than any got formerly for the same metal ; whilst two distinct lines were required, a being equal to zero in both cases, and n having the values 2.40 and 2.25 for large and small oscillations respectively.

A specimen of *iron* wire, tested in the original state, gave the results :

A. $a = 0$	B. $a = 15$
$n = 2.25$	$n = .60.$

On raising to a high temperature (bright redness) and subsequently annealing, both values of n were found to be lowered, whilst another length of the same specimen, when raised to approximately the same temperature and quenched, gave similar values. The values of n in the first case were 1.60 and .33, and in the second case 1.80 and .25.

WORK ON ALLOYS.

With rise of temperature, brass has been shown (see former paper) to behave in a way which contrasts strongly with aluminium. In the case of brass, one value of n , and thus one straight line, is got at the ordinary temperature ; this one line, although changing in inclination to the axis, persisting up to a limiting temperature in the region of 370° C. When this temperature is reached the one line breaks into two, and two values of n are always got thereafter on further increase of temperature. With aluminium, on the other hand, two values of n are observed at the ordinary temperature, these two values gradually converging with rise of temperature, and finally coinciding at the same limiting temperature as before. It was thought, then, that an alloy of the two metals might give interesting results, and that temperature changes might be got which would prove interesting in relation to those already obtained for the separate constituents.

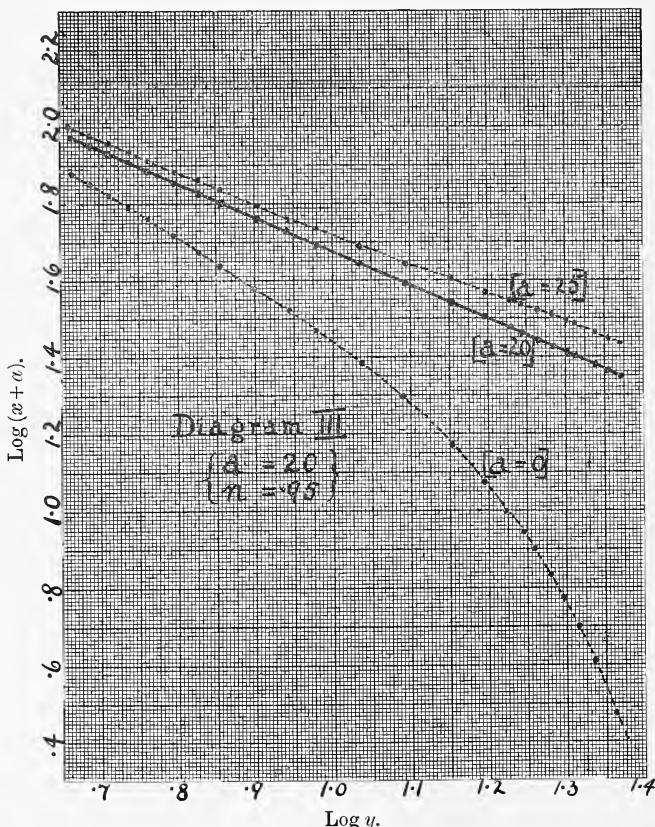
Three separate brass-aluminium alloys were made ; the brass was

melted first (a furnace with mechanical blower being used) under charcoal, glassy borax was added as a flux, and the aluminium was then gradually stirred in by a wooden rod. Two of the products were too brittle to draw, but a wire of 15 cms. length and 5 mm. diameter was got from the third casting. This was subsequently analysed in the chemical department and gave as constituent proportions, copper 74·4, aluminium 6·2, and zinc 19·6 per cent. This wire when oscillated and tested gave the values:

$$\begin{aligned} A. \quad a &= 1 \\ n &= 1.75 \end{aligned}$$

$$\begin{aligned} B. \quad a &= 1 \\ n &= 1.55. \end{aligned}$$

Repeated experiments gave similar values. The wire was then raised successively to temperatures of 300° C., 375° C., 400° C., 425° C., 450° C.,



and 500° C., but after cooling no change in the constants was obtained. A steady state as regards change by temperature must then have been reached in the heating undergone while the wire was being drawn.

A series of four *copper-aluminium* alloys were also cast and drawn in

the form of wires, of length 10 cms. and diameter 1 mm. It was found that any casting outside the limits 92 per cent. copper with 8 per cent. aluminium on the one hand, and 88 per cent. copper with 12 per cent. aluminium on the other, was useless for drawing purposes; but that within these limits wires could be fairly easily got with careful management. In all cases but that of alloy (1) one value of a was found sufficient to group the points on one line, whilst a uniformly decreasing average value of a and n was got with decreasing percentage of copper, these percentages being determined only roughly. The values were:

- (1) 92 per cent. Cu - 8 per cent. Al : $a = 6$ and $30 : n = .75$ and 1.70
- (2) 90 " Cu - 10 " Al : $a = 20$: $n = .95$
- (3) 89 " Cu - 11 " Al : $a = 10$: $n = .65$
- (4) 88 " Cu - 12 " Al : $a = 5$: $n = .58.$

A specimen of Heusler alloy—of composition 62 per cent. copper, 25 per cent. manganese, and 13 per cent. aluminium—was also cast, but the alloy was found to be far too brittle to draw.

Diagram III. shows the nature of the curve got with alloy (2).

EFFECTS OF MAGNETIC FIELD ON CONSTANTS FOR IRON.

To induce magnetism in the wire it was held within a long coil through which a current was run, and a magnetic field thus established in the neighbourhood of the wire. After removal from the coil the wire underwent a process of "ageing," in order that a steady magnetic state should be reached. The plan adopted was to torsionally oscillate it in the torsion apparatus formerly described.* Preliminary experiments were made to see if this "fatigue" in itself would alter the constants. The specimen of iron wire taken gave originally two sets of values :

A. $a = 2$	and	B. $a = 12$
$n = 1.75$		$n = .70$
$b = 430$		$b = 102.8.$

This wire was then subjected to ninety minutes torsional fatigue and again tested, giving the values :

A. $a = 0$	B. $a = 30$
$n = 1.85$	$n = .60$
$b = 169$	$b = 92.5.$

The only constant seen to change much is the b of line A, *i.e.* for the large oscillations. Repeated experiments showed this large increase of b to be real.

* Proc. Roy. Soc. Edin., vol. xxxi. p. 440.

With another specimen, giving similar original values, and subjected to fifteen minutes' fatigue, values were got :

$$\begin{array}{ll} \text{A. } a = 1 & \text{B. } a = 30 \\ n = 1.80 & n = .55 \\ b = 225 & b = 96. \end{array}$$

This value of b seems to be distinctly nearer the original value. This was confirmed by an experiment where five minutes' fatigue only was given. Here again all the constants were similar except the b of the first line, whose value was now 327. The value of b (169), obtained after ninety minutes' torsion, was found to tend to regain the normal value after the wire had been allowed to stand for a few hours to recover from fatigue. After one hour's recovery no appreciable difference was observed, but after one day's recovery the values were for the first part of the line,

$$\begin{aligned} a &= 0 \\ n &= 1.90 \\ b &= 266 ; \end{aligned}$$

and after two days' recovery,

$$\begin{aligned} a &= 0 \\ n &= 1.90 \\ b &= 371. \end{aligned}$$

With fatigue, then, the first value of b is the only one of the constants to be markedly changed. In the following experiments five minutes' torsion was in each case given for ageing, and the change again produced in the first value of b is too great to be accounted for by the effect of such fatigue. In each case the length of the wire used was far greater than that of the portion actually tested. This portion was cut from the middle of the original length, and a suspension of one minute in the coil was in each case given.

Again no change was found in the values of a or n , but the value of b for large oscillations was found to vary considerably. The results were :

Current.	b of Line A.
.1 Amp.	588
.2 Amp.	728
.3 Amp.	946
.4 Amp.	519
.5 Amp.	748
1.0 Amp.	1200.

Whilst no uniformity of value is got, a large increase on the whole is observed.

OBSERVATIONS ON A COMPOUND WIRE.

It was formerly seen* that a specimen of copper wire when stretched to the limiting tension of rupture gave one line only, with the values

$$a = 18, \quad n = 1.05,$$

and also that zinc† had a similar curve with a straight line got when

$$a = 0, \quad n = .60,$$

and the effect of electrolytically depositing a gradually increasing coating of zinc on a specimen of stretched copper was observed. Twelve experiments in all were carried out, a deposit of .8 gram of zinc being added in stages by the decomposition of a solution of $ZnSO_4$. The wire was thoroughly cleaned and momentarily dipped in acid before being immersed in the solution, and it was found that a fairly uniform coating could be got until .8 gram of zinc had been added to the copper wire (of initial weight .69 gm.). After this point the depositions became irregular and non-adherent. It was found on plotting that the straight line originally got for copper gradually and uniformly altered, giving a lesser and lesser value of n , until the value approached that of zinc itself. Accurate values of n could not be got, however, since a wavy curve was got in each case, due to pendulum swings set up in the oscillator on account of the slight irregularities of deposition. By taking a copper core of very small diameter an attempt was made to obtain practically a deposited zinc wire; but, although such a wire was obtained, the lightest oscillating ring obtainable with suitable moment of inertia was of sufficient weight to snap the wire.

* *Proc. Roy. Soc. Edin.*, vol. xxv. p. 437.

† *Ibid.*, p. 438.

(Issued separately May 19, 1913.)

XIX.—Note on a Comparison of Records of Atmospheric Electric Potential at Two Stations in Edinburgh. By G. A. Carse and G. Shearer, Edinburgh University; and H. Jameson, Royal Observatory, Edinburgh.

(MS. received February 26, 1913. Read March 3, 1913.)

THE electrograms (continuous records of the atmospheric potential) were obtained by means of electrographs of the Kelvin water-dropper type at the Physical Laboratory, Edinburgh University, and the Royal Observatory, Blackford Hill, and as the inspection of the records showed some remarkable similarities, a more detailed comparison of the records was instituted, the results of which are given in this paper.

We are indebted to the Astronomer Royal for Scotland, Professor Sampson, for access to the Observatory records.

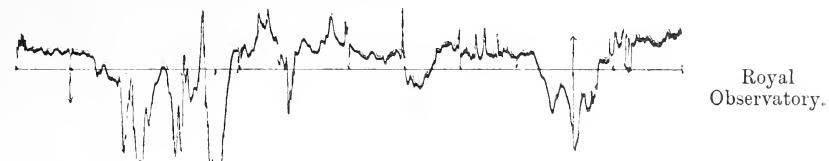
In the comparison of the curves considerable difficulty was met with on account of the difference in sensitiveness of the electrometers at the two stations. Often, when the curves appeared to agree excellently, it was found impossible to obtain definite numerical results of this agreement on account of the extreme difficulty of picking out with any degree of certainty corresponding points on the two curves.

The plan adopted was to arrange the curves in three groups, A, B, C, A containing those which showed good agreement, B fair agreement, and C bad or no agreement. This was carried out for the months January to June 1912, and the results are given in the following table. All days were compared with the exception of those days on which one or other of the

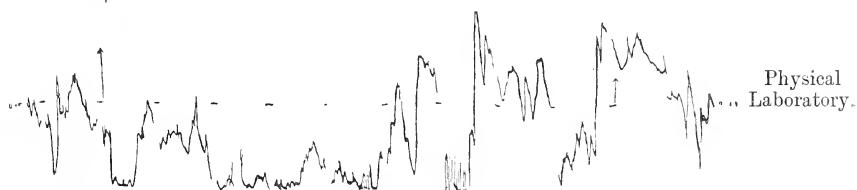
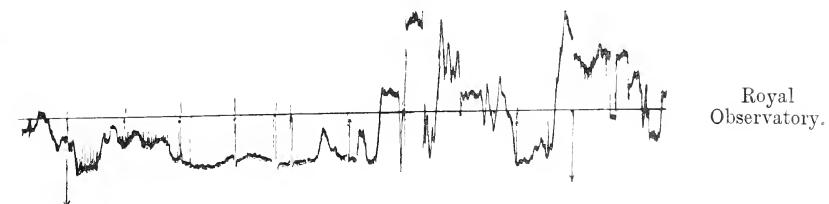
TABLE I.

1912.	Percentage of			No. of Curves compared.
	A.	B.	C.	
January	55	36	9	22
February	45	36	30	20
March	36	36	28	28
April	36	36	28	28
May	52	31	17	29
June	26	58	16	19

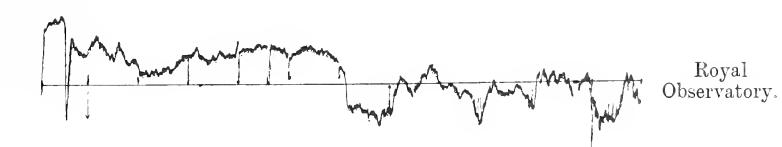
TYPE A.



March 4 and 5, 1912.

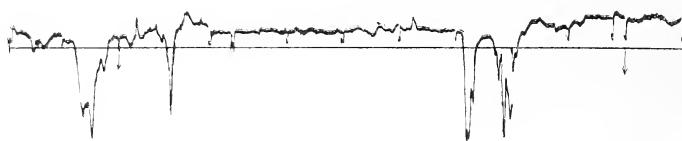


January 16 and 17, 1912.

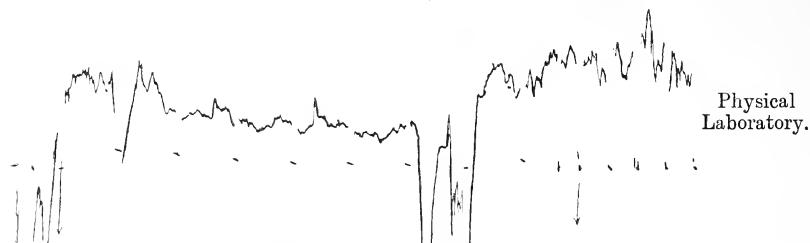


January 15 and 16, 1912.

TYPE B.



Royal
Observatory.

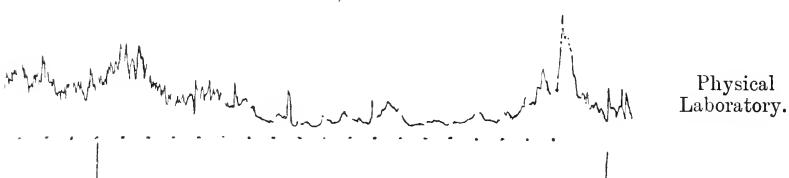


Physical
Laboratory.

March 14 and 15, 1912.



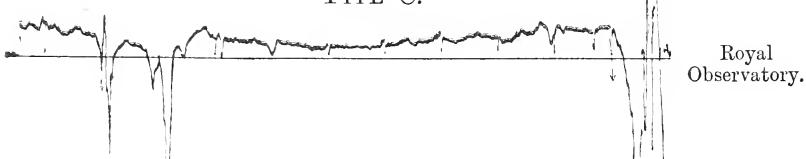
Royal
Observatory.



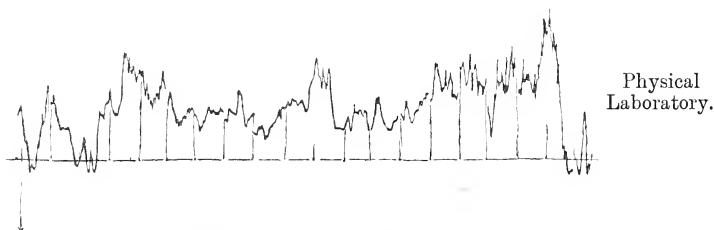
Physical
Laboratory.

April 19 and 20, 1912.

TYPE C.



Royal
Observatory.

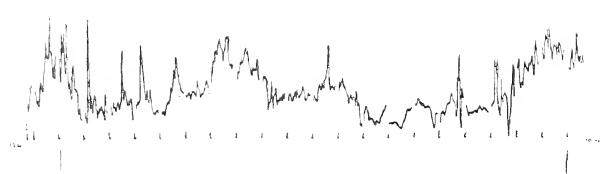


Physical
Laboratory.

March 19 and 20, 1912.



Royal
Observatory.



Physical
Laboratory.

April 23 and 24, 1912.

electrographs was not working on account of mechanical breakdowns. The curves reproduced show examples of the types of agreement referred to as A, B, and C. In order to facilitate the comparison, the time scales have been reduced to the same unit.

It will thus be seen that for the six months, January to June 1912, 42 per cent. of the 146 curves investigated came under group A, 37 per cent. under group B, and 21 per cent. under group C.

Kähler,* in a series of papers in the *Meteorologische Zeitschrift*, gives the result of a comparison of electrograph curves at three stations, two separated by 100 metres at Potsdam, and the third distant 1·4 km. He found a good agreement between the curves obtained from the various stations.

As the Physical Laboratory is situated in the centre of Edinburgh and is surrounded on all sides by buildings and factories, there is always present in the air in the neighbourhood a considerable quantity of soot and dust particles.

A comparison between the records obtained there and those at Blackford Hill Observatory should be of interest, as the Observatory is outside the town, and the air, if not free from soot, etc., is at least much clearer than that in the neighbourhood of the Laboratory. The two stations differ also in height above sea-level, the Observatory being about 420 feet above sea-level, and the water-dropper 19 feet above the ground, while the Laboratory is about 245 feet above sea-level, the water-dropper 34 feet above the ground. The distance between the Observatory and the Laboratory is 1·8 miles (2·9 km.), the Laboratory being north of the Observatory.

While the curves obtained at the two stations differ from one another, the difference, generally speaking, is in the magnitude rather than in the direction of the variations. It is comparatively seldom that the potential is falling at the Observatory while rising at the Laboratory, or vice versa. In the magnitude of the variations, however, there is frequently a wide difference between the two stations.

It is to be noted that the curves which show the best agreement are those which indicate a very disturbed state of the atmosphere. It is very seldom that a negative potential gradient is found at the one station accompanied by a positive gradient at the other. While the potential is negative, and on days when there is frequently a change from positive to negative values of the gradient, the curves are often almost identical. The occurrence of negative potential takes place most frequently during

* *Meteor. Zeit.*, 155-162, 289-299, xxv., 1908; 10-17, 347-355, xxvi., 1909.

rain, and it is perhaps to be expected that the curves would agree best under these circumstances; but when the wild and rapid variations, often ranging from +500 volts/metre to -500 volts/metre, are considered, it is remarkable that the two stations, so distant from one another and in such different surroundings, should give curves which agree down to small details.

As the change from positive to negative is often a very sudden one, it has been found possible to get some numerical data with regard to the times at which corresponding changes took place.

The following table gives the results of the examination of some curves, and it will be seen that the points which from the curve obviously correspond to one another occur at very nearly the same time at the two stations:—

TABLE II.

Date, 1912.	Times of Corresponding Points.		Type of Potential Gradient.	Wind.	Remarks.
	Physical Laboratory.	Royal Observatory.			
Jan. 6	6.20 p.m.	6.18 p.m.	Neg.	E.	Calm, snow-shower in evening.
„ 6-	8.38 p.m.	8.30 p.m.	Neg.	E.	Fresh breeze.
„ 6	9.11 p.m.	9.11 p.m.	Neg.	N.E.	Fresh breeze.
„ 7	8.30 a.m.	8.25 a.m.	Pos.	N.	Breeze, frosty, fine.
„ 11	12.08 a.m.	12.06 a.m.	Neg.	E.S.E.	Gale, dull.
„ 11	12.28 a.m.	12.30 a.m.	Neg.
„ 11	1.14 a.m.	1.13 a.m.	Neg. { Neg. during 13 mins.		
„ 11	1.27 a.m.	1.26 a.m.	Neg. } at both stations.		
„ 11	4.09 a.m.	4.11 a.m.	Neg.	S.S.E.	...
„ 11	6.42 a.m.	6.40 a.m.	Neg.
„ 11	8.06 a.m.	8.12 a.m.	Neg.	S.S.W.	Moderate breeze.
„ 11	8.29 a.m.	8.34 a.m.	Neg.
„ 13	2.58 a.m.	3.06 a.m.	Neg.	S.	Fresh breeze, fine.
„ 13	3.40 a.m.	3.45 a.m.	Neg.	...	"
„ 13	3.54 a.m.	3.57 a.m.	Neg.	...	"
„ 13	4.06 a.m.	4.06 a.m.	Neg.	...	"
„ 14	3.15 p.m.	3.12 p.m.	Neg.	S.	Moderate breeze, rain.
„ 14	4.44 p.m.	4.46 p.m.	Neg.	S.S.W.	"
„ 14	5.12 p.m.	5.21 p.m.	Neg.	S.S.W.	Light breeze.
„ 15	1.11 p.m.	1.11 p.m.	Neg.	S.E.	Breeze, damp, overcast.
„ 16	12.21 a.m.	12.20 a.m.	Neg.	S.S.E.	Gale, showery.
„ 16	2.23 a.m.	2.26 a.m.	Neg.	S.S.E.	"
„ 16	3.04 a.m.	3.00 a.m.	Neg.	S.S.E.	"
„ 16	7.50 a.m.	7.54 a.m.	Neg.	S.S.E.	Gale, rain.

The times of apparently corresponding points have been measured to within two minutes, but too much stress must not be laid on the time differences on account of the difficulty, already mentioned, of picking out

corresponding points. Kähler* found a mean lag of 3·9 minutes, and showed that this lag depended on cloud-drift, and to a lesser degree on wind direction and strength.

A difference of 4 minutes between the two stations might be due to errors in measuring the curves, and in the few cases where there seems to be evidence for a lag this is probably due to the two points measured not being exact points of correspondence.

There is thus no conclusive evidence for a lag due to wind or cloud drift. More accurate means of measuring time, and also fuller data with regard to cloud drift, would be necessary for a proper investigation of the question. The prevailing wind in Edinburgh being westerly, and the two stations being north and south of one another, it is not to be expected that the mean lag would be a very large one. Assuming a mean velocity of wind for these disturbed days at 20 miles per hour, and the distance between the stations as 1·8 miles, then with a south wind we would expect a lag of the Laboratory curve behind the Observatory curve of about 5 minutes, while with any other wind direction the lag would be smaller.

The differences of magnitude of the variations which are evident between the two stations are probably due to the difference between the purity of the air, any change of conductivity in the air being accompanied by a greater effect at the Blackford Hill than at the Laboratory, on account of the very much larger number of soot and dust particles present in the neighbourhood of the latter station. These act as nuclei to which the ions get attached, their velocities being thus considerably reduced.

There is one other difference between the stations which may have some effect in giving rise to differences in the curves. The water-droppers at the two stations differ in height above the ground. It has been shown that in Paris† considerable differences are obtained in the curves showing variation of potential at different heights above the ground. Any change which was merely a change in the layers of the air nearest the ground might affect one station and leave the other unaffected.

Generally speaking, however, the influence of the town air is much smaller than might have been expected, and the curves obtained in the town and those at a station on the outskirts of the town distant 1·8 miles are more or less similar, the difference being, generally speaking, difference of magnitude.

* Kähler, *loc. cit.*

† Chauveau, *L'Électricité atmosphérique*, Paris, 1902.

XX.—Changes of Electrical Resistance accompanying Longitudinal and Transverse Magnetizations in Nickel. By Professor C. G. Knott, D.Sc.

(Read January 20, 1913. MS. received January 23, 1913.)

IN 1903 I communicated to the Society a paper on the relation between magnetization and resistance of nickel at high temperatures (1). In this paper the magnetization was along the direction in which the resistance was measured. A second paper, in which the magnetization was transverse to the direction in which the resistance was measured, was communicated in 1906 (2). In these later experiments a flat coil of nickel wire was used; and it was necessary to use very high fields before an appreciable change of resistance was obtained. The results indicated that, in a strongly magnetic metal like nickel, the magnetization, and not the magnetizing force, was the determining factor. It followed that the change of resistance accompanying the application of a transverse magnetic force must depend, like magnetization, upon the form of the material. In order to magnify the resistance effect I resolved to use a fairly broad strip of nickel instead of a thin wire. The chief difficulty of measurement would then lie in the smallness of the resistance of such a strip. Still, with a sufficiently delicate galvanometer this difficulty might be overcome. Accordingly, I obtained from Germany a quantity of thin nickel sheeting of pure nickel, and proceeded to investigate its properties in exactly the same way as had already been applied in the case of nickel wires.

As I had expected, the change of resistance of a nickel strip when magnetized transversely—that is, in the direction of the width of the strip—became easily measurable in moderate fields.

To find how the width influenced the magnitude of the effect I took three strips of widths 8, 4, and 2 cm., rolled them into compact coils, and placed each axially in turn in the air-gap of an electromagnet. The resistance changes due to the action of approximately the same magnetic field were as the numbers 62, 47, and 22·5. These numbers can be accepted only as a rough quantitative indication; for the part of the air-gap occupied by the coil varied in length according to the width of the nickel strip which formed the coil, and consequently the average field could not be the same for all. As is well known, the strength of the field at different points

along the axis of an air-gap of the length required to admit all the coils is not by any means uniform.

It then occurred to me that some interesting results might be obtained if the nickel were subjected to a combination of the two fields—the one along the direction in which the resistance was measured, and the other at right angles thereto. A few rough experiments towards the end of July 1905 convinced me that the investigation was worth following up.

Accordingly, in December of the same year, with an improved form of apparatus, I began the first series of experiments. Experience suggested further improvements; and the experiments which are now to be described were carried out between May 14 and June 27, 1906.

My intention was to carry out similar experiments with iron strips. Unfortunately, the arrangement had to be taken to pieces in consequence of the removal of the Natural Philosophy and Applied Mathematics Departments from the Old University to the new quarters in Infirmary Street, an event which involved so many readjustments and developments that it was never found possible to continue the present research on the lines originally laid down.

As the results obtained with the nickel seemed to be both novel and interesting, I propose to discuss them now by themselves, with the hope, however, that further results in the same line of study may be obtained before long.

As already indicated, the main idea of the experiments is to subject the magnetic metal to a combination of magnetic fields acting in mutually perpendicular lines, and to measure the associated change of resistance for each combination.

A strip of nickel of width 2·1 cm. and of resistance 0·0378 ohm was doubled over and rolled up like a tape measure so as to form an anchoring core for a transformer-wound coil. The neighbouring turns of the nickel were insulated by interleaved asbestos sheeting, and the magnetizing coil was wound round in two layers, there being 57 turns in the inner layer and 51 in the outer. In its final form the coil formed a roughly elliptical anchor ring, whose external diameters were 4 and 3·5 cm., and interior diameters 1·7 and 1·3 cm.

If we regard this as electromagnetically equivalent to a circular anchor ring of radius 1·31 cm., we find in accordance with the usual formula that one ampere passing through the magnetizing coil will produce in the interior an average field of 16·5 gauss.

This coil was then placed axially in the gap of a solenoid consisting of six large coils placed end to end, three on each side of the gap. The field

within the gap for various strengths of currents passed through the solenoid was measured by means of a bismuth coil. The lines of magnetic force due to this field ran across the nickel—that is, transversely to the magnetic field established in the anchor-ring coil enclosing the strip.

The method of experiment was practically identical with the method used in my former papers on change of resistance with temperature. The nickel strip formed the greater part of one arm of a Wheatstone bridge, an approximate balancing being secured by adjustment of the point of contact on a stretched wire. The combined system of conductors forming the Wheatstone bridge was made part of a circuit through which a small steady current was passed from a secondary cell. When this current was flowing steadily through the circuit, one of the known resistances in the bridge was altered slightly in a definite manner by introducing a large resistance shunt in parallel with a portion of the resistance. The deflection obtained on the galvanometer, being due to a measurable disturbance in the balance, was essentially a standardising of the deflection. This calibrating shunt being thrown out of connection, the magnetic force or forces were now brought into action on the nickel strip. The balance was again disturbed on account of the magnetization of the nickel.

The comparison of this deflection with the former deflection due to the known change of resistance in the other arm gave a ready means of calculating the proportionate change of resistance in the arm containing the nickel.

Thus let P, Q, M, N be the resistances of the arms of the Wheatstone bridge, N being the one of which the nickel strip forms a part. Then the current in the galvanometer is given by the formula

$$C = E(PN - QM)/R,$$

where E is the electromotive force and R is a function of the resistances which make up the system. In the experiments now under discussion, the galvanometer resistance was about 20 ohms, and the resistance of the branch containing the cell fully 6 ohms, while the resistances P, Q, M, N were all less than 1 ohm. As was shown in my former paper, this ensured that any slight change in M or N did not appreciably affect the value of R .

A known change dM in M will therefore produce a corresponding change in the current, whose value will be, very approximately,

$$dC = -EQdM/R.$$

Similarly, a small change dN in N will produce a corresponding change in the current of value

$$dC' = +EPdN/R.$$

Hence

$$\frac{PdN}{QdM} = -\frac{dC'}{dC} = -s,$$

a ratio given immediately by the experimental readings.

But initially, since C is small, the balanced Wheatstone-bridge ratio holds to a sufficient approximation, namely,

$$P/Q = M/N;$$

and hence finally we have the very simple relation

$$dN/N = -sdM/M,$$

where everything on the right-hand side is known. If now n is the resistance of the nickel which forms part of the conductor whose resistance is N , the change dn is the same as dN , and the change in unit resistance is to be calculated from the expression

$$\frac{dn}{n} = \frac{N}{n} \frac{dN}{N} = -s \frac{N}{n} \frac{dM}{M}.$$

In the present experiments the resistance of N was 0·0478 and that of the nickel strip 0·0378, giving the ratio N/n equal to the reciprocal of 0·791.

As the experiments were carried out, the deflections were obtained by reversing the steady current through the system. The following two sets of numbers, taken just as they were jotted down in the laboratory book, will show how the observations were made and so far reduced:—

EXPERIMENT OF JUNE 6, 1906. CALIBRATION EXPERIMENT.

Shunt.	Readings.				
∞	131	131·9	83·5	84	
50	101	101·3	114·1	114·5	
∞	131·2		83·8		
Deflection	= <u>30·1</u>	<u>30·2</u>	<u>30·4</u>	<u>30·6</u>	$= 121·3 \div 127 = .955$

The first reading 131 is obtained with positive current without the shunt resistance in connection. The shunt 50 is then introduced, and the reading changes to 101. The shunt is thrown out, and the reading becomes 131·2. It is put in again, and the galvanometer reads 101·3 (passing up the second column). Finally, with the shunt thrown out, the reading is 131·9. The current is then reversed and an exactly similar set of readings are taken. These appear in the fourth and fifth columns, the fifth being read up as in the case of the third. The fourth row of figures gives the deflections, each of which is the difference of the "shunt" reading and the mean of the "non-shunt" readings which precede and follow. The sum of these four deflections is taken and divided by the reducing factor

127, which is calculated from the resistances in the circuit, and this reduction gives 10,000 times the value of what I have called above dM/M .

In the continuation of the experiment of June 6 which follows, H stands for the longitudinal field and T for the transverse field, "longitudinal" meaning along the length of the strip whose resistance is being measured, and "transverse" at right angles to this direction. The successive readings of the galvanometer which fill the first three columns of numbers now run across the page, each set of three forming one measurement. The first and third are the readings obtained with the steady current in the positive direction, the second that obtained with the current reversed. The fourth column contains the deflections, which are the differences between the second column numbers and the mean of the corresponding first and third column numbers.

EXPERIMENT OF JUNE 6, 1906—*continued.*

T=8 cells. H=11 cells.

Cyclic Field.	T=0.	Readings.	1st Difference.	2nd Difference.	
+ H	132·8	84	132·7	- 48·8	
	130	111·8	124	- 15·2	+ 24·3
	121	91	121·3	- 30·2	
- H	123·3	117	120	- 4·7	+ 22·2
	117	94	118	- 23·5	_____
				Sum	+ 46·6 ÷ 955 = + 48·7
					+ T = 11·7 on galv.
+ H	137	76·2	138	- 61·3	
	150	91	145·3	- 56·7	- 2·1
	129	81·5	129·5	- 47·8	
- H	142·5	98·2	138	- 42·1	- 1·4
	122	88·4	122	- 33·6	_____
				Sum	- 3·5 ÷ 955 = - 3·7
					+ H = 11 cells.
- T	108	132·5	103	+ 27	
	128	111·3	124	- 14·7	- 58
	92	148	85	+ 59·5	
+ T	111·3	129	104	+ 21·3	- 58·5
	71	168	65	+ 100	_____
				Sum	- 116·5 ÷ 955 = - 122
					H = 0.
- T	58·2	147	62·2	+ 86·8	
	83	128	87·2	+ 42·9	- 33·9
	72·5	141·1	76	+ 66·8	
+ T	94	121·7	96·8	+ 26·3	- 33·7
	82	134·2	84·3	+ 51·1	_____
				Sum	- 67·6 ÷ 955 = - 70·8

The first group of five measurements is obtained with no transverse field, the longitudinal field being put on and removed twice with change of direction between the first and second applications. The transverse field is then applied and kept steadily in action for the next five successive sets of measurements, the longitudinal field being put on and off with reversal of direction as before. In the next group the longitudinal field is in its turn applied steadily, while the transverse field is put on and off just exactly as the longitudinal field was manipulated during the first and second groups of measurements. Finally, in the fourth group the longitudinal field is thrown off altogether, and the transverse field applied and removed by itself in a cyclic manner, as was done with the longitudinal field in the first group.

The field which is put on and off twice in succession with reversal of direction will be distinguished as the "cyclic field"; and the other, which for the time is being maintained, as the "steady field."

The application of the cyclic field is indicated at the left-hand margin, the presence or absence of the steady field at the head of each group of five measurements.

It will be noticed that the steady field is applied only in one direction. In the early sets of experiments observations were made with the steady field, first in one direction, and then in the other. The effect, however, is quite independent of the direction of the field. This is clearly shown by the fact that the second differences in the fifth column of numbers are almost exactly the same for the two directions of the cyclic field. These second differences are calculated on the same principles as the first differences. Each is the difference between the deviation of exact balance when the cyclic field is on and the mean of the preceding and succeeding deviations from exact balance when the field is off.

The closeness in value of the second differences in each group is in fact a check on the accuracy of the experiment. This is well brought out in the experiment shown. In the majority of cases the same satisfactory result was obtained, although in some instances the agreement was not quite so good.

This agreement between the two values of the second differences is the more striking when it is noticed that the first differences in each group vary greatly among themselves. If no other resistance-changing cause were present than the application and removal of the magnetizing forces, the first, third, and fifth in each column of first differences should be the same; and similarly the second and fourth should be the same. But in each series there is a steady change, which is at once explained as due to the heating effect of the current which establishes the longitudinal field.

The current in the solenoid which establishes the transverse field is too far distant from the nickel to have any direct temperature effect upon it.

In the first and second groups of five measurements, the current in the anchor-ring coil lasts only for a fraction of the whole time, producing an increase of resistance represented by 25.3 ($= 48.8 - 23.5$) in the first group and by 27.7 in the second. In the third group, however, the longitudinal field current is flowing the whole time; and the corresponding change of resistance is represented by the number 73. These differences are all in the same direction, increasing positive or decreasing negative. Roughly speaking, the heating current in the first and second groups of experiment acts in a periodic manner for two-fifths of the whole time of the five measurements, whereas in the third experiment the current flows for the whole time. Now, two-fifths of 73 is 29.2, which is as close to 27.7, or even 25.3, as we could reasonably expect.

Finally, in the fourth group the differences are positive and diminishing, indicating therefore a decreasing resistance by an amount represented by the number 35.7. But this is quite in line with the general explanation; for now there is no heating current in the anchor-ring coil, which begins to cool down towards its original temperature at about half the rate of its previous rate of heating. It is no doubt this heating effect which makes the experimental errors somewhat considerable in a few instances.

The final reduction in the notebook is to the resistance change dN/N , which is the ratio of the sum of the two final differences to the calibration number already obtained. This particular way of making the reduction is necessary because the first difference 30.1 in the calibration experiment corresponds with half the first differences in the other experiments, since these are obtained by the reversal and not merely by the making or breaking of the steady current through the Wheatstone bridge. Hence, to the sum of the four values of the calibration difference used in making the reduction will correspond the sum of the two final values of the deflections due to the magnetic change of resistance. The method is of course simply a convenient way of comparing means.

Each change of resistance is expressed in terms of 100,000 ohms, and each final value is deduced from fifteen individual readings on the galvanometer scale. The four groups of observations of the kind indicated above may be regarded as forming one complete experiment for a given combination of longitudinal and transverse field. The four changes of resistance measured are produced by the longitudinal field (h) acting cyclically alone, by the same field acting cyclically when the transverse field (t) is maintained, by the transverse field (t) acting cyclically alone, and by the same

field acting cyclically when the longitudinal field (h) is maintained. These changes will be represented by the notation H , $H(T)$, T , and $T(H)$. Thus $H(T)$ means the result as regards resistance change due to the longitudinal field (h) acting on what might be called the T -condition.

These reductions to the change per 100,000 of the branch of the Wheatstone bridge containing the nickel strip are given in practically chronological order in the Appendix. The last or unit figure in these numbers is of course quite within the errors of observation. In the final reductions this last figure is omitted and the changes of resistance are estimated per 10,000. The reductions to changes of resistance of the nickel itself are given in Table A. These numbers are obtained from the numbers in the Appendix by division by the number 0.791.

TABLE A.

The tabulated numbers are changes of unit resistance $\times 10^4$.

Longitudinal Fields.		Transverse Fields.								
		50	110	170	270	430	580	700	815	
10.6	H	+12	+ 9	+ 13	+ 11	+ 12	+ 11	+ 8	+ 11	I.
	T(H)	-10	-48	- 78	- 90	-102	-106	-108	-108	
	T	-13	-35	- 64	- 75	- 84	- 88	- 88	- 91	
	H(T)	+10	+ 3	0	0	- 1	- 1	- 2	- 2	
17.7	H	+23	+24	+ 23	+ 26	+ 23	+ 26	+ 22	+ 25	II.
	T(H)	-15	-57	- 91	-112	-116	-124	-125	-112	
	T	-12	-42	- 64	- 75	- 88	- 89	- 89	- 88	
	H(T)	+19	+19	+ 2	0	+ 1	- 3	- 2	- 3	
28	H	+39	+40	+ 38	+ 37	+ 38	+ 39	+ 37	+ 39	III.
	T(H)	-15	-65	-106	-126	-131	-147	-144	-157	
	T	-11	-42	- 63	- 77	- 86	- 87	- 87	- 92	
	H(T)	+32	+19	+ 5	0	- 3	- 5	- 6	- 4	
37	H	+52	+49	+ 49	+ 58	+ 62	+ 50	+ 49	+ 47	IV.
	T(H)	-29	-66	-112	-144	-154	-159	-162	-160	
	T	-11	-41	- 64	- 88	- 90	- 92	- 93	- 92	
	H(T)	+42	+26	+ 10	- 2	- 5	- 7	- 6	- 7	
47.4	H	...	+61	+ 58	+ 58	+ 60	+ 60	+ 59	+ 61	V.
	T(H)	...	-71	-124	-154	-168	-172	-178	-184	
	T	...	-35	- 65	- 78	- 88	- 91	- 91	- 94	
	H(T)	...	+21	+ 13	- 8	- 12	- 12	- 14	- 12	
57	H	+61	+66	+ 55	+ 67	+ 75	+ 53	+ 63	+ 67	VI
	T(H)	-28	-85	-121	-192	-187	-182	-195	-184	
	T	-13	-40	- 67	- 84	- 95	- 91	- 95	- 93	
	H(T)	+39	+31	+ 16	0	- 16	- 17	- 16	- 16	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	

This table is composed of six large groups of four rows and eight columns, any one group giving the measurements associated with a particular value of the longitudinal field. The first row of each group gives the values of H , the second the values of $T(H)$, the third the values of T , and the fourth the values of $H(T)$. Thus, with the exception of the third row of each group, every row has to do with a particular value of the longitudinal field h . Similarly, with the exception of the numbers of the first row in each group, the numbers in any one column have to do with a particular value of the transverse field t . The values of the longitudinal field are entered down the left-hand margin, and the values of the transverse along the top of the table. The nature of the particular field or combination of fields is indicated in the second column.

In all cases each field whose effect on the resistance was being studied was applied and removed twice in succession, with change of direction between the two applications. The nickel was thus subjected to a magnetic cyclic state as regards the magnetizing force which is being applied and removed. In the concomitant changes of resistance there was no evidence of hysteresis. Such effects, indeed, could hardly have been looked for in the case of the transverse field, seeing that the width of the nickel strip was short compared to its breadth as a cylindrically shaped coil. It was otherwise, however, with the field applied longitudinally along the coiled strip, which formed the core of the anchor-ring magnetizing coil. We cannot doubt that there was magnetic hysteresis of the usual kind; but no hysteresis was observed in the cycle of resistance change accompanying the cycle of magnetization.

A glance through the numbers tabulated in Table A shows that the resistance is increased in the direction of magnetization, and decreased in the perpendicular direction. This was established in 1856 by Thomson (Kelvin), and has been frequently verified by other experimenters. In the table, all the numbers in the H rows are positive, and those in the T rows are negative. The numbers in any one H row should be the same; for they are the changes of resistance due to the action of the same field. Similarly, the T numbers in any one column represent the change of resistance due to the application of one particular transverse field, and should also be the same. It will be noticed that the numbers corresponding to each H or each T fluctuate somewhat, and this fluctuation may be taken as a guide to the possible errors of the experiment.

In Table B the mean values of the different measurements of each H and of each T are recorded, with the average error attached. The

contiguous columns of "calculated" values contain numbers calculated according to an empirical formula given below.

TABLE B.

Longitudinal Field.	Resistance Change $\times 10^4$.		Transverse Field.	Resistance Change $\times 10^4$.	
	Obs.	Calc.		Obs.	Calc.
10·6	+11 \pm 1	10	50	-12 \pm 5	-15
17·7	+24 \pm 1	23	110	-39 \pm 2	-45
28	+38 \pm 1	40	170	-65 \pm 2	-65
37	+52 \pm 4	51	270	-80 \pm 4	-79
...	+50 \pm 1	...	430	-88 \pm 3	-87
47·4	+60 \pm 1	59	580	-90 \pm 2	-90
57	+64 \pm 7	64	700	-91 \pm 3	-91
...	+63 \pm 3	...	815	-91·5 \pm 1	-91·5

In each case the change of resistance increases with the magnetizing force, tending to a limit in the higher fields. This is particularly well marked in the case of the transverse field, whose values are much greater than those of the longitudinal field.*

The magnitude of the change of resistance is greater for a longitudinal field than for a transverse field of the same strength. This, however, may be largely a question of relative dimensions. Were the strip as wide as it is long, the transverse effect would probably be as large as the longitudinal effect under the same field.

The general march of the change of resistance with magnetizing force is shown graphically in the curves marked H and T in the figure on p. 216. The values from which these are drawn will be explained later. Inspection will show that the numbers of Table B agree very closely with the curves.

It will be seen at a glance that the curves are of the same general type as those which show the march of induction with magnetic force. Reasoning from this similarity, I have tried to fit to the curves an empirical formula of the type

$$R(b + h^2) = ah^2,$$

where R is the increase per 10,000 due to the field h , a and b being constant coefficients. The formulæ obtained are, for the longitudinal field, h ,

$$R(789 + h^2) = 80h^2,$$

and for the transverse field, t ,

$$R(12800 + t^2) = -93t^2.$$

* In a recent measurement with the same strip in much higher fields, the change of resistance was found to pass through a maximum in about field 900.

The values of R calculated from these expressions for the various fields involved are inserted in Table B for comparison with the observed values. The agreement is good, except perhaps for the lowest transverse field. The formulæ, of course, apply only within the limits of the fields used in the experiments.

In these experiments with the nickel strip I did not obtain an increase of resistance in the lower transverse fields, such as was observed by Grunmach (4) and Heaps (5) in their experiments with nickel wires. The transverse fields to which they had to subject their fine wires were as a rule much higher than the fields which I was able to get with my arrangement. The range of value of the transverse fields which I used corresponds nominally with the fields below 700 or 800 with which they obtained in general an increase of resistance. In their experiments the decrease of resistance did not set in to any marked degree till field 1000 was exceeded. Both these experimenters used very thin nickel wires; and it was not till the transverse field reached the value of 2000 or 3000 that the decrease of resistance was of the same magnitude as that obtained in my experiments with the nickel strip. It was indeed with the object of increasing the magnetic induction in transverse fields that I chose the strip instead of the otherwise more convenient wire. It is interesting to remember in this connection that in his early experiments Kelvin used thin plates and not wires.

Thus we see that, just as the magnetization itself depends upon the form of the material in a given magnetic field, so in regard to the change of resistance in a transverse field the width in the direction of the magnetizing force is an important determining factor.

These comparisons show that I am here dealing with a phenomenon which is only in a broad and general sense similar to that which was being studied by Grunmach about the same time. The immediate object of my experiments was not, however, to study the relation between resistance and magnetization, but to study the interplay of the two mutually perpendicular fields in their effect on resistance. I now pass to the consideration of this effect of the two cross magnetizing forces in various combinations.

I shall choose for detailed discussion one of the forty-seven complete groups of experiment given in Table A, say, the one corresponding to longitudinal field 47·4 and the transverse field 170. This combination is conveniently represented by the symbol V.3, V. referring to the fifth group and three to the third column. In this group V.3 the values of the changes of resistance per 10,000 are

$$H = +58, T = -65, H(T) = +13, T(H) = -124.$$

Here the longitudinal field is 47·4, and the transverse field is 170. Each of these alone produces much the same amount of change of resistance, the one positive and the other negative. When the cyclic longitudinal field is superposed upon the steady transverse field, the effect of the former, though still positive, is much less than when there is no maintained transverse field. We might reasonably enough express this by saying that the transverse field, which by itself produces a decrease in the resistance, continues when maintained to impose a kind of molecular restraint against an influence which would by itself increase the resistance. But in the neighbouring experiment, V. 4, the effect of the longitudinal field when superposed upon the steadily maintained transverse field is a further *decrease* of resistance. There is not merely a restraining action against the normal influence of the cyclic field; there is a change in the method of molecular yielding, the superposed longitudinal field having the same general effect as an additional transverse field.

An examination of the whole set of experiments shows that when the nickel strip is magnetized transversely by a force not exceeding 250 or thereabouts, the change of resistance due to the superposition of a given longitudinal magnetic field is markedly less than when the longitudinal field acts alone. For a transverse field somewhere in the neighbourhood of 250 or a little lower, there is no further resistance change when a longitudinal field is superposed. As the maintained transverse field is increased above this critical value, the effect of the superposed longitudinal field is to *decrease* the resistance, not to increase it. *And this decrease increases with the longitudinal field.*

It should be noted that the change of resistance due to the superposed longitudinal field is measured from the resistance which the material has under the influence of the steady transverse field. Now this transverse field has already produced a decrease of resistance or increase of conductance in a direction perpendicular to its lines of force. When along this direction of increased conductance the lines of force of a second field are made to act, the conductance of the nickel is decreased or increased according as the steadily acting transverse field is less or greater than a particular critical value. The further removed the transverse field is from this critical value, the greater numerically is the change of conductance associated with the superposed longitudinal field.

Consider now the effect of a steadily maintained longitudinal field upon the change of resistance due to a cyclically applied transverse field. In the case V. 3 already cited we see that the transverse field which acting alone

gives a decrease of resistance of 65 in 10,000, produces when superposed cyclically upon the steadily acting longitudinal field a *much greater diminution* of resistance, namely, 124. As a glance through the table will show, the combination $T(H)$ is of the same sign as T , and is, with the single exception of I. 1, always greater, in most cases much greater, than the corresponding T . There is nothing here at all in harmony with the view first suggested as to the relations of H and $H(T)$. In the relations of T and $T(H)$, the steadily maintained longitudinal field, which by itself invariably decreases the conductance, becomes when associated with the cyclically applied transverse field the cause of a greater increase in the conductance than is obtained by the transverse field acting alone. Instead of the steady longitudinal field being a restraint upon the effect of the superposed transverse field, it makes the nickel, as regards its change of resistance, even more sensitive to the influence of the transverse field.

The effect of the longitudinal field acting alone on the resistance 10,000 is to change it to $10,000 + H$; and the effect of the transverse field acting alone is to change 10,000 to $10,000 + T$, where in these experiments T is always negative.

Now $T(H)$ is the result of the transverse field superposed on the condition H . Hence the original resistance 10,000 becomes, under the combined fields,

$$(10,000 + H) \frac{10,000 + T(H)}{10,000} \\ = 10,000 + H + T(H) + \frac{H \times T(H)}{10,000}.$$

On account of the comparative smallness of H and $T(H)$ compared to 10,000, the last term may be neglected.

Similarly when the longitudinal field is superposed on the transverse field, producing a change of resistance $H(T)$ per 10,000, the original resistance 10,000 becomes

$$10,000 + T + H(T) + \frac{T \times H(T)}{10,000},$$

where as in the previous case the last term is negligible.

Thus the *total* changes per 10,000 of resistance of the nickel under the combined fields are given by the sums

$$H + T(H) \quad \text{and} \quad T + H(T).$$

If the effects of the combined fields were simply superposable, these sums should be equal to $H + T$. A brief inspection of the table will show that, with the single exception of the case I. 1, the sums $H + T(H)$ and $T + H(T)$

are always negatively greater or positively smaller than the sum $H+T$. The relation is brought out very clearly when the differences are tabulated. But since $H+T(H)-(H+T)$ is the same as $T(H)-T$, and $T+H(T)-(H+T)$ the same as $H(T)-H$, it is simpler to calculate these differences directly from Table A and form a new table. The results are shown in Table C, in which, however, it has not been thought necessary to give in detail the values obtained from the columns 5, 6, 7, and 8. These which are associated with transverse fields higher than 400 are so similar that they are sufficiently represented by the means of the four sets.

TABLE C.—DIFFERENCES OF CHANGES OF RESISTANCE, OBTAINED BY SUBTRACTING $H+T$ FROM, (a), $H+T(H)$ AND, (b), $T+H(T)$.

Mean $H =$	Mean $T =$	12	39	65	80	(90)	
11	a b	+ 3 - 2	- 15 - 6	- 14 - 13	- 15 - 11	- 16 - 13	I.
24	a b	- 5 - 4	- 15 - 5	- 27 - 21	- 37 - 26	- 31 - 26	II.
38	a b	- 4 - 7	- 23 - 21	- 43 - 35	- 49 - 37	- 57 - 43	III.
51	a b	- 18 - 10	- 25 - 23	- 48 - 39	- 56 - 60	- 67 - 56	IV.
60	a b	...	- 36 - 40	- 59 - 45	- 76 - 66	- 84 - 72	V.
63	a b	- 15 - 22	- 45 - 35	- 54 - 39	- 108 - 67	- 93 - 80	VI.
		(1)	(2)	(3)	(4)	(5678)	

Down the left-hand margin and along the top the mean values of H and T are also given, for a reason which will appear immediately.

It will be seen at a glance that the a -differences and the b -differences follow the same general law of change, increasing numerically as negative quantities as we pass from left to right along each row, or from top to bottom along each column. A close inspection shows that the increase

is fairly regular. The irregularities which occur are not surprising when we bear in mind that we are dealing with the differences of small quantities of which the experimental errors are considerable. If, confining our attention to either the a - or the b -differences, we plot each row in terms of the corresponding values of T , and each column in terms of the corresponding values of H , we get a series of zigzag lines which radiate in a roughly rectilinear manner from the origin. Let us assume that the a -differences tabulated in Table C can be represented in the form AHT , and that the b -differences can be represented similarly in the form BHT , where A and B are coefficients, each of which is the ratio of the sum of the corresponding set of differences to the sum of the products of the H 's and T 's.

Thus we find that each a -difference has the value

$$a\text{-diff.} = 0.0169 HT,$$

and that each b -difference is given by

$$b\text{-diff.} = 0.0138 HT.$$

These may be written in the form

$$T(H) = T(1 + 0.0169 H)$$

$$H(T) = H(1 + 0.0138 T),$$

where H is always positive and T is always negative.

Thus $T(H)$ is always negative and numerically greater than T , the ratio of the two being $1 + 0.0169 H$.

Again, $H(T)$ is positive like H but less than the value of H in the ratio of $1 + 0.0138 T$ to unity until T attains the value -72 ($= -1/0.0138$). When the transverse effect has this critical value, which occurs in the neighbourhood of field 220 (see Table A), the values of $H(T)$ vanish. For numerically higher values of T , $H(T)$ becomes negative and goes on increasing numerically for increasing values of both H and T .

We may now substitute for T and H in the formulæ just given their values in terms of the magnetizing forces t and h . The formulæ then become

$$T(H) = - \frac{93 t^2}{12,800 + t^2} \left(1 + \frac{1.35 h^2}{790 + h^2}\right)$$

$$H(T) = + \frac{80 h^2}{790 + h^2} \left(1 - \frac{1.28 t^2}{12,800 + t^2}\right).$$

These two formulæ embody the average results already shown in Table A. By their means we can calculate the values of the changes of resistance for various convenient combinations of the two fields, as given in the following tables:—

TABLE D.—VALUES OF CHANGES OF RESISTANCE PER 10,000 DUE TO CYCLIC LONGITUDINAL FIELDS (h) SUPERPOSED ON VARIOUS VALUES OF STEADY TRANSVERSE MAGNETIZATION.

\diagdown t	$h =$	10	20	30	40	50	60
0	+9	+27	+43	+54	+61	+66	
100	+4	+12	+19	+24	+27	+29	
200	+0.3	+0.8	+1.3	+1.6	+1.8	+2	
300	-1.1	-3.2	-5.1	-6.4	-7.2	-7.8	
400	-1.7	-5.0	-8	-10.1	-11.4	-12.3	
500	-1.9	-5.8	-9.2	-11.6	-13.1	-14.1	
600	-2.2	-6.5	-10.4	-13.1	-14.8	-16.0	
800	-2.3	-6.9	-11.0	-13.8	-15.6	-16.9	

TABLE E.—VALUES OF CHANGES OF RESISTANCE PER 10,000 DUE TO CYCLIC TRANSVERSE FIELDS (t) SUPERPOSED ON STEADY LONGITUDINAL MAGNETIZATION.

\diagdown h	$t =$	100	200	300	400	500	600	800
0	-41	-70	-81	-86	-88	-90	-91	
10	-47	-80	-93	-99	-101	-104	-105	
20	-60	-102	-118	-126	-128	-131	-133	
30	-71	-121	-140	-149	-152	-155	-157	
40	-78	-134	-155	-164	-168	-172	-174	
50	-83	-142	-164	-175	-179	-183	-185	
60	-87	-148	-171	-182	-186	-190	-192	

The significance of these numbers is shown clearly in the representative graphs of the accompanying figure. The cyclic fields to which the changes of resistance are due are measured horizontally, and the corresponding changes of resistance vertically. The values of the steady magnetizing forces are indicated on the figure by numbers placed to the right of the curves. The curves marked H, which lie in the upper part of the diagram, correspond to the first of the two tables just given. They show the effect of the cyclic longitudinal field acting alone. For the lower values of the transverse field t , the resistance increases with application of the longitudinal field. But for a certain value of the transverse field, about 220, the change of resistance is zero. For higher values of steady transverse field the effect of the superposed cyclic longitudinal field is to decrease the resistance; and this decrease of resistance increases numerically as the longitudinal field is taken stronger.

The curves marked T, which are all on the negative or lower side of the horizontal axis, correspond to the second of the two tables. The striking feature is that the decrease of resistance is numerically increased when the transverse field is superposed on a steady longitudinal field, and that this

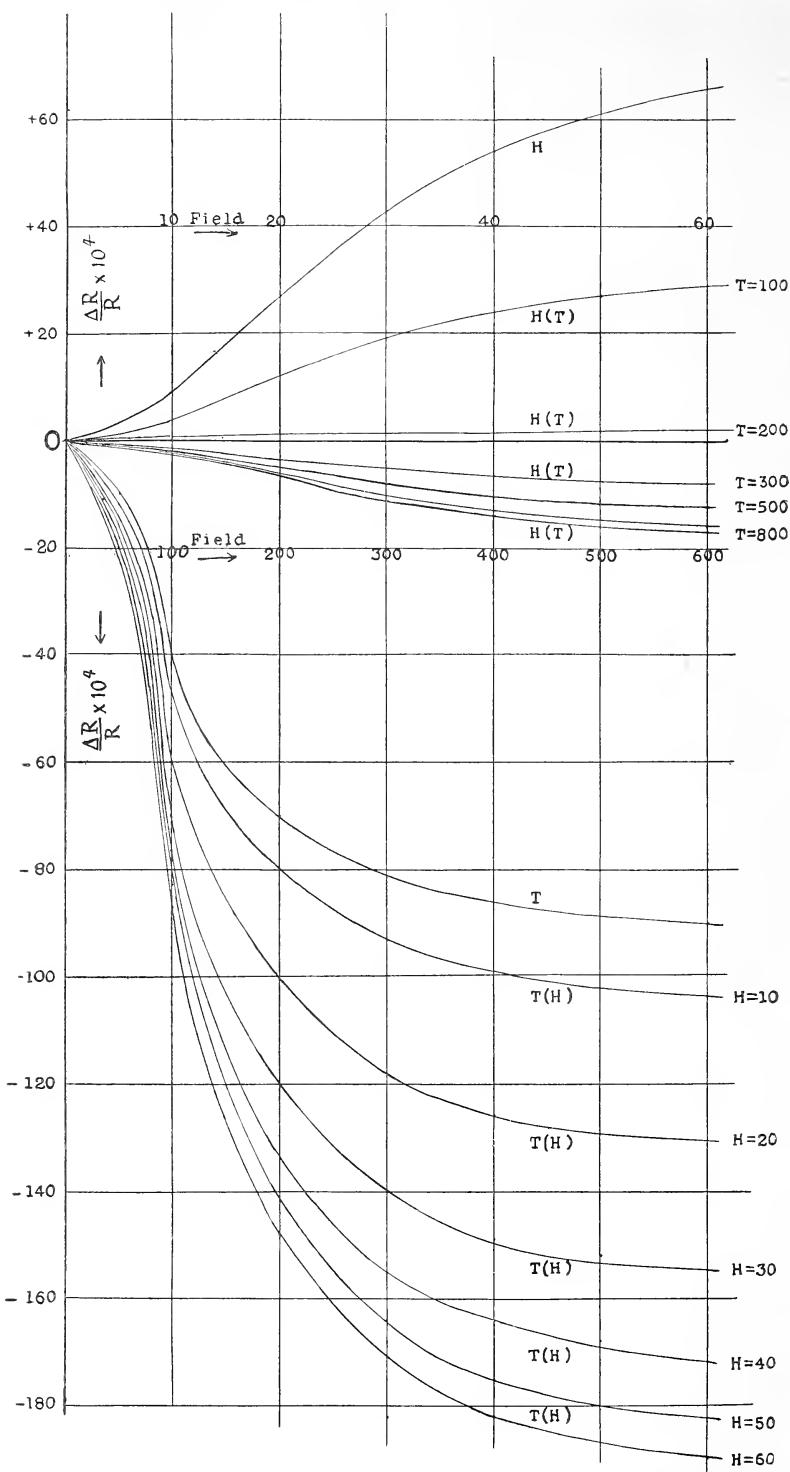


FIG. 1.

enhanced effect is greater as the steady longitudinal magnetization is increased.

Each curve, it will be noticed, is a kind of iso-magnetic curve. Along each H curve the transverse magnetic field is constant; and along each T curve the longitudinal magnetic field is constant.

With these graphs in view, we may summarise the phenomena in these words :

1. When a nickel strip is conveying a current, its conductance is diminished in a longitudinal magnetic field, increased in a transverse magnetic field.

2. Reversal of either magnetic field does not change the accompanying effect on the conductance.

3. When a cyclic longitudinal field is superposed upon a steady transverse field of magnitude less than a certain critical value, the diminution in the conductance is less marked as the transverse field increases and practically vanishes when this critical value of transverse field is reached.

4. When the steady transverse field exceeds this critical value, the superposed cyclic longitudinal field causes an increase in the conductance; and this increase in conductance becomes more marked as the longitudinal field is taken greater.

5. When a cyclic transverse field is superposed upon a steady longitudinal field, the increase in the conductance is augmented. Not only does the increase of conductance grow greater with the stronger transverse field, but it also grows greater as the steady longitudinal field is increased.

It may be noted in passing that it is not possible to connect in any way the change of resistance with the changes of length and volume which nickel experiences when magnetized. In a longitudinal field of 50 units nickel is shortened by about 5 in 1,000,000, and expands linearly at right angles by about 2 in 1,000,000 (6). These small changes would produce a diminution of resistance of about 9 in 1,000,000. This is quite negligible in comparison with the measured changes, and besides is in the opposite direction.

It is evident that we are dealing with a remarkable molecular and intermolecular effect. Now the one fundamental theory in electricity which at present can claim any success in getting behind electrical and magnetic phenomena is that which exploits the negatively charged electron. As early as 1900, Sir J. J. Thomson (7), in his memoir presented to the Congrès International de Physique, showed how, on certain plausible assumptions, the electron theory of conductance established a relation between magnetic force and electrical resistance. His conclusion was that

the effect of a transverse field was to increase the resistance. But in nickel the resistance is diminished in a transverse field. To bring the electron theory into accord with fact, Professor E. P. Adams (8) has modified Sir J. J. Thomson's theory by introducing a term depending on the change of molecular configuration, and has obtained an expression which may be either positive or negative according to the relative values of the different terms.

The following brief summary of his original theory was given by Sir J. J. Thomson in the *Proceedings of the Cambridge Philosophical Society* (9):—

"According to this theory, the current through a metal is carried by means of corpuscles, those small negatively electrified particles which constitute the cathode rays, which are given off by incandescent metals and also by metals when exposed to ultra-violet light. These corpuscles are assumed to be distributed throughout the volume of all metals, being produced by the corpuscular dissociation of the molecules. These particles, like the particles of a gas, are supposed to be moving rapidly in all directions, their kinetic energy, like that of the molecules of a gas, being proportional to the absolute temperature. Under the action of an electric field these charged corpuscles acquire a drift in a definite direction—the opposite direction to the electric force, since their charge is negative. This drift of the corpuscles under the electric field constitutes the current through the metal. If n is the number of corpuscles per unit volume of the metal, u the velocity of drift in the negative direction of x , e the charge on a corpuscle, then the intensity of the current parallel to the axis of x is equal to neu . If X is the electric force, m the mass of a molecule, t the average time between two collisions of a corpuscle, u is equal to $tXe/2m$; if λ is the mean free path, c the velocity of mean square, $t = \lambda/c$; thus"

$$\text{the current} = X \frac{1}{2} \frac{ne^2}{m} \frac{\lambda}{c} = X \times \text{conductance},$$

and the resistance has the value $2mc/(ne^2\lambda)$.

Of the quantities involved in this expression, the mean free path λ is the one which seems most likely to be influenced by an applied magnetic force. As worked out in the papers already referred to, the explanation depends on the length of the mean free path or the average time of free motion from encounter to encounter of the negative corpuscles with the neutral molecules of the material.

The theory so far as it goes is obviously incomplete. If it cannot in an obvious manner explain the general nature of the phenomenon, it can

hardly be expected to co-ordinate the more complex relations established in this paper. With an experimental knowledge of these relations it may, however, be possible to gain some insight into the mutual action of the molecules and the electrons or corpuscles which carry the electric charge and constitute the current.

In a magnetic metal like nickel we seem compelled to regard the molecules as small electro-magnets consisting of whirls of negatively charged corpuscles. The simplest molecular magnet of this kind is the doublet, in which the negatively charged corpuscle rotates with high speed round the more massive positive associate, whose speed is correspondingly small. In the unmagnetized condition of the metal these molecular whirls will form magnetically neutral groups, so that the intermolecular spaces will be comparatively free from magnetic force. When, however, an external magnetic field acts upon the metal, the groups are more or less broken up and the molecular whirls tend to set so as to face in a definite direction. The intermolecular spaces will become filled with tubes of magnetic force tending to thread the doublets or whatever particular form the molecular magnets have. Any electron in rapid motion through this intermolecular space will be driven by this established magnetic field in a direction at right angles to the magnetic force and to its own velocity.

In the first place, let the conductor be magnetized longitudinally, that is, in the direction in which the resistance is being measured. The tubes of magnetic force will tend to pass along either in the same direction as, or in the opposite direction to, the drift of the negatively charged electrons. These electrons will tend to describe helices round the lines of magnetic force, and will be driven from the regions where the magnetic force is stronger to the regions where it is weaker. Being thus driven out of the lanes of strong magnetic force which pass through the hearts of the molecular magnetic whirls, they may well be imagined as colliding more frequently with the more obstructive parts of the molecules. The free path will therefore be diminished and the resistance increased. Whether this may be accepted as a reasonable explanation or not, it is clear that the effect of a longitudinal field upon the molecular grouping must be such as to put impediments in the paths of the drifting electrons.

Under the influence of the transverse field, the tubes of magnetic force in the intermolecular spaces run across the nickel strip at right angles to the drift of electrons by which the current is conveyed.

The negative electrons will tend to pass towards one side, and the more massive positive electrons to pass towards the other. Will this remove

impediments? It is not easy to see how it can, unless the rearrangement of the molecules opens up the channels along which the electrons may pass.

This seems to require that the forms of the magnetic whirls are such that they present less obstruction when they are facing across the strip than when they are facing along it. This supposition would explain the two main effects. But how far will it go in explanation of the complex results which are the subject of this paper?

On any of the more usual conceptions of the molecular movements which accompany magnetization, a longitudinal field will always tend to turn the magnetic whirls so as to face more in the direction of the length of the strip. Similarly, if the magnetic molecules are turned so as to face along the strip under the influence of the longitudinal field, the effect of the superposed transverse field will be to turn the magnetic whirls round so as to face obliquely across the strip. But, if the molecules present less obstruction when they face across the strip than when they face along it, it is not easy to see how intermediate orientations, such as would be produced under the combined influence of the two fields, should be accompanied by a still further increase of conductance. In short, no simple theory of orientation of magnetic molecules can at all fit in with the facts. We must look for an explanation to some more complex process in which the grouping of the magnetic molecules, and even the internal structure of each magnetic molecule, are of fundamental importance.

Before further discussion, however, it will be useful to make similar experiments with iron or steel ribbon. In their magnetic behaviour iron and nickel present both similarities and contrasts; and it cannot fail to be of interest to compare their properties in regard to the present line of inquiry also. This comparison I hope to make in a few months.

APPENDIX.

RESULTS AS REDUCED IN LABORATORY NOTE-BOOK, ARRANGED APPROXIMATELY
ACCORDING TO DATE IN THE YEAR 1906.

The numbers in the columns headed Resistance Change give the changes of resistance, estimated per 100,000, in the branch N which contains the Nickel strip.

h and *t* represent respectively the longitudinal and transverse fields.

Date and Fields.	Cyclic Field.	Steady Field.	Resistance Change.	Date and Fields.	Cyclic Field.	Steady Field.	Resistance Change.
May 15 $h=10\cdot6$ $t=815$	<i>h</i> <i>h</i> <i>h</i> <i>t</i> <i>t</i> <i>t</i>	none <i>t</i> none none <i>h</i> none	+ 69 - 17 + 102 - 706 - 854 - 733	May 17 $h=17\cdot7$ $t=815$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>t</i>	+ 199 - 22 - 695 - 884
$h=10\cdot6$ $t=700$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 63 - 16 - 699 - 854	$h=17\cdot7$ $t=700$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 171 - 15 - 704 - 986
$h=10\cdot6$ $t=580$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 89 - 10 - 695 - 839	$h=17\cdot7$ $t=580$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 203 - 20 - 701 - 978
$h=10\cdot6$ $t=430$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 98 - 11 - 661 - 807	$h=17\cdot7$ $t=430$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 182 + 6 - 693 - 920
May 16 $h=10\cdot6$ $t=270$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 83 - 10 - 593 - 713	$h=17\cdot7$ $t=270$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 208 + 21 - 596 - 886
$h=10\cdot6$ $t=170$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 89 + 30 - 510 - 613	$h=17\cdot7$ $t=170$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 178 + 18 - 510 - 720
$h=10\cdot6$ $t=110$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 74 + 24 - 278 - 379	$h=17\cdot7$ $t=110$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 186 + 61 - 317 - 448
June 19 $h=10\cdot6$ $t=50$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 90 + 78 - 83 - 101	June 19 $h=17\cdot7$ $t=50$	<i>h</i> <i>h</i> <i>t</i> <i>t</i>	none <i>t</i> none <i>h</i>	+ 183 + 150 - 93 - 116

APPENDIX (*continued*).RESULTS AS REDUCED IN LABORATORY NOTE-BOOK, ARRANGED APPROXIMATELY
ACCORDING TO DATE IN THE YEAR 1906.

The numbers in the columns headed Resistance Change give the changes of resistance, estimated per 100,000, in the branch N which contains the Nickel strip.

h and t represent respectively the longitudinal and transverse fields.

Date and Fields.	Cyclic Field.	Steady Field.	Resistance Change.	Date and Fields.	Cyclic Field.	Steady Field.	Resistance Change.
May 18 $h = 28$ $t = 815$	h	none	+ 304	May 25 $h = 37$ $t = 815$	h	none	+ 371
	h	t	- 30		h	t	- 56
	h	none	+ 308		t	none	- 722
	t	none	- 723		t	h	- 1264
	t	h	- 1240				
	t	none	- 717				
$h = 28$ $t = 700$	h	none	+ 295	$h = 37$ $t = 700$	h	none	+ 387
	h	t	- 49		h	t	- 51
	t	none	- 690		t	none	- 737
	t	h	- 1142		t	h	- 1279
$h = 28$ $t = 580$	h	none	+ 307	$h = 37$ $t = 580$	h	none	+ 395
	h	t	- 40		h	t	- 54
	t	none	- 685		t	none	- 728
	t	h	- 1161		t	h	- 1260
$h = 28$ $t = 430$	h	none	+ 297	June 6 $h = 37$ $t = 430$	h	none	+ 487
	h	t	- 21		h	t	- 37
	t	none	- 679		t	none	- 708
	t	h	- 1036		t	h	- 1220
$h = 28$ $t = 270$	h	none	+ 294	$h = 37$ $t = 270$	h	none	+ 458
	h	t	- 4		h	t	- 15
	t	none	- 608		t	none	- 697
	t	h	- 1000		t	h	- 1140
$h = 28$ $t = 170$	h	none	+ 303	$h = 37$ $t = 170$	h	none	+ 390
	h	t	+ 37		h	t	+ 75
	t	none	- 502		t	none	- 503
	t	h	- 837		t	h	- 887
$h = 28$ $t = 110$	h	none	+ 315	$h = 37$ $t = 110$	h	none	+ 385
	h	t	+ 153		h	t	+ 286
	t	none	- 335		t	none	- 326
	t	h	- 518		t	h	- 524
June 19 $h = 28$ $t = 50$	h	none	+ 309	June 19 $h = 37$ $t = 50$	h	none	+ 415
	h	t	+ 254		h	t	+ 333
	t	none	- 86		t	none	- 90
	t	h	- 122		t	h	- 228

APPENDIX (*continued*).

RESULTS AS REDUCED IN LABORATORY NOTE-BOOK, ARRANGED APPROXIMATELY ACCORDING TO DATE IN THE YEAR 1906.

The numbers in the columns headed Resistance Change give the changes of resistance, estimated per 100,000, in the branch N which contains the Nickel strip.

h and t represent respectively the longitudinal and transverse fields.

Date and Fields.	Cyclic Field.	Steady Field.	Resistance Change.	Date and Fields.	Cyclic Field.	Steady Field.	Resistance Change.
June 8 $h=47\cdot 4$ $t=815$	h h t t	none t none h	+ 479 - 96 - 743 - 1453	June 12 $h= 57$ $t= 815$	h h t t	none t none h	+ 528 - 127 - 738 - 1453
$h=47\cdot 4$ $t=700$	h h t t	none t none h	+ 465 - 112 - 722 - 1411	$h= 57$ $t= 700$	h h t t	none t none h	+ 498 - 127 - 748 - 1539
$h=47\cdot 4$ $t=580$	h h t t	none t none h	+ 474 - 96 - 717 - 1364	$h= 57$ $t= 580$	h h t t	none t none h	+ 417 - 134 - 724 - 1438
June 11 $h=47\cdot 4$ $t=430$	h h h t t	none t none none h	+ 472 - 94 + 456 - 693 - 1332	$h= 57$ $t= 430$	h h h t t	none t none none h	+ 512 - 142 + 568 - 749 - 1482
$h=47\cdot 4$ $t=270$	h h t t	none t none h	+ 461 - 66 - 615 - 1219	$h= 57$ $t= 270$	h h t t	none t none h	+ 529 - 83 - 661 - 1518
$h=47\cdot 4$ $t=170$	h h t t	none t none h	+ 455 + 100 - 516 - 977	$h= 57$ $t= 170$	h h t t	none t none h	+ 437 + 123 - 530 - 956
$h=47\cdot 4$ $t=110$	h h t t	none t none h	+ 484 + 164 - 278 - 564	$h= 57$ $t= 110$	h h t t	none t none h	+ 468 + 244 - 320 - 676
				June 18 $h=57$ $t=50$	h h t t	none t none h	+ 481 + 312 - 100 - .219

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- (7) J. J. THOMSON, "Indications relatives à la Constitution de la Matière fournies par les Recherches récentes sur le Passage de l'Électricité à travers les Gas," *Congrès international de Physique*, vol. iii., 1900.
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- (9) J. J. THOMSON, "On the Theory of Electric Conduction through thin Metallic Films," *Proc. Cam. Phil. Soc.*, vol. xi. p. 120, 1901.

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XXI.—On the Deviation of the Torsional Oscillations of Metallic Wires from Isochronism. By Professor W. Peddie.

(Read December 18, 1912. MS. received January 21, 1913.)

§ 1. In former papers (*Trans. R.S.E.*, 1898; *Phil. Mag.*, 1894) it was shown that the period of the inward oscillation of an iron wire was distinctly greater than the period of its outward oscillation over the same range, and that the complete period was slightly greater when the range was relatively great than when it was small. In these respects the material deviated from the conditions of isochronism which are characteristic of the simple harmonic vibrations of a material which practically follows Hooke's Law during distortion.

As the method of observation which was formerly employed did not give much more than qualitative results, and was quite inadequate for the determination of the actual law of oscillation at all stages, a different method was employed in the work described below. A description of that method, and of the apparatus, is first given; and then, by means of curves, the experimental results obtained in the cases of three different materials are shown, and the nature of their deviation from those characteristic of simple harmonic motion is exhibited. Finally, a theoretical discussion is given of a molecular condition under which the observed laws of torsional oscillation might be simulated.

EXPERIMENTAL DISCUSSION.

§ 2. In fig. 1 the disposition of the apparatus is exhibited. Above, on the right, a horizontal wooden arm carrying the torsion head can be seen. The head is coned strongly, and works in a similarly coned bearing fitted in the wooden arm. A pin-vice, carefully centred, clamps the upper end of the wire, and the oscillator is clamped to its lower end by a similar vice. The oscillator consists of a circular brass ring of rectangular section, and equal brass pins, arranged equidistantly and concentrically round the ring at angular intervals of 10° , project downwards from it. Beneath it is placed an ebonite disc which rests by screw feet on a vertically adjustable stand. Narrow radial slits are cut in the upper surface of the disc, their middle points lying on a circle of the same radius as that of the circular row of pins in the oscillator, but the angular distance of successive slits is 12° instead of 10° .

Each consecutive set of five slits are connected together by a copper strip on the under side of the disc, and each strip is joined by a copper

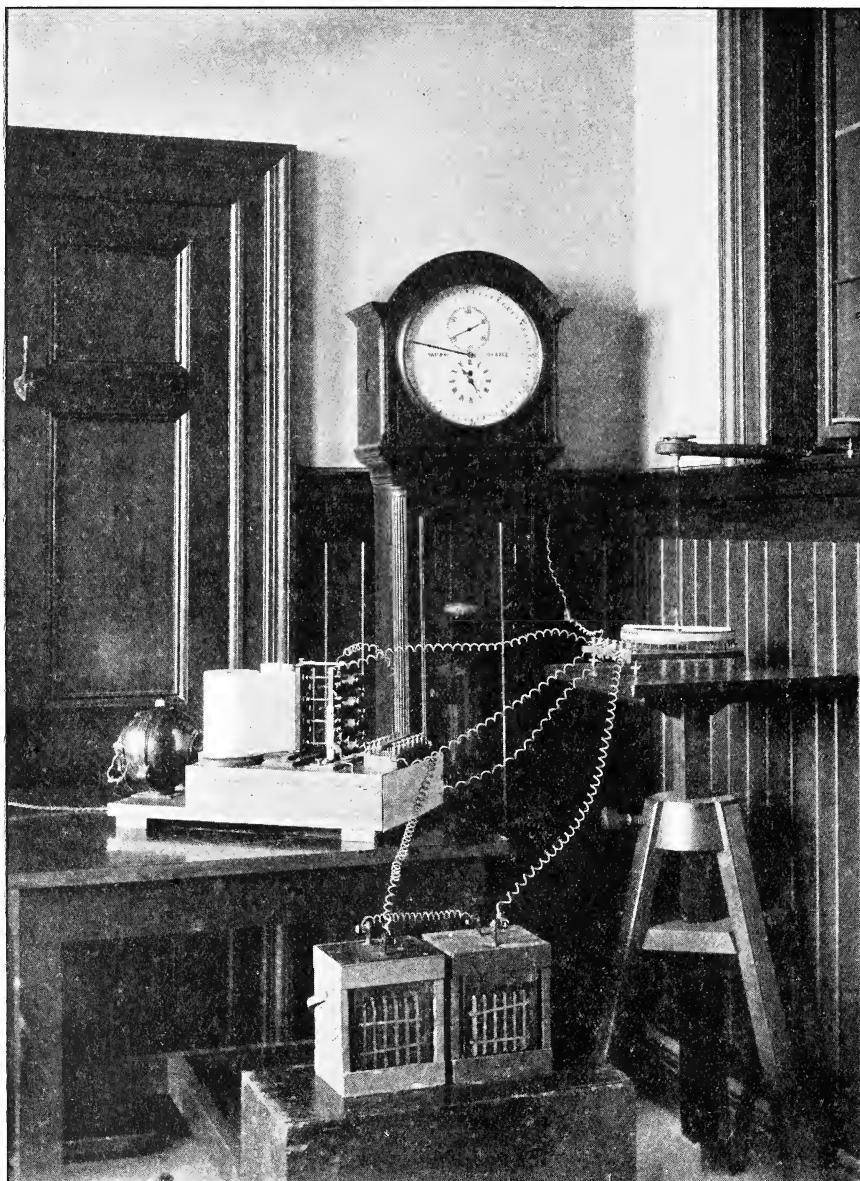


FIG. 1.

wire to a binding screw. If the slits are filled with mercury, so that the surface bulges out above the plane of the disc, electrical contact can be established between various pairs of slits by means of the pins attached

to the rotating oscillator. Six recorders of a seven-record chronograph can be placed one in each of the six circuits furnished by the copper strips, while the seventh is connected with a clock. Thus it is possible to register the instants at which the oscillator attains successive angular positions at intervals of 2° .

In the actual experiments it was found that registers at intervals of 10° were sufficient, so the first slit only in each of the sets was filled with mercury. Further, three alternate sets were joined together metallically by means of a wire connecting the corresponding binding screws, and the remaining three were connected each to one of three registers of the chronograph. The connecting wires, including those to the clock and the accumulators, are seen in fig. 1. The figure also shows the chronograph* and the small motor which drives the paper strip. The strip passes between two brass plates (the front one seen dark against the paper) both of which are pierced by seven circular holes, the holes in one plate facing those in the other. The records are made upon the paper as it passes these holes. The drums on or off which the paper is rolled have their axes vertical, and no two holes are in one horizontal line. Thus the records are kept distinct. Each record is made by a needle point attached to a shaft which carries on its other end the armature of a small electro-magnet. When the armature is attracted by the magnet, in spite of the control of a spring, the needle point is driven into the paper and makes a small sharply defined hole. The needle point is attached to the shaft by a joint so that it can turn horizontally in the direction in which the paper is drawn. This prevents tearing of the paper. Finally, the cessation of the magnetising current allows the withdrawal of the needle by the controlling spring, and the needle point, being also under the control of a spring, reassumes the direction of the shaft to which it is attached. Four of the horizontal shafts can be seen against the dark background of the brass plate in fig. 1, and the armatures attached to them are seen nearer the edge of the clock. The switch, with two rows of seven binding screws, is visible, against the side of the clock, at the right-hand edge of the wooden base of the chronograph.

Fig. 2 gives a diagrammatic plan of the connections. It is evident that chronographs 3, 5, and 7 should give, apart from instrumental irregularities, identical records. But these records are established by contacts made at slits whose angular positions differ by 120° . Hence errors due to slight want of symmetry of construction, or to slight

* The cost of the chronograph and motor was defrayed by part of a grant given by the Trustees of the Carnegie Fund.

pendulum oscillations, etc., can be largely eliminated by taking the mean of the three results. As a matter of fact, in each of the cases illustrated by the graphs, one of the three registers had failed to work; yet the graphs are almost entirely free from visible irregularities.

§ 3. Each record consists of a linear series of needle-pricks in the paper. These can be readily seen if viewed under oblique illumination. The clock record is needed only to test the regularity of the motion of the paper, and so to supply a time scale.

The punctures are spaced close together when the motion of the oscillator is rapid; they are wide apart when the range is near its

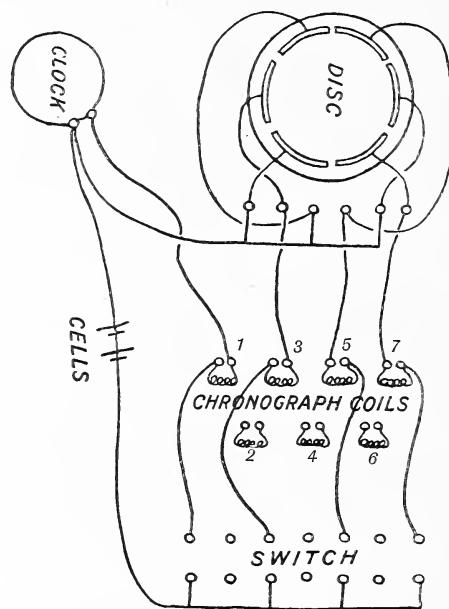


FIG. 2.

maximum value. The position of the maximum is very easily found. In general it is attained at some instant between the recording of the two successive points which are most widely separated on the paper. But it might happen to be attained at some instant between the recording of two near points which have much larger intervals immediately outside them. In either case, when a number of multiples of 10° are included in the range, it may be assumed that the maximum was reached at a point on the record midway between the pair of points referred to; for the times of in and out oscillation over a given small range in the neighbourhood of the maximum cannot differ greatly. Hence we get the time of a complete semi-oscillation.

In taking a record, the oscillations are started by means of a single out-and-in motion of the torsion head. The motor is started in sufficient time for its motion to become steady before the first maximum is reached, and the chronograph switch is closed also before the first maximum is reached. The reading of a scale fixed round the rim of the oscillator is taken at the first and second maxima. The oscillations are then allowed to die down, and the reading corresponding to no twist is found. So the values of the two maximum ranges are obtained.

A diagram is then plotted with suitable range and time scales, the instant of attainment of the first maximum being called zero, while that of the second maximum is the time of the semi-oscillation as obtained from the record. The two end points on the diagram are now correctly fixed; but we do not know the instant at which the oscillation passed through its zero value, nor can we tell with accuracy from the apparatus the angular position of that zero relatively to the slits in the ebonite disc which determine the recorded angles. To surmount this difficulty, the intermediate points on the graph are plotted relatively to an arbitrary time and angle zero which cannot be far wrong. If these intermediate points do not now form a smooth, consistent graph with the end points, the whole intermediate set must be slid, without rotation, to a position which gives consistency. This was always found to be possible.

§ 4. Fig. 3 shows results obtained with an iron wire. The experiment was the first made with the apparatus. No readings of angles were made. The instants of attainment of definite angular positions, counting from the instant of attainment of the first maximum as zero, were given by the record. A cosine curve, of unit amplitude and of the observed semi-period, was then plotted by means of points corresponding to the recorded instants; and the average drop of the oscillation, per recorded interval, between the two extreme recorded points within the two successive maxima was found. The two extreme points within the maxima being assumed to be identical with the corresponding observed points, the difference of the ordinates of the two latter was divided into equal parts corresponding to the average drop per recorded interval, and the ordinates so found were plotted against the recorded times measured from the first maximum. The points so found are shown in the figure, the cosine curve being represented by the full line. It could not be predicted beforehand that these points would lie on the curve; but, with the exception of one point obviously in error, it is evident that they do practically lie on it. Since the average drop of angle used is such that the second maximum is compelled to be equal to the first, the corresponding actual drop in the

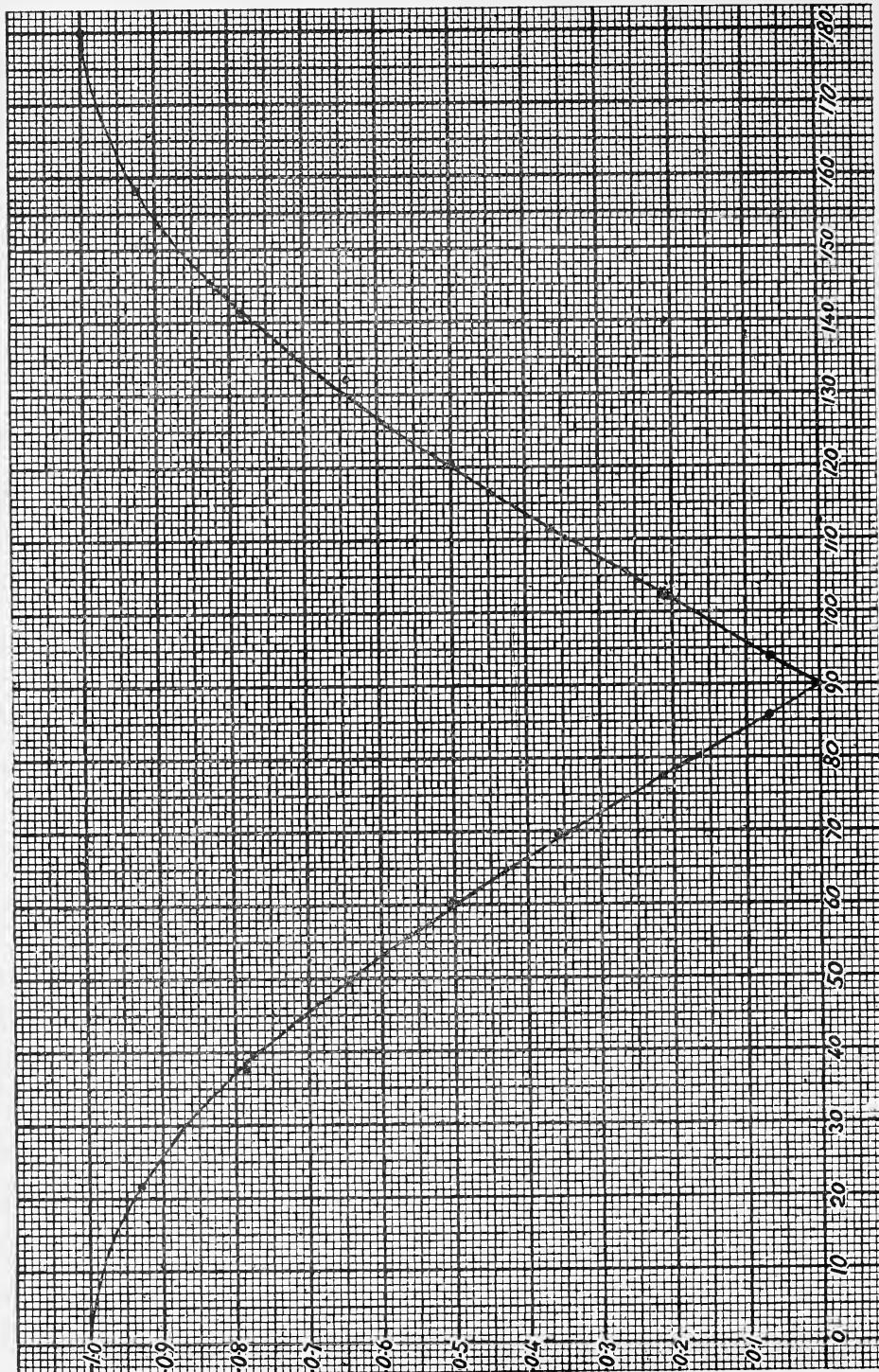


FIG. 3.

observations was really smaller than it. Thus the actual points, which could have been plotted had observations of the values of the two maxima been made, must have lain to the right-hand side of the cosine curve; and so the time of the in-motion must have exceeded that of the out-motion. In the diagram, negative ordinates have been plotted as if they were positive.

Fig. 4 exhibits results obtained with a copper wire. The second maximum value was 0·91 times the first. The full line represents a cosine curve with exponentially decaying amplitude. The separate points represent the results of observation, the points next each maximum being taken on the full curve. The difference between the times of the in and out motions is quite well-marked, and exhibits the essential difference between the internal loss of energy in a distorted metal and the loss which occurs when a viscous resistance proportional to speed is active.

§ 5. Fig. 5 shows results obtained with the same copper, the length of the wire being halved, while the maximum angle of twist was not much altered from its former value, so that the loss of energy per oscillation was greatly increased. The ratio of the second maximum angle to the first was now 0·68. The isolated points represent the results of observation, and were plotted by the process described in § 3. The difference between the times of the in and the out motions exceeds one-sixth of their average value.

Fig. 6 exhibits similar results in the case of a zinc wire, in which the dissipation of energy was so great that the amplitude of oscillation fell to about one-half of its initial value in a single semi-oscillation.

In each of these figures, the full curve represents a cosine curve whose zero point is at the point of inflection of a curve drawn freehand through the observed points. The dotted line is the axis of the cosine curve. The correspondence of the small oscillations of an elastic material such as steel with the simple harmonic law is well known; but it seems to be a very remarkable thing that, in the case of a substance possessing so great viscosity * that the amplitude falls to one-half of its initial value in a single semi-oscillation, the law of oscillation should be very accurately simple harmonic from the extremity of the range inwards to the position of set (inflection), beyond that to the zero point, and almost as far beyond the zero on the negative side as the position of set is separated from it on the positive side. After that stage is reached, the deviation from the simple harmonic law takes place very rapidly. [A slight error, in either direction, in choosing the point of inflection, diminishes the accuracy of the correspondence greatly; so that the most suitable cosine curve is readily determined.]

* Viscosity is here used merely to indicate internal resistance to relative motion of the parts.

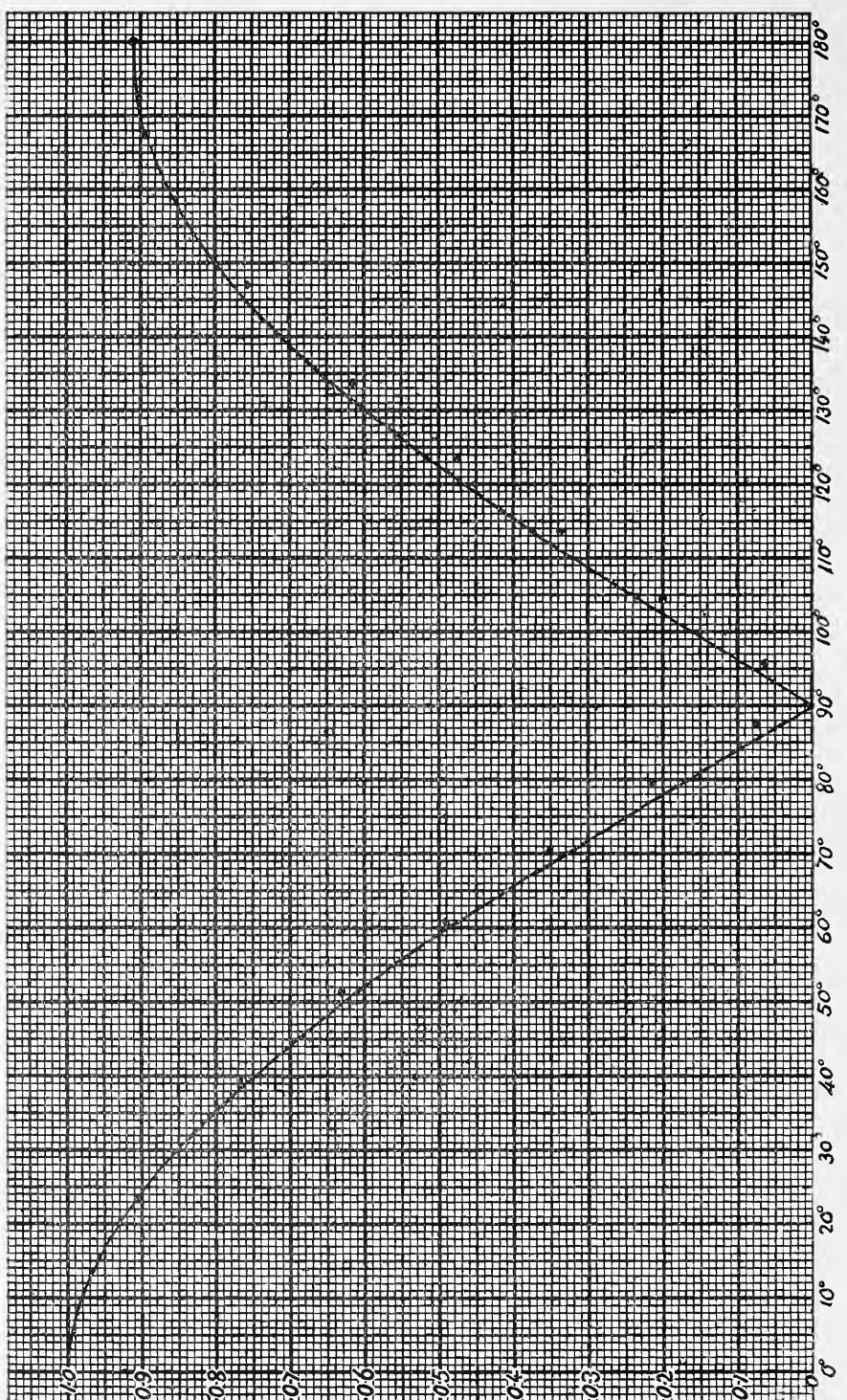


FIG. 4.

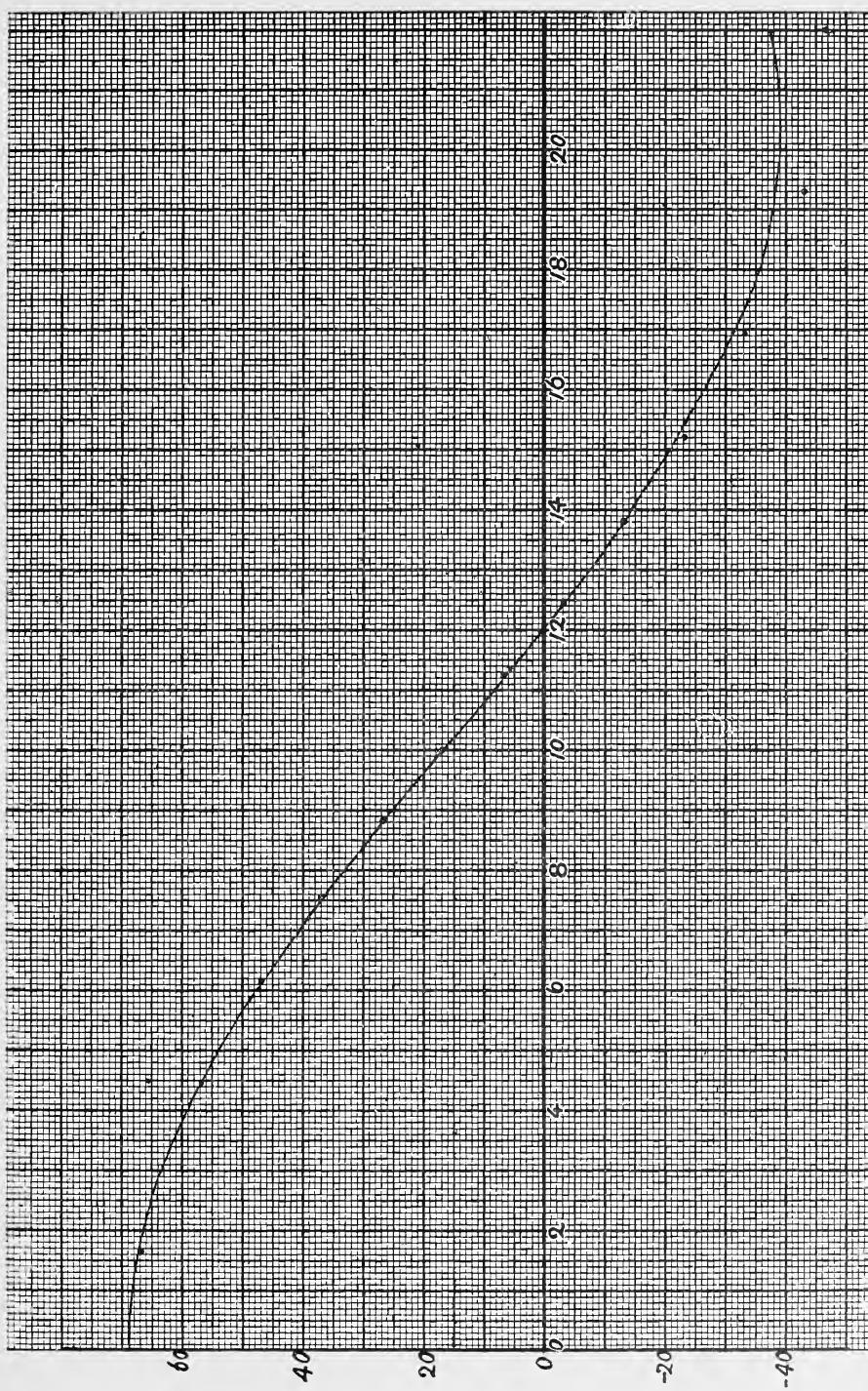


FIG. 5.

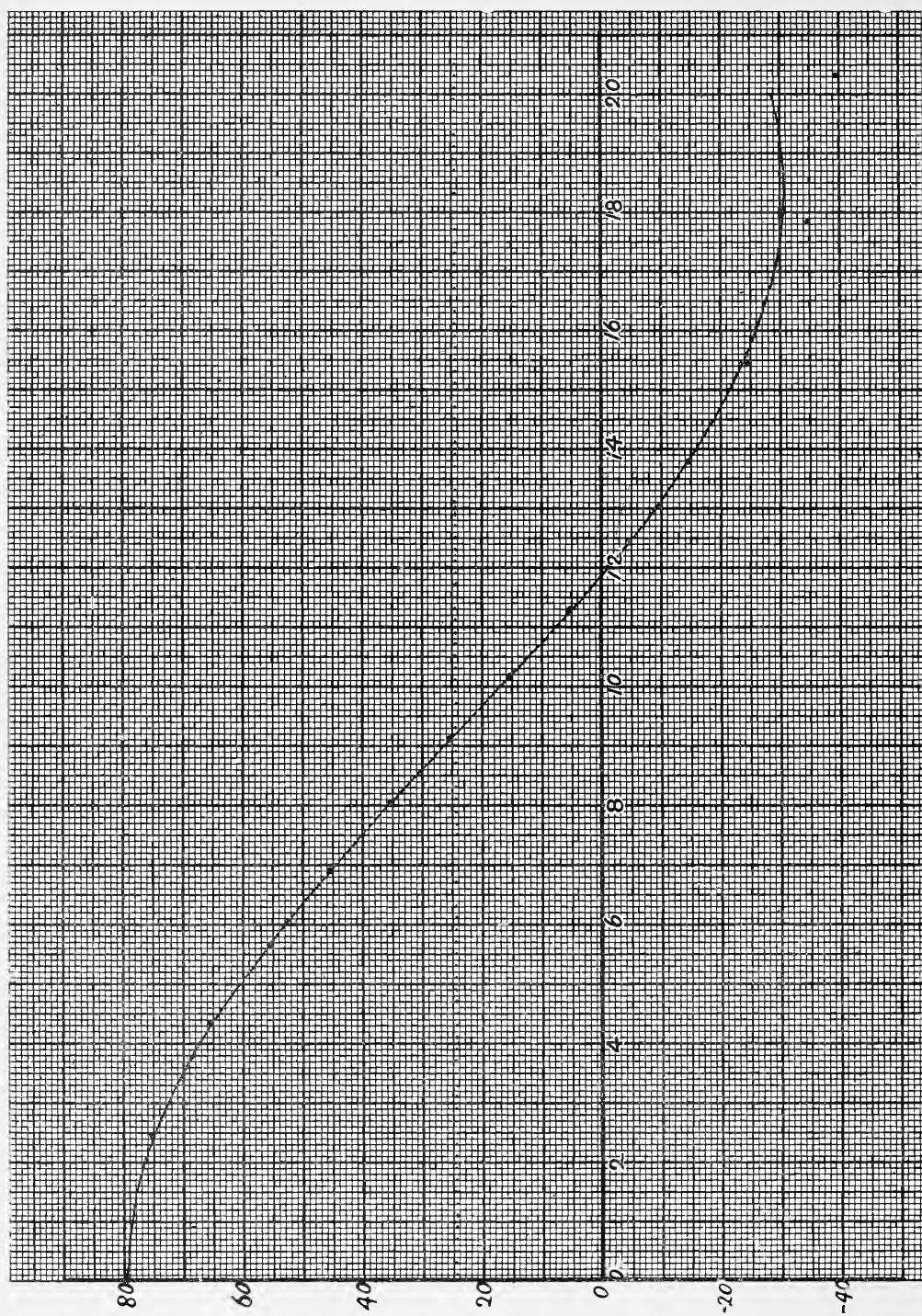


FIG. 6.

The result is very significant in connection with the view of G. Wiedemann, that the loss of energy is due to the work done in shifting the position of set from one side of the zero point to the other. Its possibility was suggested by the theoretical considerations given below (§ 9).

THEORETICAL DISCUSSION.

§ 6. The experimental data upon which a theoretical discussion must be founded are those described in the preceding paragraphs, and the now well-established fact (see Ritchie, *Proc. R.S.E.*, vol. xxxi. p. 424, vol. xxxiii. pp. 177, 183) that the law of the decrement of the range, y , of oscillation, as the number of oscillations, x , increases, is very accurately given by the relation $y^n(x+a)=b$, where n , a , and b are constants, together with other known facts regarding the statical and kinetic laws of deformation of a solid, *e.g.* those of "set." Any theory must be of "molecular" type, the loss of energy being, in great part at least, due to the rupture of molecular configurations. These may, of course, take place on a finite "crystalline" scale.

§ 7. In previous papers (*Phil. Mag.*, July 1894; *Trans. R.S.E.*, 1898) it has been shown that, when the fractional decrease of the range per oscillation is not too great, the above formula can be deduced from the assumption that the loss of energy per oscillation is proportional to a power of the angle of twist measured from the position about which the oscillations take place and towards which they converge as their amplitude decreases. In the latter paper an attempt was made to express this loss in terms of the work done in distorting molecular groups which break down when the distortion becomes too great. Two factors have then essentially to be considered: (1) the law of force to which a group is subject during distortion, and (2) the law regulating the number of groups which break down per unit volume in a given small range of distortion. Regarding the law of force, at least when the individual groups are of invisible dimensions, nothing is known except as an average, in which case Hooke's Law is fundamentally applicable. But it is probably legitimate to postulate that the number of groups of any one type, per unit volume, is so great, and that the nature of the interconnections is such, that Hooke's Law may be assumed to apply to each type on the average. The only quantity then remaining undetermined is the number of groups of given type which break in a given small range of distortion. In order to deduce the above empirical law, it is necessary to assume either that this quantity is proportional to a power of the distortion, and that the power is the same for all types, or that the quantity when averaged over all types is

proportional to a power of the distortion. The latter postulate was adopted in the paper of 1898, and is sufficient in cases in which one definite power suits over the whole range of the observed oscillations, large and small alike. But, in those cases [*e.g.* iron slightly at very small oscillations (*Trans. R.S.E.*, 1898) and other metals very distinctly under certain conditions (Ritchie, *Proc. R.S.E.*, vol. xxxi. p. 424, and vol. xxxiii. p. 183)] in which one power suits with larger oscillations and a different power suits with smaller oscillations, the change being very sharply marked, it seems to be necessary to assume that a new type of groups becomes active only when the distortion exceeds a definite limit.

It might be expected that a new type of this kind would first become effective at the surface of the wire, since the shear is greatest there, and that its effect would gradually become evident in more and more central regions as the oscillations were made greater. In this case there could not be a sudden change in the value of n at a definite range of oscillation. The change should take place more or less gradually, unless indeed the dissipation of energy became very greatly increased whenever the change did begin. In the experimental observations the suddenness of the change in the dissipation is made evident by a quite sudden alteration in the drop of the amplitude per swing, when the decreasing oscillations pass the critical stage. A way of avoiding the assumption of so great a volume density of dissipated energy in the mere surface layers as would be needed to account for the suddenness of the change can be found in the assumption that the action, though starting at the surface, actually spreads inwards to regions in which the stress is not sufficient to start it—just as the complete disintegration of a Rupert's drop follows a surface rupture, or as the tremors caused by one landslip precipitate other slips.

Another experiment, carried out by Mr Ritchie (*Proc.*, vol. xxxiii. p. 193), seems to indicate that the proper view of the action is that which regards the energy loss *when averaged over all types* of contributing groups as being proportional to a power of the distortion. In that experiment zinc was deposited electrolytically on a core of copper wire, and the value of n was found to pass *continuously* from the value characteristic of copper alone towards that characteristic of zinc alone, as the thickness of the zinc increased. In this case the zinc molecular groups and the copper molecular groups were entirely separate, except at the surface of junction. It might therefore be thought that in general the energy dissipated should be representable as a sum of two terms, one involving that power of the distortion characteristic of the one metal, the other involving the power characteristic of the other metal, while, at small

distortions, the power characteristic of the outer metal should alone appear. But, the ratio of the ns for copper and zinc being nearly 2:1, it can easily be verified that a sum of such terms cannot give results at all agreeing with the results of observation. On the other hand, one term alone, with an intermediate value of n , represents the actual results well. There seems to be a commingling of effects in regard to the dissipation of energy.

§ 8. This also gives some justification of the treatment adopted in the paper of 1898, in which Hooke's Law was postulated as an average for all of the types of groups suffering distortion and rupture, and the number of groups suffering rupture within a given very small change of twist might include many of these types. But, if we select as the characteristic of a "type" the shearing strain at which rupture just occurs, we may arrive at the same result by considering each type by itself, and then integrating over all types, provided that n be regarded as having the same value for each type. This gives an appearance of restriction, but the process derives support from the experimental fact just given above, and, in any case, it would throw light upon results which might be due to the condition of one type being dominative. For example, Mr Ritchie's discovery of a sudden change in the value of n , when certain metals are heated to definite temperatures, seems to indicate a change of molecular structure.

To make the development of any theory possible, subsidiary postulates regarding the conditions of rupture and re-formation of groups have to be made. In the former paper it was assumed that, for every group of given type which suffered rupture, a new group of the same type came into existence. This is analogous to the fundamental assumption of the kinetic theory, and is at least plausible as the expression of an average condition. In the following investigation it is also assumed that the characteristics of a type are not altered by ruptures of the constituent groups, the consequence of a rupture under strain being essentially the formation of a new (it might be the re-formation of the old) group instantaneously under no strain. In the former paper, the work was simplified by the postulate that there was no initial strain of any group, and that each old or new group of given type broke down under the same limiting strain in either direction of twist. But this condition required that all strain should again vanish when the angle of twist reattains its initial value, whereas it is known that negative twist is requisite to destroy the "set" produced by the positive twist. In the present investigation, simplification having been attained by dealing first with one type alone, this restriction is dispensed with. It is assumed that the initial condition is one of strain equilibrium,

some of the groups being even just at the point of rupture. This view, of course, is supported by all the phenomena of "after-action" or slow recovery from set.

§ 9. Consider molecular groups or small crystalline groups, all of one type, which break their configuration whenever the shear to which they are subjected exceeds a value ϕ_0 : and regard those which are contained in unit length of a cylindrical shell, of radius r and thickness dr , of the material of a wire. When the wire is twisted, say to the right, groups which, in the initial state of internal equilibrating stresses, were already sheared to the limiting extent ϕ_0 , instantly break down; and any group, already sheared to the right to the extent $(1-m)\phi_0$, breaks down when the shear $m\phi_0$ is attained. If θ be the angle of twist per unit length of the wire, the material of the cylindrical shell is sheared to the extent $\phi = r\theta$. So, if $f''(m\phi_0)d(m\phi_0)$ be the number, per unit volume, of the groups which break in the range $d(m\phi_0)$, the total number breaking in unit length of the cylindrical shell is $2\pi r dr f''(m\phi_0)d(m\phi_0)$. These would again break and retake their original position if a negative shear $(1-m)\phi_0$ were imposed. Rupture would again ensue, according to our postulate, if the positive shear $m\phi_0$, or the negative shear $(1-m)\phi_0$ (measured of course from the initial position of equilibrium as zero), were exceeded by any integral multiple of ϕ_0 .

After, say, positive shear to any extent less than or exceeding ϕ_0 , if the twist be again reduced, a position of positive set will be found somewhere between the greatest integral multiple of ϕ_0 attained and the next greatest. But, if we first limit the investigation to a material constituted of only one type of groups, and also limit it to cases in which the set does not form a large fraction of the total distortion, we need only consider at most one integral multiple of ϕ_0 in the shear.

When, after a positive excursion of any magnitude, say exceeding $\theta_0 = \phi_0/r$, the twist again becomes zero, the distribution of groups is $f''(m\phi_0)$ due to groups which were initially distorted to the positive side, together with $f''(1-m)\phi_0$ due to groups initially distorted to the negative side; and all of these are pulling the wire towards the positive position of set. The condition is now such that in any motion on the positive side of the initial zero, provided that the shear ϕ_0 be not exceeded and that no negative displacement occur, absolutely no dissipation of energy will take place in so far as these groups are concerned. Farther, if a negative shear $q\phi_0$ is now made ($q < 1$), any motion involving shear between the limits $(1-q)\phi_0$ and $q\phi_0$ on the positive and negative sides respectively, will take place without dissipation of energy. *The material could oscillate about the position*

of set with simple harmonic motion provided that neither limit was exceeded. Such a condition in complete oscillations is unknown in nature; but it was this result which suggested the comparison of the observed data with a simple harmonic graph having its zero at the position of set (§ 5).

§ 10. Hooke's Law being postulated as applicable on the average, the energy lost in the range $d(m\phi_0/r)$ is proportional to the square of $m\phi_0$ multiplied by the number of groups which break in that range. In this way the whole amount of energy dissipated in an oscillation may be determined; but we shall proceed by estimating the force.

Suppose now that the oscillations have decayed until the last positive (right, say) maximum shear was $q_n\phi_0$. No energy is dissipated until the subsequent negative (left, say) shear exceeds $(1-q_n)\phi_0$. Consider the negative shear $q\phi_0$ greater than this. Sheared to the left, and therefore pulling inwards, there are those groups which were, in the initial state, sheared to the left within the limits $(1-q)\phi_0$ and ϕ_0 , and which have therefore broken within the range of shear 0 to $q\phi_0$. Postulating Hooke's Law and neglecting the constant factor, the pull per unit volume of the cylindrical shell exerted by these is got by integrating

$$-f''(m\phi_0)d(m\phi_0) \cdot (q - m)\phi_0, \quad m \text{ from } 0 \text{ to } q. \quad . \quad . \quad (1)$$

Also, in that range are the groups originally there and which were to break at $(1-m)\phi_0$ to the right. These give rise to the pull

$$\begin{aligned} & -f''(1-m)\phi_0d(1-m)\phi_0 \cdot (q - m)\phi_0, & (1-m) \text{ from } (1-q) \text{ to } 1, \\ \text{or} \quad & +f''(1-m)\phi_0d(1-m)\phi_0 \cdot [(1-q) - (1-m)]\phi_0 & , , \\ \text{or} \quad & f''(m\phi_0)d(m\phi_0) \cdot (1 - q - m)\phi_0, & m \text{ from } (1-q) \text{ to } 1 \end{aligned} \quad (2)$$

Sheared to the left also are those groups originally sheared to the right within the range $q\phi_0$ to ϕ_0 , and which broke in the twist to the right within the range 0 to $(1-q)\phi_0$. Let the latter shear be $m\phi_0$. The total shear is therefore $(q+m)\phi_0$, and so we obtain

$$-f''(m\phi_0)d(m\phi_0)(m+q)\phi_0, \quad m \text{ from } 0 \text{ to } 1-q. \quad . \quad . \quad (3)$$

Finally, sheared to the left, there are those groups which were initially sheared to the left within the range 0 to $(1-q)\phi_0$, and which would break at the shear $(1-m)\phi_0$ to the left, $(1-m)$ lying within q and 1. So we get

$$\begin{aligned} & -f''(1-m)\phi_0d(1-m)\phi_0 \cdot (q + m)\phi_0, \\ \text{or} \quad & -f''(1-m)\phi_0d(1-m)\phi_0 \cdot [(1+q) - (1-m)]\phi_0, \quad (1-m) \text{ from } q \text{ to } 1, \\ \text{or} \quad & -f''(m\phi_0)d(m\phi_0) \cdot (1 + q - m)\phi_0, \quad m \text{ from } q \text{ to } 1 \end{aligned} \quad . \quad . \quad . \quad . \quad (4)$$

Adding (1) and (4), we have

$$-\phi_0 f''(m\phi_0) d(m\phi_0) - q\phi_0 f''(m\phi_0) d(m\phi_0) + m\phi_0 f''(m\phi_0) d(m\phi_0) \quad . \quad (5)$$

Adding (2) and (3), we have

$$\phi_0 f''(m\phi_0) d(m\phi_0) - q\phi_0 f''(m\phi_0) d(m\phi_0) - m\phi_0 f''(m\phi_0) d(m\phi_0) \quad . \quad (6)$$

Adding (5) and (6), we get finally, on integration,

$$\begin{aligned} & -2q\phi_0[f'(r\theta_0) - f'(0)] + \phi_0[f'(q\phi_0) - f'(1-q)\phi_0] \\ & = -2r\theta[f'(r\theta_0) - f'(0)] + r\theta_0[f'(r\theta) - f'(0)] \quad . \quad . \quad . \quad (7) \end{aligned}$$

and the same result holds if q be less than $1 - q_n$.

§ 11. If the expression (7) were applicable, the second bracket becomes zero when $\theta = \theta_0/2$, and the motion would be simple harmonic with an amplitude equal to the first maximum angle attained below the value $\theta_0/2$. Such a condition does not obtain in nature; but it is not impossible that the kind of action under discussion may be fundamentally concerned in the process of "accommodation" described by G. Wiedemann (*Phil. Mag.*, 1880), according to which a wire, if twisted to and fro a few times between a definite positive and negative angular limit, obeys Hooke's Law much more closely than before within that limit.

But the form of the second term in the second bracket precludes the possibility of f being a power of the angular distortion, as it must be if the energy loss is to be practically representable as proportional to a power of the angular distortion. That term arises from the lower limit in the first term of (6). The simplest way of retaining the general mode of treatment followed, while getting rid of that term, is to modify the postulates in such a way as to add a term $\phi_0 f''(m\phi_0) d(m\phi_0)$, $m=0$ to $m=1-q$. This implies the assumption that the total back pull is less than that given above. Such a diminution might be due, for example, to the members of the groups contributing to the expression (3) acting differently when the reversed shear exceeds some value $(m+gq)\phi_0$, say where $g < 1$; or it might be due to the interaction with other groups.

Instead of the expression (7) we should then get

$$-2r\theta[f'(r\theta_0) - f'(0)] + r\theta_0[f'(r\theta) - f'(0)] \quad . \quad . \quad . \quad (8)$$

§ 12. If we suppose that the commingling of effects alluded to at the end of § 7 is felt to the very core of the wire, the whole moment of the force due to unit length of the cylindrical shell then becomes the integral, from $r=0$ to $r=a$, of the quantity

$$-8\pi k\theta r^3 dr[f'(r\theta_0) - f'(0)] + 4\pi kr^3 dr\theta_0[f'(r\theta) - f'(0)],$$

where a is the external radius of the wire, and $2k$ is the Hooke's Law coefficient. Using the notation $f = H'''$, this becomes

$$\begin{aligned} & -8\pi ka^4\theta \left\{ \frac{1}{a\theta_0}H'''(a\theta_0) - \frac{3}{(a\theta_0)^2}H''(a\theta_0) + \frac{6}{(a\theta_0)^3}H'(a\theta_0) - \frac{6}{(a\theta_0)^4}[H(a\theta_0) - H(0)] \right\} \\ & + 4\pi ka^4\theta_0 \{ (\alpha\theta)^{-1}H'''(\alpha\theta) - 3(\alpha\theta)^{-2}H''(\alpha\theta) + 6(\alpha\theta)^{-3}H'(\alpha\theta) - 6(\alpha\theta_0)^{-4}[H(\alpha\theta) - H(0)] \} \\ & - \pi ka^4H'''(0)(\theta_0 - 2\theta) \quad . \quad (9) \end{aligned}$$

If we now postulate a solution

$$H(x) = Ax^p,$$

we have $H(0) = 0$, and also $H'''(0) = 0$ since $p = n + 5 > 4$. Then (9) becomes

$$-4\pi kAa^4(p-1)(p-2)(p-3)[\theta_0 \cdot 2(a\theta_0)^{p-4} - \theta_0(a\theta)^{p-4}] \quad . \quad . \quad . \quad (10)$$

which is Hooke's Law modified by a term varying as a power of the distortion. It therefore leads to the observed law of dissipation of energy.

And it also gives the observed experimental results regarding the law of oscillation (§ 5). In the inward motion from a maximum shear $\mu\phi_0$, the motion is simple harmonic to the position of set, beyond that to the initial zero, and beyond that to an extent which may reach the range $(1-\mu)\phi_0/r$ or not, according to the nature of the action which changes (7) into (8).

If there is any inferior limit to shear at which commingling of effects ceases, at least one type of groups which dominated the action ceasing to break, a sudden change in the power of the distortion could occur when the range of oscillation diminished to the corresponding value. Thus it may be that a sudden change in the value of n in the equation $y^n(x+a) = b$ is due to the existence of a dominating type of groups which is effective above a certain range of oscillation and becomes ineffective below it. The temperature or stress effects described by Mr Ritchie would then be explainable, by the production or destruction of one type of groups. The shifting, with change of temperature, of the limiting range of oscillation at which the value of n suddenly altered, seems to indicate a change of the limiting shear at which one type of groups breaks down. This is naturally to be expected in the neighbourhood of a temperature at which a change in crystalline form takes place.

§ 13. The necessity for the view that commingling of effects takes place, even in the sense of the sharing of the action between groups in which the strain is sufficient itself to effect rupture and those in which it is not sufficient, at once appears, if we integrate (8) only throughout the region in which the former condition holds. The limits for $r\theta$ are then $\phi_0/2$ and $a\theta$, while those for $r\theta_0$ are $\phi_0\theta_0/2\theta$ and $a\theta_0$. Instead of (10) we then obtain

$$-4\pi kA(p-1)(p-2)(p-3) \left\{ \frac{2\theta}{\theta_0} \left[(a\theta_0)^p - \left(\frac{\phi_0\theta_0}{2\theta} \right)^p \right] - \frac{\theta_0}{\theta^4} \left[(a\theta)^p - \left(\frac{\phi_0}{2} \right)^p \right] \right\} \quad (11)$$

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an expression which is not reconcilable with the experimental data regarding the loss of energy.

§ 14. When the angle of oscillation is so small that crystalline groups of "finite" size do not break down, if the loss of energy becomes proportional to the second power of the distortion, it is easy to account for the exponential law of decrement first established by Kelvin.

When the angle of oscillation is so large that integral multiples of θ_0 are included in it, the preceding postulates regarding the action lead to a periodic expression for the force, the set being very large. Some experiments by Mr Stewart, Carnegie Scholar, University College, Dundee, whose results are not yet published, seem to indicate the existence of a periodic condition even when the set is negligible. It might be negligible, even though the shear greatly exceeded ϕ_0 for the groups undergoing rupture, provided that another type of groups did not undergo rupture at all.

If we are to take account of the presence of a continuous series of types of groups, we must multiply (10) by $\psi(\theta_0)d\theta_0$ and integrate over any range specified.

SUMMARY.

1. The oscillations of a viscous wire exhibiting set are simple harmonic with regard to the position of set throughout the whole range of the in-motion and a considerable part of the out-motion. In the remainder of the out-motion the deviation from the simple harmonic condition is great.

2. The period of in-motion to the zero is distinctly greater than that of the out-motion from it.

3. A theoretical deduction of these results, and of the law of loss of energy per oscillation, can be founded on the assumption that that loss is due to the rupture of strained molecular (which may be finite crystalline) groups.

4. A sudden change in the oscillation parameters when the decreasing maxima pass through a definite magnitude is explainable by the existence of a dominant type of groups which cease to rupture when the decreasing strain reaches a definite value; and the known peculiar temperature and stress effects are explainable as depending on well-recognised conditions of structural changes.

(Issued separately June 6, 1913.)

XXII.—The Effect of Thermal Treatment and the Effect of Longitudinal Strain in inducing a Sensitive State in certain Magnetic Materials. By Margaret B. Moir, M.A., B.Sc., Carnegie Research Fellow in the University of Glasgow. *Communicated by Professor A. GRAY, F.R.S.*

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It has been pointed out by several experimenters, notably by Ewing,* and Gray and Ross,† that a specimen of steel freshly annealed is in a peculiar magnetic condition. Thus, if a specimen, thoroughly demagnetised, is annealed and then tested, a certain I-H curve is obtained. If it be then demagnetised, and tested again, a second I-H curve is obtained which lies definitely below the first; and any further tests after demagnetisation produce a repetition of the second curve, which is characteristic of the specimen. To obtain the first curve again, the specimen has to be annealed from the same temperature as before, and a test made before demagnetisation. It has been shown also that this “sensitive state” of the material, as it has been called, can be induced not only by heating to a high temperature, but, to some extent at least, by any change of temperature, and that it is while the temperature is actually changing, not while it remains constant at any value, that it is induced.

Though much has been done by previous experimenters, there still remains a wide field for further investigation, and, at the suggestion of Dr J. G. Gray, the author began in January 1911 a detailed examination of a number of steels, with a view to finding how they compared with one another when subjected to similar thermal treatment, and how differing thermal treatment affected the same specimen.

After considerable progress had been made in this research, it was suggested by Dr Gray, with a view to obtaining further information about the nature of the “sensitive state,” that the effect of strain in inducing it should also be investigated. That the “sensitive state” could be induced by application or removal of longitudinal stress had already been pointed out by Ewing,‡ but no attempt had been made to compare the amounts of sensitive state induced under different circumstances and in different materials.

* J. A. Ewing, “Experimental Researches in Magnetism,” *Trans. Roy. Soc.*, clxxvi. p. 630.

† *Proc. Roy. Soc. Edin.*, xxviii. pp. 239 and 615 (1908).

‡ *Trans. Roy. Soc.*, clxxvi. p. 580 *et seq.*

This research was accordingly undertaken in conjunction with that previously begun, the general aim of the work being to ascertain to what extent the two treatments produced similar effects.

The present paper therefore falls into two parts:—

I. An account of the effect of thermal treatment in inducing sensitive state in five different specimens, and a comparison of the results obtained for the different specimens.

II. An account of the effect of longitudinal strain in inducing sensitive state in five different specimens, and a comparison of these results among themselves, and with the results given in Part I.

I. EFFECT OF THERMAL TREATMENT IN INDUCING SENSITIVE STATE.

The specimens used in this investigation were with one exception in the form of cylinders 20 cm. in length, and about 0·9 cm. in diameter, the exception being a piece of steel wire of the same length, but only about 0·2 cm. in diameter.

The following table gives the composition of the specimens:—

Description of Material.	Percentage Composition.					
	Carbon.	Mn.	Si.	P.	S.	Tungsten.
Medium carbon steel	0·80	0·20	0·075	0·012	0·02	...
Hard steel	1·321	0·339	0·143	2·745	0·023	...
Steel wire	0·755	0·660	0·066	0·027	0·017	...
Cast iron	3·15	0·15	0·13
Tungsten steel	0·51	0·13	0·19	4·01

Previous to being tested, each specimen was carefully annealed from 900° C. in a Fletcher gas furnace, care being taken to exclude air from contact with the specimen during the process. For the magnetic tests, a Gray-Ross magnetometer* provided with an electric furnace† was employed, and in the case of each specimen the procedure was as follows:—

The specimen was first carefully demagnetised by what is known as the process of reversals—that is to say, it was submitted to the action of an alternating magnetic field of gradually diminishing strength; the initial value of this field being great and the final value zero, the specimen was left devoid of residual magnetism and magnetic history.

* *Proc. Roy. Soc. Edin.*, vol. xxix. p. 182, 1909.

† *Proc. Roy. Phil. Soc. Glasg.*, vol. xii., 1910.

The specimen was then, while in its position in the solenoid, raised by means of the electric furnace to a temperature of about 50° C., put through a magnetic test, demagnetised by reversals to remove the sensitive state, and tested again. The difference between the first and second readings is a measure for any field strength of the sensitive state induced by the previous thermal treatment. The specimen was then demagnetised once more, and the temperature raised another 50° C., and the process repeated, and this continued till a temperature of about 600° C. was reached. This enables the amount of sensitive state induced by a rise of about 50° C. at different parts of the temperature scale to be determined.

The same process was then repeated at intervals of 100° C. instead of 50° C., and tests were also made to determine the effect of a rise of temperature from room temperature to about 100°, 200°, 300°, 400°, 500°, and 600° C., the effect of fall of temperature from these temperatures to room temperature, and also the effect of rise and fall of temperature without intermediate demagnetisation.

The values of the sensitive state are expressed in the usual manner as the percentage by which the magnetic intensity for the specimen in the sensitive condition exceeds the normal value, and the figures for each specimen are taken for the particular field for which for that specimen the effect is about a maximum.

The effects produced by successive rises of 50° C. and 100° C. are shown graphically by ordinates erected at the temperature points at which readings are taken. This method was adopted as showing more clearly than a continuous curve could do exactly what measurements were obtained, and also because the successive rises were not in every case exactly the same, and therefore the process could hardly be described as a continuous one.

The other effects, however—of rise of temperature, of fall of temperature, and of rise and fall of temperature—are all shown graphically in curve form with the temperatures as abscissæ and the percentage sensitive state as ordinates.

Discussion of Results.

Medium Carbon Steel.—Fig. 1 (i.) shows the effect of successive rises of temperature of about 50° C., and illustrates very clearly how greatly the effect of a certain rise of temperature depends on its position on the temperature scale; for while a rise from 100° C. to 150° C. produces a sensitive state of nearly 40 per cent., a rise from 200° to 250° C. has an entirely negligible effect. The most susceptible part of the temperature

scale is from 0° C. to 200° C., especially about 150° C.; after that, further rises of 50° C. produce no sensitive state till a temperature of 400° C. is reached, after which further successive rises of 50° C. produce sensitive state of about 5 per cent.

Fig. 1 (ii.), showing the effect of successive rises of 100° C., gives exactly the same division of the scale into susceptible and non-susceptible parts—the greatest effect being produced in the interval 100°–200° C., which induces a sensitive state of over 40 per cent.

Fig. 2 (i.) shows curves (a), (b), and (c)—(a) showing sensitive state due to rise, (b) sensitive state due to fall, and (c) sensitive state due to rise and fall of temperature. (a) shows a marked maximum about the region of 200° C., and lies above both (b) and (c) up to a temperature of 300° C., when it crosses (b) and finally lies below (c) also; (b) remains pretty steadily between 20 per cent. and 25 per cent. for all temperatures; and (c), which is below (b) for all temperatures, rises rapidly at first, then slowly till a temperature of 400° C. is reached, when it falls somewhat, then rises once more.

Hard Steel.—Fig. 1 (iii.) shows the effect of a rise of 50° C. to be, as in the previous specimen, greater for the rise from 100° to 150° C. than at any other part of the scale, though the effect in this case is only a little more than half as great; but in this specimen, on the other hand, a rise of 50° C. at any part of the temperature scale, except above 500° C., produces some effect. The effects of rises of 100° C. are somewhat similar to, but less marked than, those due to rises of 50° C.

The (a), (b), and (c) curves for this specimen are somewhat similar in general form to those of the last, but the maximum in the (a) curve does not appear till about 400° C., and is much less marked, the maximum value reached being only 21 per cent. in this case, as compared with 38 per cent. in the other. Indeed, the sensitive state induced in this specimen by all varieties of thermal treatment employed is much less marked than in the case of medium carbon steel.

Steel Wire.—The effect of increasing the temperature by 50° or 100° C. is still less marked in this case than before, but the greatest effects still show at 100–150° C. and 100°–200° C. The (a), (b), and (c) curves are again somewhat similar, but with much less definite turning-points; and the (a) curve, so little is its maximum marked, never rises above the (b) curve, which lies throughout the range below the 15 per cent. line.

Cast Iron.—The effect of increasing the temperature by 50° or 100° C. is, in the case of cast iron, greater than for steel wire, but not so great as for hard steel. This specimen, however, shows one point of difference from

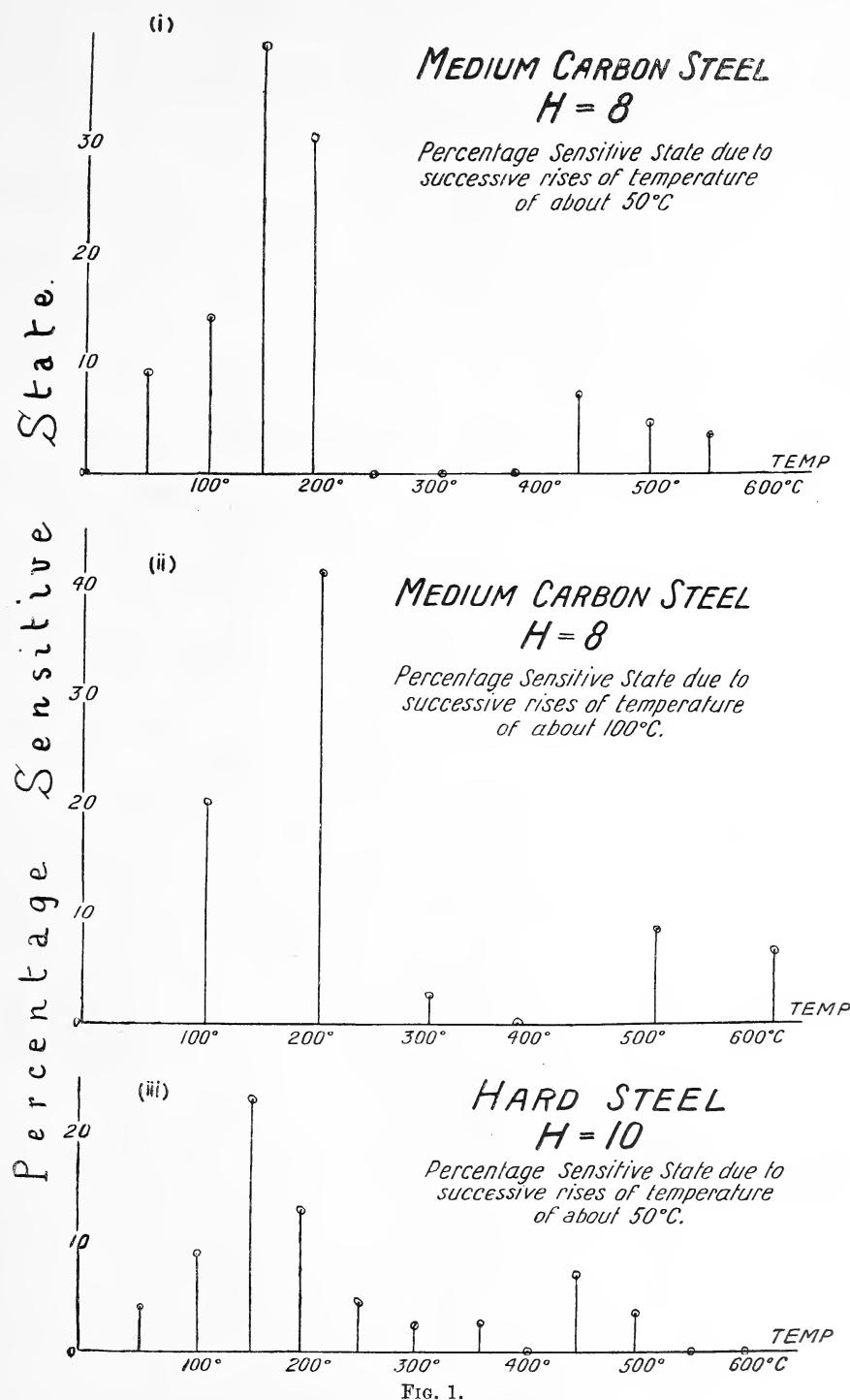


FIG. 1.

those previously considered—the most effective part of the temperature scale in producing "sensitive state" being for it 150° – 200° C., instead of, as

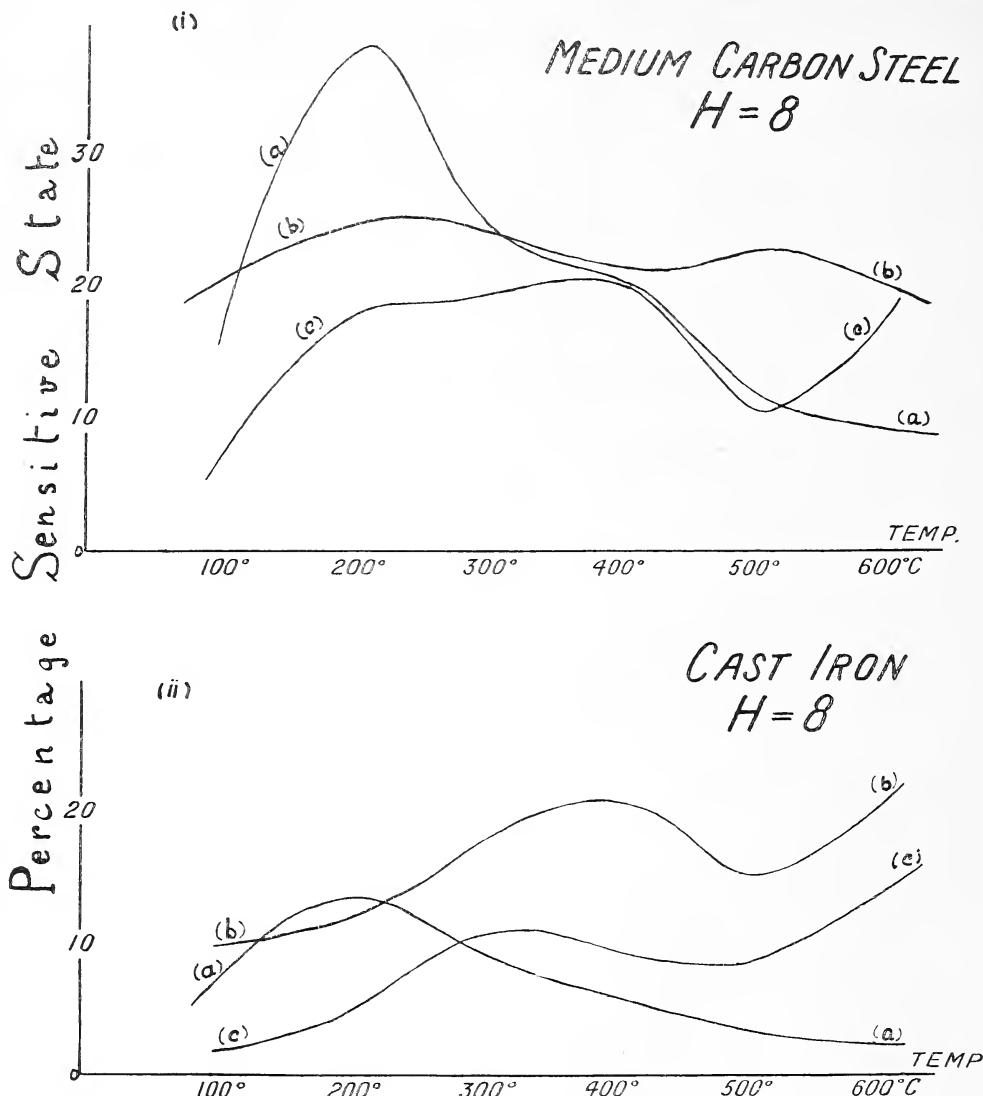


FIG. 2.

- (a) Percentage sensitive state due to rise of temperature.
- (b) " " " fall " "
- (c) " " " rise and fall " "

in all the other cases, 100° – 150° C. As might be expected, 100° – 200° C. is still the most sensitive interval for rises of 100° . While the (a) curve for this specimen (fig. 2 (ii.)) resembles that previously shown in rising to a maximum value (of only 13 per cent., however) about 200° , and then falling

off steadily, the (*b*) curve differs considerably, as, instead of maintaining a fairly steady value, it rises from 10 at 100° C. to a maximum of over 20 at 400°, then falls and rises again. The (*c*) curve lies entirely below the (*b*) curve, and is similar to it in form.

Tungsten Steel.—This specimen, with which we leave the carbon series, displays some very decided differences in behaviour from those previously examined.

First of all, the specimen shows much less sensitive state than any of the others, and the most susceptible part of the scale is now about 300° C.—from 300° to 350° C. for rises of 50° C., and from 200° to 300° C. for rises of 100° C. Also for low temperatures the specimen is very insensitive, rises of 50° having no effect in producing sensitive state below the 300° to 350° C. rise, while, in rises of 100° C., 0° to 100° C. has no effect; the maximum sensitive state produced in any of these intervals is less than 2 per cent. Of the (*a*), (*b*), and (*c*) curves, the (*b*) curve shows much the more important effect, rising from zero for a fall of temperature from 100° C. to room temperature to 8 per cent. for a fall from 600° C. to room temperature; while the (*a*) curve, beginning to rise at the same point, never rises above 3 per cent. The (*c*) curve, again, only begins to rise after a rise and fall of more than 300° C., and it also never rises above 3 per cent.

Coming now to compare the results given us by these five specimens, we note first of all the very great difference between the four belonging to the carbon group and the tungsten specimen. Not only is the sensitive state induced in the latter much less marked, and the most sensitive part of the temperature scale much higher up, but rises of temperature at the beginning of the scale have no effect at all.

Between the different members of the carbon group very strong resemblances appear. The part of the scale which is most pronounced in producing sensitive state is in each case very nearly the same, being only a very little later in the case of cast iron. It would appear, then, that the addition of a marked quantity of carbon tends to raise the temperature of maximum susceptibility, though the addition of smaller quantities has no appreciable effect.

Considered generally, the specimen of the four showing the least sensitive state is the specimen of steel wire, while that showing most is the specimen of medium carbon steel. As the percentages of carbon in the two are very nearly the same, .755 per cent. and .80 per cent., it is probable that the smaller sensitive state is connected with the greater percentage of manganese in the steel wire specimen. The specimen of hard steel shows next greatest effect, and cast iron comes third in the list.

Taking maximum values of sensitive state reached in the temperature range chosen, we find for fall of temperature, for steel wire 15 per cent., for medium carbon steel 25 per cent., for hard steel 24 per cent., and for cast iron 21 per cent.; while for rise of temperature, for the same four specimens, we have 10, 38, 22, and 13 per cent. These maximum effects appear, of course, at very different temperatures.

Again, looking at the (a), (b), and (c) curves generally, we find that as a rule the (b) curve lies above both the (a) and (c) curves, and for lower temperatures the (a) curve lies above the (c) curve, while for higher temperatures the reverse is the case. That is to say, if we consider any given temperature, we can generally induce a greater sensitive state by rendering the specimen neutral at that temperature and then allowing it to cool than by either heating it up to that temperature and then testing, or by heating and then cooling before testing. Of these two methods, the former is likely to be the more efficacious if the temperature considered lies below 400° C., the latter if it lies above.

II. EFFECT OF LONGITUDINAL STRAIN IN PRODUCING SENSITIVE STATE.

A Gray-Ross magnetometer was employed in this investigation also, but, as it was desired to apply longitudinal stress to the specimen while it was in such a position that it could be tested magnetically, a solenoid in a vertical instead of in a horizontal position was used, and the one-pole method was adopted.

A long narrow solenoid, about 70 cm. in length, was attached vertically by means of a framework to the magnetometer board, so that one end was about 10 cm. above the level of the magnetometer needle. An oblong hole in the magnetometer board and in the table below it allowed the solenoid to pass through, and also allowed of its being moved nearer to or further from the needle as required. The solenoid carried, in addition to the ordinary coil used for magnetising the specimen, a second coil through which a small current could be passed to balance, within the solenoid, the effect of the earth's vertical field.

In order to fix the specimen in the solenoid in such a way that it might be readily removed and adjusted, and at the same time be firmly enough fixed not to alter its position when subjected to stress, it was screwed at each end into a short length of thick brass wire. Of these brass wires, one could be clamped firmly into position just above the upper end of the solenoid, while to the other was attached a strong brass hook from which could be hung the weights applied, so that the specimen, when in position

in the solenoid, hung from the upper brass wire with its ends about equally distant from the ends of the solenoid. Also the upper brass wire could be moved through a small distance up and down at the clamp till that position was obtained which brought the pole of the specimen most nearly opposite to the magnetometer needle.

To apply longitudinal stress to the specimen, a large can was obtained, capable of holding about a cubic foot of water, and fitted with a gauge, and also a tap near the bottom. This can was hung on the hook attached to the lower brass wire, and a gentle stream of water was allowed to flow into it, until the desired weight, which was read off on the gauge, was obtained. In this way any desired stress could be applied with an almost entire absence of vibration—a very important point, as it has been shown that vibration greatly diminishes the sensitive state. To decrease the weight by any fixed amount, it was only necessary to turn on the tap near the bottom of the can, and allow the required amount of water to flow out.

The five specimens tested were all steel wires, 50 cm. in length and about $\frac{1}{10}$ inch in diameter. This thickness of wire, which is moderately large, was chosen so that the effect of small stresses might be the better examined. A weight of 1 kilogramme applied to such a wire gives a stress of 19·74 kilos per square cm. Of the five specimens used, four, supplied by Messrs W. N. Brunton & Son, form a graded carbon series having respectively 0·15 per cent., 0·55 per cent., 0·8 per cent., and 1·2 per cent. carbon in their composition. The fifth is interesting as being a specimen of the same wire as was used in Part I., and the results obtained by the two methods are therefore for it directly comparable.

Each specimen was, before being tested, carefully annealed as in the previous case, and then stretched several times by the application and removal of the maximum weight to be employed. Unless this was done, it was found that the specimen was not in a stable condition, and that repetition of the same processes did not produce the same results.

The specimen was then placed in position in the solenoid, carefully demagnetised, subjected to a certain strain, tested, demagnetised, and tested again, and from the two curves obtained the percentage sensitive state due to increase of stress from 0 to w was determined. It was then demagnetised again, the strain was removed and the sensitive state due to decrease of stress from w to 0 obtained, and this was repeated for various values of w from 0 up to 20 kilogrammes.

The effect of the cycle 0- w -0 was also determined, and also the effect of increasing or decreasing the stress by successive differences of 2·5 and of 5 kilos.

In the course of the carrying out of the tests the specimen was very frequently removed, in order that the zero might be tested, a proceeding which was found to be very necessary, as the small changes in the position of the zero which took place in the course of a test were sometimes found to be large enough to make the readings for sensitive state quite unreliable, and a large number of readings had frequently to be taken before agreement could be obtained.

Before going on to the discussion of the results obtained for the different

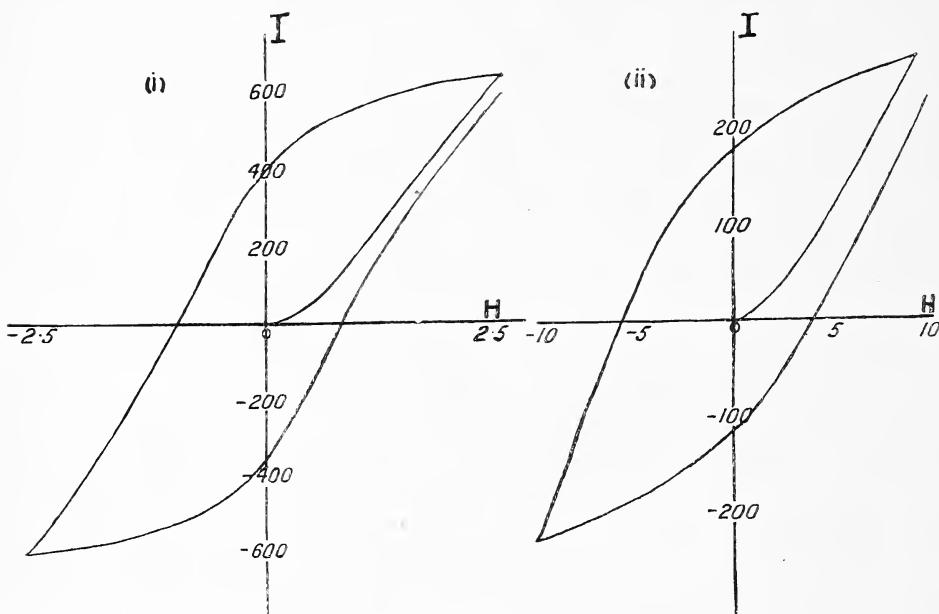


FIG. 3.

specimens, it may be worth while pointing out how very close is the analogy between the effect of strain and the effect of thermal treatment.

Fig. 3 (i.) shows a hysteresis curve for Specimen I. (0·15 per cent. C) taken after the specimen had been demagnetised with a load of 10 kilos on, and the load had been removed. As will be seen, the hysteresis curve is not closed, as it would have been had the specimen been demagnetised just previous to being tested, and the value of the intensity of magnetisation corresponding to the maximum field employed is distinctly less after even one reversal of the field. Indeed, the first reversal removes about 50 per cent. of the total improvement produced by the alteration of stress. If we compare with this curve that shown in fig. 3 (ii.), a reproduction from a paper by Gray and Ross on the effect of thermal treatment in inducing

sensitive state, we see the remarkable similarity of effect produced by the two methods of treatment.

Passing now to the discussion of the results obtained for the different specimens, we have in fig. 4 (i.) the most interesting results for—

Specimen I. (0·15 per cent. C).—The three curves shown are lettered as in the first part of the paper, the (a) curve showing the effect of increasing

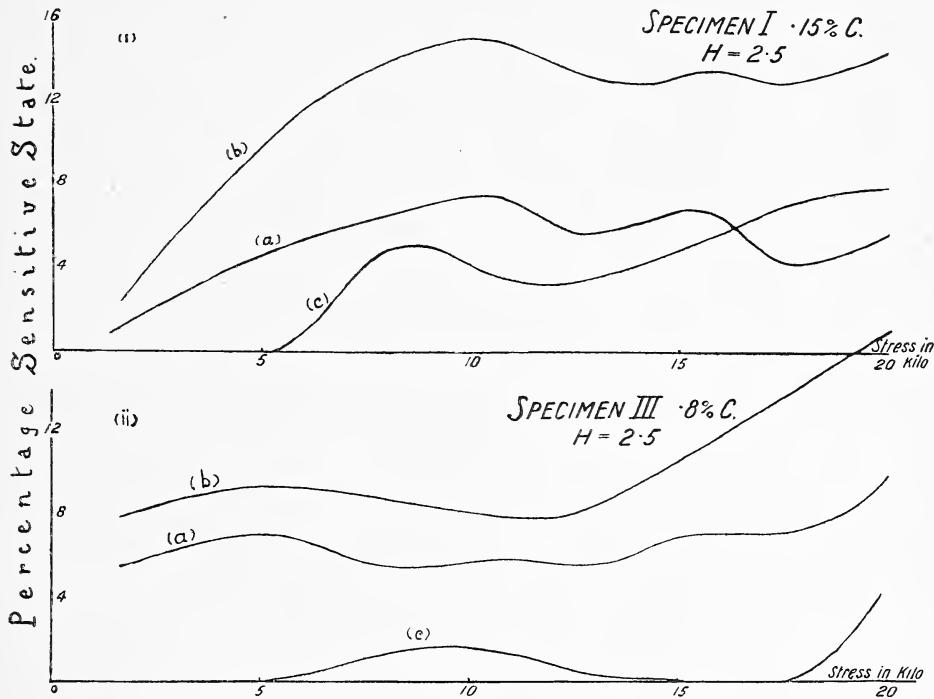


FIG. 4.

- (a) Percentage sensitive state due to application of stress.
- (b) " " " removal " "
- (c) " " " application and removal of stress.

the stress from 0 to w , the (b) curve showing the effect of decreasing the stress from w to 0, and the (c) curve showing the effect in producing sensitive state of the cycle 0- w -0.

As the stress is increased from 0 to 10 kilos, both (a) and (b) curves rise rapidly to a maximum. Further increase of stress is followed by a slight drop in the value of the sensitive state, but a second though lower maximum occurs about 15 kilos, and at the end of the range the curves are rising once more. The maxima and minima of the two curves occur at almost the same points, and the (b) curve lies entirely above the (a) curve, the percentage sensitive state due to decrease of stress being, indeed, about

twice as great as that due to the corresponding increase. The (c) curve shows only one maximum and one minimum in the range, and lies below both (a) and (b) till a stress of 15 kilos is passed, when it crosses (a), but remains below (b).

Specimen II. (0·55 per cent. C).—Application of stress to this specimen gives on the whole much higher percentage values of sensitive state than in the case of the previous specimen, but the first maximum appears in the (b) curve only at 12·5 kilos, and in the (a) curve not till 15 kilos. Both curves show also a minimum value within the limits of the range, and are rising at the end of it; but this time too the turning-point appears later in the (a) curve than in the (b) curve. The (b) curve lies entirely above the (a) curve, and both of these are above the (c) curve, which does not at any point of the range rise above a value of 4 per cent.

Specimen III. (0·8 per cent. C).—The sensitive state values for this specimen, fig. 4 (ii.), are much lower than those reached in either of the previous specimens, so far as the range examined goes, and, though there is a slight indication of a maximum near the beginning of the range in both the (a) and (b) curves, the values of the sensitive state remain in each fairly constant till a stress of 12·5 kilos is reached, after which both curves rise steadily. Neither has reached a maximum, however, within the limits of the range. The (c) curve shows only very small values, and lies below both the others for the whole range.

Specimen IV. (1·2 per cent. C).—The (a) and (b) curves for this specimen show smaller values of sensitive state than any of the others. Both remain steady for low values of the stress, and only rise very slowly as the stress is increased, the (b) curve lying entirely above the (a) curve throughout the whole range. The sensitive state induced by the stress cycle 0-w-0 is negligible for all values of w from 0 to 20 kilos.

Specimen V. (0·755 per cent. C).—This specimen shows on the whole more sensitive state than Specimen III., and less than Specimen II., a result quite to be expected if the sensitive state produced depends on the carbon content of the material. The form of the (b) curve is very similar to that of the (b) curve of Specimen II., but the (a) curve has its turning-points at lower instead of at higher stresses, as was the case with that specimen. The (c) curve lies throughout the range below the 2 per cent. line.

The figures showing the effect of increasing or decreasing the stress by successive differences of 2·5 and 5 kilos have not been shown, as they are hardly of sufficient interest to merit separate discussion. As a rule, effects of decrease rather than of increase of stress were examined, as the (a) and (b) curves showed decrease to be the more effective in producing sensitive

state. Perhaps the most interesting fact in this connection is that in every case examined the greatest effect was produced by the last decrement of all, the final removal of the applied stress—the final removal of even a small stress being much more effective than the removal of a larger stress when some weight was still left on.

Comparing now these results among themselves, particularly the results obtained for the first four specimens, which form a graded series, we notice, first of all, that in each the (b) curve, representing the effect of removal of stress, invariably lies above the (a) curve, which exhibits the effect of application of stress, and as a general rule the (c) curve lies below both. Also the (a) and (b) curves pretty nearly follow the same course, rising and falling together, except in the case of Specimen II. (0·55 per cent. C), where the rise and fall in the (a) curve come later than in the (b) curve, and Specimen V. (0·755 per cent. C), where the reverse is the case.

Comparing now the general values of the sensitive state reached within the chosen range by each of the specimens, we find that higher values are reached by Specimen II. than by any of the others, and that these high values correspond to greater stress-values than do the high values of Specimen I. Specimen V. shows lower values than Specimen II., Specimen III. much lower values, and Specimen IV. lower values still; but all these specimens show increasing values towards the end of the range, and therefore probably show a maximum effect for stresses beyond the limits of the chosen range.

All these facts, then, seem to point to the probability that, with an increasing percentage of carbon in the composition of the steel, a higher possible sensitive state *may* be induced, but that to induce it greater stresses are required as the percentage of carbon rises. That a harder steel should require the application of a greater stress to produce a maximum effect is only what might have been expected, but further investigation is required, in which, by the use of thinner wires or greater applied stresses, increased strains may be produced, in order to determine definitely to what extent increasing percentage of carbon in the steel goes along with increasing values of the sensitive state induced.

Coming now to a comparison of the effects produced by the two methods, we find some marked similarities between them, though the fact that the phenomena have only been examined over a very limited range in the second set of experiments prevents a complete comparison being made.

It may be noted, however, that in both cases the (b) curve is generally above the other two, *i.e.* that removal of stress and cooling correspond in producing a greater effect than the other processes; while application of

stress and heating correspond in producing a greater effect than the cyclic operations of application and removal of stress, and heating and cooling, for all stresses (except in the case of Specimen I.) and for low and moderate temperatures.

On the other hand, the effects produced by strain are in general much less marked than those produced by thermal treatment, and the field for which the effect is a maximum is in the former case much lower.

The phenomena in the two cases, then, while showing decided similarities, are far from being identical, and at the present stage of the research it is impossible to come to any conclusion as to whether there is a connection between them or not.

The author desires to take this opportunity to express her indebtedness to Dr J. G. Gray for advice and assistance in the carrying out of the experiments, and to Professor Gray for the interest he has taken in the work.

(Issued separately July 4, 1913.)

XXIII.—Some New Types of Spectrophotometer on the "Flicker" Principle. By James Robert Milne, D.Sc.

(Read March 21, 1910. MS. received April 27, 1912.)

IN a previous paper * I have described a new form of spectrophotometer in which the comparison of relative brightness is effected by bringing the two beams accurately edge to edge: the present paper is for the purpose of describing some spectrophotometers in which the comparison is effected by allowing the two beams to fall alternately on the eye in rapid succession.

I. It will perhaps conduce to clearness if I refer to a subsidiary matter first—namely, the question of the production of a beam of light suitable for use in light-absorption measurements. Unless the beam consists of nearly parallel rays, † and has the same intensity at each point of its cross-section, it is clear that measurement of the relative brightness of two parts of it, one of which has passed through the absorbing substance, will not lead to a satisfactory determination of the absorption. The only way of obtaining a reasonably parallel beam is by the use of what amounts to a collimator having the usual slit replaced by a somewhat large round hole, and hence possible variations of method merely turn on different methods of lighting up this hole. I find that both the electric arc and limelight are completely unsuited for the purpose, the ensuing beam of light from the collimator lens usually possessing darker parts which are evident to the eye if the beam be allowed to fall on a white surface, and which may be recorded photographically on the surface of a sensitive plate momentarily exposed to the light of the beam and afterwards developed in the usual way. A gas or other flame focussed by means of a lens on the hole of the collimator gives only a weak beam of light, unless, indeed, acetylene gas be employed. I worked with the latter for some time with quite satisfactory results, but it is messy and troublesome, and I have now discarded it in favour of the Nernst electric lamp. Before this had been generally introduced, I tried a strip of electrically heated platinum placed immediately outside the hole in the collimator, and this is very simple and satisfactory, but it requires a somewhat heavy current, and cannot safely be used at a temperature

* "Some General Principles of the Absorption Spectrophotometer, and a New Form of Instrument which embodies them," *Proceedings of the Optical Convention*, p. 178, 1905.

† In regard to this point see, e.g., Ewan, T., "On the Absorption Spectra of Some Copper Salts in Aqueous Solution," *Phil. Mag.* (5), xxxiii. p. 326, 1892.

sufficient to give good illumination in the violet. The Nernst filament is, of course, unmeltable and seems to be almost ideal for the purpose. I have found it best to use those employed for lantern projection work, which require to be heated with a Bunsen to make them start.

II. The first type of flicker spectrophotometer I constructed is shown diagrammatically in fig. 1, and is a simple modification of the instrument already referred to.* In that instrument a double-image prism was employed to bring the two beams accurately edge to edge; in this case a prism of greater power is used, which has the effect of completely superposing them. The two beams X and Y, whether proceeding from two different light-sources as in "comparison spectrophotometry," or from a single light-source as in "absorption spectrophotometry," are focussed by the lens L_1 on the collimator slit. After passing in turn through the collimator lens L_2 , the dispersing prism P_1 , and the telescope lens L_3 , they are brought to a focus at the double-image prism P_2 , and thence pass on as one through the nicol N into the small telescope T , and thence to the observer's eye. In the plane of the spectrum immediately in front of P_2 there is placed, as usual, a shutter with a slit which permits only the light of the desired part of the spectrum to pass. On looking into the telescope, which it is convenient to focus on a screen pierced with two exactly similar apertures for the two beams and placed between L_2 and P_1 , the observer sees these two apertures in monochromatic light and superposed on each other. To make a measurement, the opaque disc shown in fig. 2 is placed anywhere between L_2 and L_3 , and by its rotation alternately cuts off the light from each beam. In these circumstances, unless the two beams are equally bright, a strongly marked flickering sensation is produced, and this furnishes a delicate criterion of their equality of brightness. The relative intensity of the two beams can be altered as required by rotating the nicol N , the reading on the circular dial of which, when the flickering has ceased, gives the required measurement after the usual "reduction." I find it most convenient to drive the disc by means of a little water turbine, as such a motor can be made perfectly steady-running and practically noiseless. The water-tap, or a brake on the axle, furnishes a ready means of adjusting the speed, and should be arranged to be near the observer's hand, as the best speed depends on such variable factors as the colour of the light, the intensity, etc.

The apparatus thus arranged suffers, like most polarising spectrophotometers, from the error caused by the partial polarisation in the vertical direction imparted to the light by the prism P_1 . This error can be done

* *Loc. cit.*

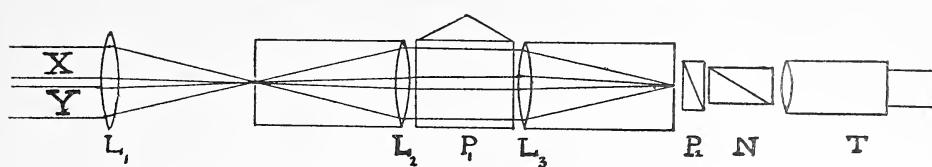


FIG. 1.

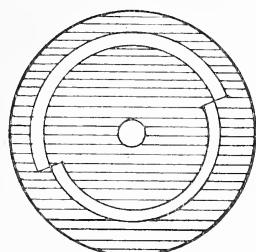


FIG. 2.

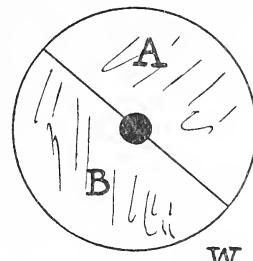


FIG. 4.

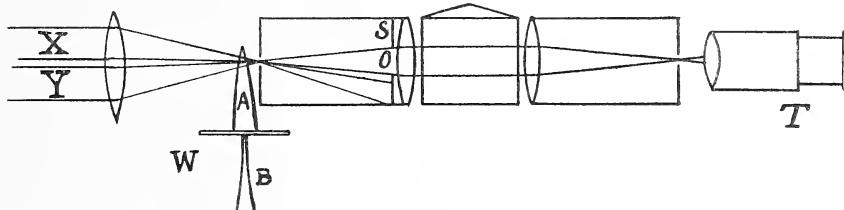


FIG. 3.

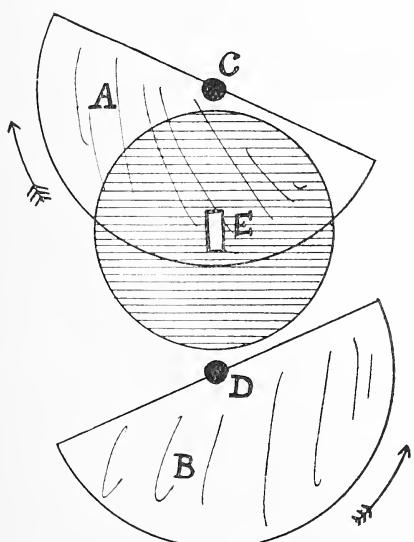


FIG. 5.

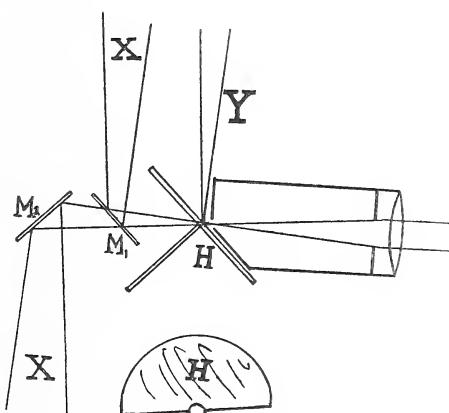


FIG. 6.

These figures are not to scale.

away with by using a double-image prism so constructed that the two beams are not, as is usual, polarised vertically and horizontally, but instead in two directions inclined at 45° to the vertical. I would add the general remark that, while polarisation spectrophotometers are popular on account of their great convenience, they possess the grave fault of being wasteful of light. This is due, of course, to the fact that in polarising a beam of light more than half the intensity is inevitably lost. And although polarisation affords a ready means of altering light intensities, the accuracy of the measurement obtained when a double-image prism is employed is somewhat uncertain, because such prisms do not really divide a beam of light into two equally intense beams, but give rise also to faint additional beams. This may easily be verified by examining any bright light-source through such a prism, when, in addition to the two chief images that are formed, various others will be seen.

III. These reasons induced me to turn my attention to the construction of a flicker spectrophotometer which should be independent of polarisation.

Two different types of arrangement suggest themselves, one depending on the deviation caused by a prism, the other on reflection by a mirror.

(1) An instrument employing the first of these principles is diagrammatically shown in fig. 3, where W is a sort of glass wedge wheel, consisting of two semicircular discs A and B, fig. 4, A being one half of a convergent lens, and B one half of a divergent lens of equal power. The axle of W is situated parallel to the collimator tube intersecting the (imaginary) prolongation of the slit, and (indifferently) either above or below the latter. The axle is rotated by means of a small pulley fixed on it, driven by a belt from the little water motor mentioned before. It will be seen that the transition of the passage of the light entering the collimator slit from the one half lens A to the other B takes place very rapidly. When A is in front of the slit the beams X and Y are deflected downwards; when B is in front of the slit they are deflected upwards. The result is that the aperture O in the screen S placed at the end of the collimator is lighted alternately first by one beam and then by the other; and therefore, unless the two beams are of equal brightness, the observer, on looking through the small telescope T, which is focussed on the aperture O, is conscious of a flickering effect. Evidently it is not essential that the lens wheel should be placed at the collimator slit, and as a matter of fact I originally put it in a position in front of the observing telescope T; but there are two drawbacks to this. First of all, it is less convenient because it means mounting both lens wheel and driving motor on the movable arm of the spectroscope, instead of having the lens wheel alone on the immovable

collimator and the motor anywhere near, on the table, etc. Also, I find that it is an excellent plan to get rid of the momentarily discarded beam at the earliest possible place in the optical train, because this gets rid of diffuse accidental reflections later on, which are often a source of serious trouble. To aid this, I have replaced the usual narrow collimator tube by a large blackened rectangular box, in which the momentarily discarded beam is effectually lost. The lens wheel just described was only tried after various other arrangements had been discarded, the objection to it being its apparent lack of symmetry. A symmetrical arrangement of two similar glass wedges set opposite ways and given a reciprocating motion had to be abandoned because of its mechanical awkwardness. This was succeeded by the device shown in fig. 5, where A and B are halves of the *same* convergent lens, and, turning on axles at C and D geared together by two toothed wheels, come alternately in front of the collimator slit E. The symmetry, however, is not perfect, because, in order that they may clear each other, one of the halves must be nearer the slit than the other; and besides, there is the objection that toothed wheels always give rise to a certain amount of noise. It is therefore fortunate that it turns out on examination that the asymmetry of the lens wheel which would otherwise be so satisfactory does not cause any alteration in the relative intensity of the two beams, or at all events does not do so if the beams are regarded as made up of sensibly parallel rays. The proof of this statement is simple but a little tedious, and hence is omitted.

(2) After the preceding description, the type of instrument employing reflection instead of refraction will be very readily understood. In this a "mirror wheel" consisting of a semicircular disc of mirror glass H, fig. 6, takes the place of the lens wheel of fig. 3, and when the mirror wheel covers the collimator slit the beam Y is reflected at 45° into the slit, and then when the slit is uncovered the other beam, X, passes into it. As before, the (imaginary) prolongation of the axis of the half disc intersects the (imaginary) prolongation of the slit either above or below the latter, but in this case the end of the collimator must be levelled away as shown, in order that the half disc may come sufficiently near. There is no difficulty about this, as the movable jaw of the slit, with its accompanying screw, etc., can be put on the other side, where there is plenty of room. Complete symmetry is secured by reflecting X as shown at a mirror placed either as M_1 or as M_2 . It will be observed that in the case of the mirror wheel the two beams are at the same horizontal height; in the case of the lens wheel they are one above the other; and hence it is convenient to have the instrument provided with a wheel of each sort to be employed as

occasion may require. A simple variant of the mirror wheel just described is a *complete* glass disc silvered, however, only over a semicircle, the beam X being transmitted through the clear half. This arrangement has the curious property of being unsymmetrical *except when the two beams are of equal intensity*, as will readily be discovered. The result is that it will give true readings, but with a slight loss of sensitiveness.

IV. It remains to call attention to some general principles regarding flicker spectrophotometers.

(1) In substituting one beam for the other, each ray of the beam just withdrawn must have its place exactly taken by the corresponding ray of the new beam, otherwise a sensation of flickering will persist when the intensities of the two beams are made equal. Before I discovered this I had a great deal of trouble in a certain case where the two beams came from different light-sources, because, as I found afterwards, the image of the aperture O, fig. 3, formed on the retina of the eye by the two beams respectively was in the one case formed by a narrower cone of rays than in the other. The arrangement of a suitable diaphragm in the path of the wider beam at once got rid of the difficulty.

(2) Not only must the corresponding rays of the two beams agree exactly in position, but they must agree also in brightness if flicker is to cease entirely at the position of balance. For example, if one of the two beams has its central rays somewhat less bright than its marginal rays, while the other beam is quite homogeneous, then it is impossible to make a satisfactory measurement. It is often easier to discover than to remedy such a difficulty. Two beams which would give quite good results with an ordinary spectrophotometer, in which they are merely brought into contact edge to edge, may prove unworkable with a flicker instrument because of their non-uniformity of brightness. It may perhaps be urged that such beams ought not to be used in any circumstances, and that therefore this peculiarity of the instrument should not be considered a drawback.

(3) It will be noticed that all the above instruments have been designed so that the transition from the one beam to the other is very sudden. This I have done in consequence of some experiments which I made to find whether a sudden or a gradual replacement of the one beam by the other gave rise to the more decided flickering. The evidence was conclusively in favour of a transition as sudden as could be arranged for, and a psychologist whom I questioned on the subject informed me that a change of brightness may be expected to be more noticeable if quickly accomplished.

SUMMARY.

First of all there is discussed the best method of obtaining an intense beam of parallel rays having each ray of equal brightness, as the production of such a beam is a pre-requisite of all spectrophotometric measurements of light absorption. Next there are briefly described three new types of "flicker" spectrophotometer, the first of these depending on polarisation, the second on refraction, and the third on reflection. All three are quite successful : which it is best to use depends on the circumstances of the case. Finally, three general governing principles are laid down which would seem to apply equally to all types of flicker photometer.

There remains the pleasant duty of expressing my thanks to the Trustees of the Carnegie Trust for the various grants which have made these somewhat expensive investigations possible.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF EDINBURGH.

(Issued separately July 14, 1913.)

XXIV.—The Scattering of Light. By James Robert Milne, D.Sc.
(With Two Plates.)

(Read March 21, 1910. MS. received May 1, 1913.)

§ 1. INTRODUCTION.

IN most optical text-books a reference will be found to "Christiansen's experiment."* Christiansen passed a beam of light through a liquid containing a transparent insoluble powder, the refractive index of the powder being the same as that of the liquid for one, and only one, colour of light. For that colour, accordingly, the mixture is optically homogeneous, but for all other colours it is highly heterogeneous and the light is scattered, with the result that the directly transmitted light is approximately all of one wave-length, and with increasing thicknesses of mixture becomes more and more so. Lord Rayleigh has shown experimentally that the selective action of such a mixture may be surprisingly efficient, and in an experiment carried out by him with improved arrangements and a layer of mixture 15–20 mm. thick, he estimates that the extreme differences of wave-length in the transmitted light amount to no more than two and a half times the distance between the two D lines.†

It has for some time seemed to me that it might be worth while to follow up this experiment, and further consideration has only served to show how wide is the field thus suggested. Not only is the choice of materials, both solid and liquid, very large; but the conditions of the experiment, as regards temperature, thickness of layer, size of particles, etc., admit of almost infinite variation. In each case, however, it is clear that the results must depend on quite definite laws, and although these may be somewhat complicated, it is reasonable to expect that they could be elucidated by a sufficient series of well-considered experiments. No particular difficulty need be anticipated in carrying out the necessary photometric and other measurements. And the outcome of such an investigation would probably be of considerable interest both from a theoretical, and also perhaps from a practical point of view. Hitherto nothing of this kind appears to have been done, at least of a systematic nature; such investigations on the subject as I am aware of have all

* *Wied. Annal.*, Band xxiii. p. 298, Nov. 1884. Description by Christiansen of his original experiment.

† "On an Improved Apparatus for Christiansen's Experiment," Lord Rayleigh, *Phil. Mag.*, xx. p. 358, 1885; also *Nature*, lx. p. 64, 1899.

been made incidentally in connection with researches directed to quite other ends. Thus, for example, in photographic research, when measurements are to be made of the densities of negatives, it is necessary to have regard to the fact that such negatives not only absorb light but also scatter it; and this has led to experiments on the scattering of light by negatives, and to the consideration of certain related matters.* And again, the introduction of various modern illuminants in which a great intensity of light proceeds from a source of small dimensions has created a demand for diffusing screens,† and a certain amount of data is available in regard to these. The scattering, however, is often obtained either by irregular reflection,‡ or by particles embedded in the glass which are so small as to be of dimensions comparable with the wave-length of light itself,§ and neither of these cases, of course, has any connection with Christiansen's experiment.

In view then of the scantiness of the existing information on the subject, and of the desirability of adding to it, I began a series of experiments; and the purpose of this paper is to give some account of the results that have been obtained up to the present time.

§ 2. APPARATUS.

On reference to fig. 1 it will be seen that the light from the ground-glass plate G_1 under examination is reflected at the mirror M , and condensed by the lens L_2 so as to pass into the slit of the spectrophotometer S by which its intensity is measured. This measurement is effected by balancing its light against that from another piece of ground glass G_2 illuminated by the movable Nernst lamp N_2 , which can be pulled backwards and forwards at will along the rails R . The light from G_2 passes above the mirror M and is likewise condensed by the lens L_2 into the slit of the spectrophotometer. The latter is an instrument on the flicker

* *E.g.* Sir W. de W. Abney, *Jour. Soc. Chem. Ind.*, 31st July 1890, p. 772, "On the Accuracy of the Grease-Spot Photometer for Measuring the Density of Photographic Plates, and a Note on the Sector Photometer." "A Demonstration of Scatter in Turbid Liquids," W. Scheffer, *Brit. Jour. Phot.*, p. 941, 9th Dec. 1910. C. Jones, *Photographic Journal*, 1898-1899, and André Callier, p. 200, 1909. *Theory of the Photographic Process*, Sheppard and Mees, pp. 38 and 114. See also *The Brit. Jour. of Photography* for 30th Aug. and 13th Sept. 1912, in connection with the scatter error in wedge photometry.

† See, *e.g.*, *Electrical Photometry and Illumination*, H. Bohle, p. 153 *et seq.*

‡ On the subject of diffuse reflection, see, *e.g.*, Trotter, *Illumination, its Distribution and Measurement*, p. 93 *et seq.* *Outlines of Applied Optics*, P. G. Nutting, p. 165 *et seq.* F. H. Gilpin, *Trans. Ill. Eng. Soc.*, v. 854-874, Dec. 1910. H. R. Wright, *Phil. Mag.* (5), No. 49, p. 119, 1900.

§ See, *e.g.*, *Physical Optics*, Wood, chapter on "Scattering by Small Particles."

principle having a "glass-wedge wheel" immediately in front of the collimator slit (as described in the paper preceding this), and it was found to be both sensitive and convenient in use. To make a measurement, the observer moves backwards and forwards a small trolley P running on rails arranged just under his hand, which, being coupled by a cord to the other trolley N_2 carrying the Nernst lamp, has the effect of altering the intensity of the comparison beam from G_2 , and in this way a position can be found for P such that the intensities from the two plates are made equal. When this is brought about, a fountain pen attached to P is depressed, thus

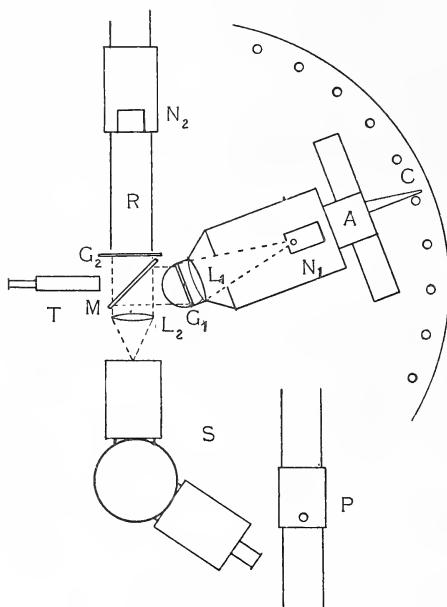


FIG. 1. Diagrammatic only, and not to scale.

leaving a record of the position by means of a dot of ink on a long strip of card fastened between the rails of P . The distances of these dots of ink from a fiducial line drawn on the card are equal to the distances of N_2 from G_2 , and thus form a permanent record of the intensities; the latter being, of course, inversely proportional to the squares of the distances N_2G_2 .*

Eight successive settings are always made and recorded, and the eighth depression of the pen produces an audible signal, so that the observer does not require to keep count. After each record the pen automatically moves slightly from right to left, so as to avoid the possibility of getting two dots superposed. The record of each complete observation therefore consists of a row of eight dots extending almost in

* N_2G_2 was in all cases kept large enough for this to be very approximately true.

a straight line across the card, the deviations being due to errors of setting. The necessary work of averaging is done without any measurement by a simple drawing instrument I designed for the purpose. This method of working enables both settings and reductions to be most expeditiously carried out. To save time in adjusting the telescope of the spectrophotometer to any required spectral region, a helium tube is mounted on a rotatable arm, so that it can be swung round, its light directed into the collimator, and the telescope set on the required line, without the necessity for reading a scale or even turning up the lights in the room. The angle of emission from the ground-glass plate of the light which is passing into the collimator can also be regulated quite simply in the dark. The plate G_1 is fixed to a levelling table standing on the end of the long arm A which is pivoted under G_1 so as to be movable round the pivot in a horizontal plane. A catch C can be made to engage with any one of the studs shown, whose positions are carefully adjusted so as to lock the arm at will at the angles 90° , 85° , 80° , etc., with R. On A stands a lantern case completely enclosing the Nernst lamp N_1 , furnished with an achromatic condensing lens L_1 , from which a parallel beam of light falls upon the ground glass G_1 , all the Nernst filament except a small portion at the centre about a quarter of an inch long having been screened off. It will be understood that in consequence of the above arrangements only those rays of the parallel beam incident upon the plate which are scattered as a parallel beam at one particular angle enter the collimator slit, and that this angle depends upon the position of the arm A. The various adjustments which have to be made to ensure that the beam of light falls perpendicularly on the surface of the plate, that the mirror M is in the proper position, etc., etc., are carried out by means of the usual slow-motion-screw fittings, and by the help of a small telescope T of special construction, which remains permanently in the position shown, as a means of subsequent verification from time to time. For the purpose of enabling experiments to be made in which the incident light falls *obliquely* on the glass plate, it has been arranged that the table to which G_1 is attached can be supported independently of A, so that G_1 can be turned to any desired angle with the latter. Care has been taken to design all the fittings for the levelling table, the arm A, and the catch C on "geometrical" principles, A for example standing on two rollers at the extremities of the crossbar and having at the other end as pivot a ball resting in a conical hollow. The result of this design is that no looseness has come about through wear, and yet both the levelling table and the arm with its lantern can at any time simply be lifted up and carried away for any purpose, and as simply replaced, with

no necessity for any readjustment whatever. In actual work such facilities are of considerable importance. The two Nernst lamps are run in parallel off the town mains, but as both are placed in the same circuit it is hoped that small fluctuations of voltage may affect each equally.

§ 3. RESULTS.

It is not proposed in the present paper to give a complete discussion of the following results; indeed, it has been found that various points of unexpected difficulty have been raised, the consideration of which must be postponed for the present. Meanwhile, however, certain remarks fall to be made.

TABLE I.

No.	Nature of Scattering Bodies.	Photo-micrographs.	Mean Wave-length of Light.	Plotted in Figure.
1. <i>Light Incident Normally.</i>				
1.	Plate glass, one side very roughly ground, ground side towards light.	I. 1	5876	2
2.	The same, but ground side away from light.	I. 1	5876	2
3.	Two pieces of ordinary commercial ground glass, ground sides together.	I. 2	6678	3
4.	The same.	I. 2	5876	3
5.	The same.	I. 2	5016	3
6.	Specially finely ground glass, ground side towards light.	I. 3	5876	4
7.	The same, but ground side away from light.	I. 3	5876	4
8.	Ground glass etched by acid, ground side towards light.	I. 4	5876	4
9.	Turbid solution of resin.	II. 5	6678	3
10.	The same.		5016	3
2. <i>Light Incident Obliquely.</i>				
11.	As in 6.	I. 3	5876	5
12.	As in 7.	I. 3	5876	5
13.	Repetition of 11.	I. 3	5876	5
3. <i>Theoretical Cases.</i>				
	Glass plate with cavities all over the face which is next the light.		Refractive index of Glass.	
14.	Semicylindrical cavities.		1.5	6
15.	Hemispherical cavities.		1.5	6

TABLE II.
§ 1. *Light Incident Normally.*

No.	Relative Intensities at the Undernoted Angles,						Relative Total Emission at the Undernoted Angles.						Angle of Maximum Total Emission.	Plotted in Figure.			
	0°.	2°.5.	5°.	10°.	15°.	20°.	25°.	30°.	0°.	2°.5.	5°.	10°.	15°.	20°.	25°.	30°.	
1	100	... 62.8	22.7	8.48	4.34	2.80	...	0	...	99.2	71.5	39.8	26.8	21.4	...	6°	2
2	100	... 72.7	29.95	13.2	5.865	3.69	...	0	...	96.4	79.3	52.0	30.6	23.7	...	6°	2
3	100	... 91.3	64.4	43.4	29.2	18.8	13.5	0	...	69.8	98.2	87.2	69.7	59.2	12°	3	
4	100	... 87.7	63.8	44.8	30.5	20.5	13.7	0	...	65.9	95.7	100	89.6	74.7	14°	3	
5	100	... 89.9	69.9	47.7	31.9	23.88	17.41	0	...	62.7	97.6	99.2	87.2	74.7	14°	3	
6	100	... 88.0	71.8	61.3	38.6	23.88	17.41	0	...	24.7	40.4	69.0	94.8	85.1	56.1	17°	4
7	100	... 88.2	70.8	51.6	39.55	27.5	15.23	8.64	0	37.0	59.3	86.3	99.0	90.5	62.2	16°	4
8	100	... 91.7	80.2	45.7	20.75	9.50	4.45	...	0	47.6	83.2	94.6	64.1	38.7	22.4	8°	4
9	20.0*	15.0	11.9	9.96	...	0	...	10.2	15.4	18.1	20.0†	3
10	20.0*	16.1	12.9	0	...	10.4	16.8	20.0†	3

§ 2. *Light Incident Obliquely at the Undernoted Angles.*

No.	Relative Intensities Emitted Normally.						The foregoing Intensities each Multiplied by the Sine of the Angle of the Incident Light, and by a factor to make the maximum equal to 100.						The Angle at which the Maximum Occurs.	Figure.			
	0°.	2°.5.	5°.	10°.	15°.	20°.	25°.	30°.	0°.	2°.5.	5°.	10°.	15°.	20°.	25°.		
11	100	85.2	62.7	41.6	30.3	18.5	9.40	...	0	46.5	68.4	90.5	98.1	79.1	49.8	...	5
12	100	82.8	67.7	52.7	46.7	26.3	15.3	...	0	29.1	47.7	76.0	97.6	72.5	52.2	...	5
13	100	80.3	65.9	44.3	30.4	16.92	...	0	43.8	71.9	96.4	98.4	72.3	...	13°	5	

§ 3. *Theoretical Cases.*

No.	Relative Intensities at the Undernoted Angles.						Relative Total Emission at the Undernoted Angles.						Angle of Maximum Total Emission.	Figure.			
	0°.	2°.5.	5°.	10°.	15°.	20°.	25°.	30°.	0°.	2°.5.	5°.	10°.	15°.	20°.	25°.		
148	100.000	99.378	96.687	87.605	75.147	61.713	47.885	37.927	37.565	82°82.	6
158	100.000	99.122	95.711	84.273	69.182	53.795	39.281	29.206	67.825	100.000	23.335	45.022	78.918	91.963	88.993	36.319	18°5

* Arbitrarily chosen as "20" because the value at 0° is unknown.
† Arbitrarily chosen as "20" because the maximum is out of range.

‡ This is the angle at which the first and second refractions are equal.
§ Corrected for the loss so due to reflection.

The values tabulated under the heading "Relative Intensities" are the results obtained by photometric measurements with the apparatus described above. (Nos. 14 and 15 are exceptions, being wholly the result of a theoretical calculation, as will be explained later on.) For greater convenience, in all cases the intensity at the angle zero has been arbitrarily taken as 100, and the other values reduced accordingly; except in the case of nos. 9 and 10, where the values at the angle zero could not be ascertained, and where the intensities at 5° have been arbitrarily taken as 20. These results are plotted in figs. 2, 3, 4, and 5. In every case distances along the horizontal axis represent the angle to the normal at which the light emerges from the plate, and distances along the vertical axis the corresponding intensity. In all cases it will be seen that the curves are of the same general nature, the intensity at first falling off very rapidly with increasing angle, but subsequently showing a tendency to become asymptotic to the angular axis. The appearance of some of the curves rather suggests that they might be fitted by the formula for the error curve $y = ae^{-\beta x^2}$, and I made trial of one of them, but without success. In any case this could never prove to be a general formula, as a glance at curve no. 6 will show.

The reason for including a column in the table headed "Relative Total Emission" is as follows. It must be borne in mind that at each point of the ground-glass surface light is emitted in *all* directions, and hence, if an imaginary hemisphere be described with its centre at any point of the glass surface, the entire hemisphere would be illuminated by light from that point. Furthermore, all the rays emerging from that point between, say, 5° and 6° to the normal will light up a certain zone on the hemisphere, those emerging between 6° and 7° an adjacent zone, and so on. Hence, to find the *total* emission of light at any angle, we must multiply the intensity at that angle by the area of the corresponding zone. In other words, the total emission at any given angle is proportional to the relative intensity at that angle, multiplied by the sine of the angle. And the values so found (each multiplied by a factor which makes the *maximum* total emission equal to 100), are the values tabulated under the heading "Relative Total Emission." They have been plotted in the figures, and each of the resulting curves appears in the same figure as the curve of relative intensity from which it has been derived, and from which it is distinguished by being in broken line.

It is evident that *the angle of maximum total emission affords a very fair criterion of the effectiveness of the scattering action: the greater the numerical value of the angle, the greater the scattering.*

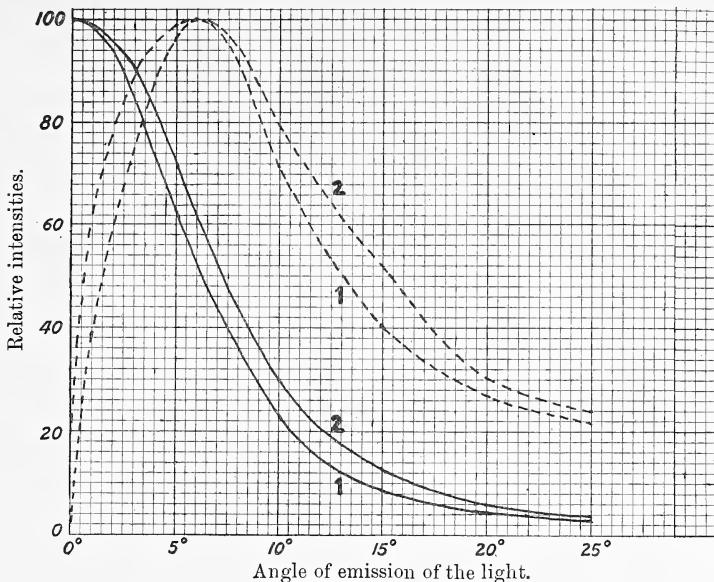


FIG. 2.

1. Plate glass, one side very roughly ground, and ground side towards light.
2. The same, but ground side away from light.

Photomicrograph, Plate I. fig. 1.

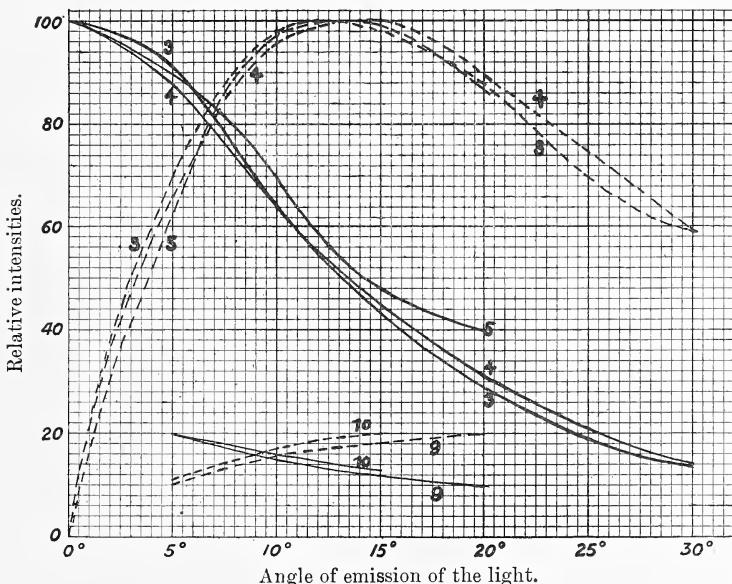


FIG. 3.

- 3 } Two pieces of ordinary commercial ground glass, λ
4 } ground sides together. $\left\{ \begin{array}{l} 6678 \\ 5876 \end{array} \right\}$ Photomicrograph,
5 } $\left\{ \begin{array}{l} 5016 \end{array} \right\}$ Plate I. fig. 2.

— Curves of relative intensity.

- - - - Curves of maximum total emission.

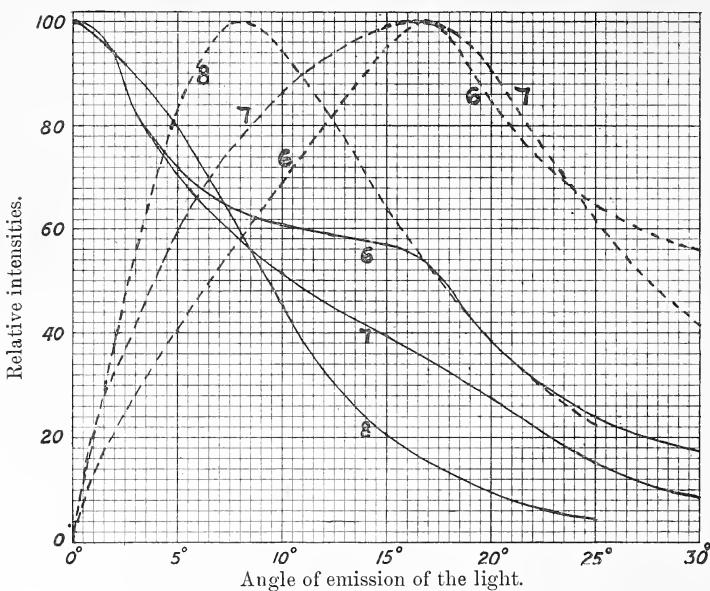


FIG. 4.

6. Extra-fine ground glass, ground side towards light. Photomicrograph, Plate I. fig. 3.
 7. The same, but ground side away from light. " Plate I. fig. 3.
 8. Ground glass etched by acid, ground side towards light. " Plate I. fig. 4,
 and Plate II. fig. 5.

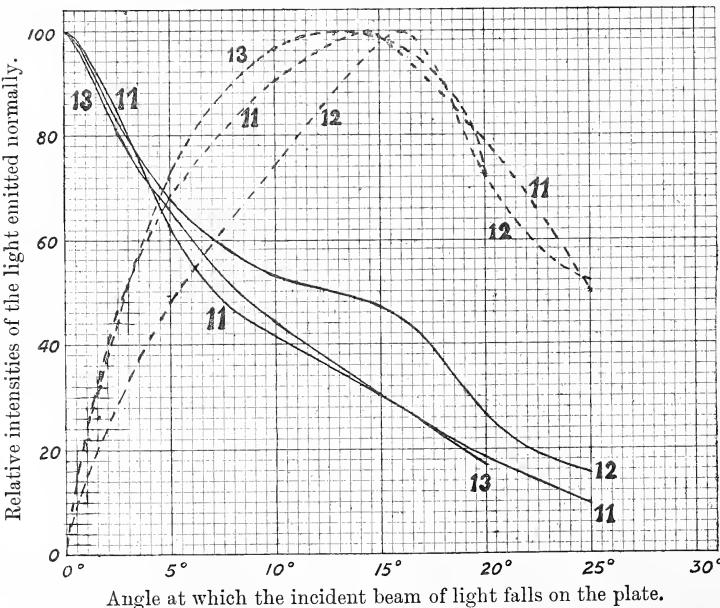


FIG. 5.

11. Extra-fine ground glass, ground side towards light, normal emission and oblique incidence.
 12. The same, but ground side away from light. Photomicrograph, Plate I. fig. 3.
 13. A repetition of 11, some time later after readjustments.
 — Curves of relative intensity. - - - Curves of maximum total emission.

In this connection it is instructive to note that the turbid solutions nos. 9 and 10 scatter the light so much that the maximum angle has not been reached by the available data (only a few measurements were made for the sake of contrast with the ground-glass results, and with some difficulty on account of the faintness of the light).

The actual determination of the angle at which maximum total emission occurs in any case is not easy. I tried determining the constants of a function of θ so that it should fit the relative intensity curve as well as possible in the neighbourhood of the region where, after multiplication by $\sin \theta$, the curve attains its maximum value. Multiplication of the formula by $\sin \theta$, and subsequent differentiation by θ , then led in the usual way to the value of θ giving the maximum value. This procedure, however, proved to be tedious, and lacking in sufficient accuracy. Finally, the maximum was determined in each case by reading off from the relative intensity curve the value of the intensity for each successive degree, multiplying it by the corresponding value of $\sin \theta$, and simply picking out the maximum from the values so obtained. The result for each experiment will be found under the heading "Angle of Maximum Total Emission."

In § 2, Table II., it may seem needless to set down the results of multiplying the relative intensities by $\sin \theta$, and the value of θ at which the maximum occurs, for of course these figures have here no direct physical significance; but it has been done simply for the sake of comparison, and to bring out more fully the nature of the functions obtained from the measurements.

In considering the figures tabulated above, the first point that suggests itself for investigation is the question of how the effectiveness of the scattering action depends on the kind of surface. It will be seen that, if we take the suggested course of using the angle of maximum total emission as our criterion, the order of the surfaces, beginning with that giving the smallest amount of scatter, is as follows: (1) coarse ground glass, (2) acid-etched ground glass, (3) ordinary ground glass, (4) specially finely ground glass, and (5) the turbid solutions of resin. (See Plates I. and II., figs. 1 (4, 5), 2, and 3 respectively. It is to be remembered, of course, that *two* plates of the "ordinary" ground glass were used.) The reason for this result is perhaps as follows. It is evident from the photomicrographs that the coarser, and less scattering, varieties of glass have pits of larger area, and it seems likely that the larger the area of the pit the less steeply do its sides slope, and therefore the less the deflection of the light from the normal direction. As the etched glass has pits with much larger areas than have any of the other sorts of glass, it would naturally be supposed,

were this explanation correct, that the etched glass would scatter least of all, which, of course, is contrary to the fact. The explanation is that an etched pit, unlike a pit due to grinding, is always of a depth proportional to its width, and hence in etched glass the steepness of the sides of even the largest pits is maintained. Lord Rayleigh, in a paper published in the *Proceedings of the Optical Convention*, p. 73, 1905, puts the matter thus: "Some very curious observations were also made upon the effect of hydrofluoric acid upon glass originally finely ground. The acid acts in such a way as to eliminate from the roughened surface all the finer irregularities, leaving only those of longer periodicity. It is not difficult to form a theory, and to illustrate that theory by drawing, especially if one takes the case of two dimensions. If one assumes, as seems reasonable, that the hydrofluoric acid always eats in normally to the surface, then if we start with any particular surface and imagine that from every point of that surface spheres are drawn having their centres in the surface and a radius proportioned to the time during which the hydrofluoric acid acts, then the envelope of all those surfaces will be the surface to be expected at the close of the operation. You see the ultimate result will be to leave the surface in the form of spherical segments, the centres of the spheres corresponding to the deepest places in the original roughly ground surface. I have a slide showing the appearance of such a piece of ground glass after treatment. It appears to be divided into a number of cells, and the wall of each spherical segment separated from its neighbour is of the nature of a ridge. Each cell is of itself absolutely devoid of structure; it appears finely polished.* Although the surface was originally finely ground all over, all the minor irregularities are gone, and we are left with the surface which in a sense might be called polished, although of course it is far from flat. In each case, if we are looking down upon the surface, the middle of these segments will be the deepest place; the ridges are raised and exceedingly sharp. According to the theory here briefly sketched they would be mathematically sharp, and such they appear to be." As regards the ground glass experimented on, the nature of the operation by which it is produced is well understood. See, for example, a paper by

* Following Lord Rayleigh, I tested the smoothness of the pits on the surface of the etched glass (no. 8) by using the pits as lenses, and the photomicrographs, Plate II. figs. 6 and 7, show the images thus obtained when the object was (1) a luminous point, and (2) a luminous cross. The definition is sufficiently good to leave little doubt as to the smoothness of the pits; and indeed the perfection of the images is somewhat surprising, in view of the fact that the pits are for the most part of a decidedly elongated shape, the length along the surface of the plate being often several times as great as the breadth, as appears clearly in the photomicrograph, Plate II. fig. 5. This fact, no doubt, accounts, partly at least, for the marked astigmatism shown by many of the images when thrown slightly out of focus.

W. Rosenhain entitled "On Abrasives and Polishing Materials."* Briefly put, the theory is as follows. The abrading material must be regarded as a collection of sharp points and edges, so that the areas of contact between the abrasive and the glass are very minute. Hence the resulting pressure per unit area is in many cases greater than the tensile strength of the glass, with the result that as the grinding progresses conchoidal fractures take place all over the surface of the latter. Grinding is to be sharply distinguished from polishing, inasmuch as polishing produces

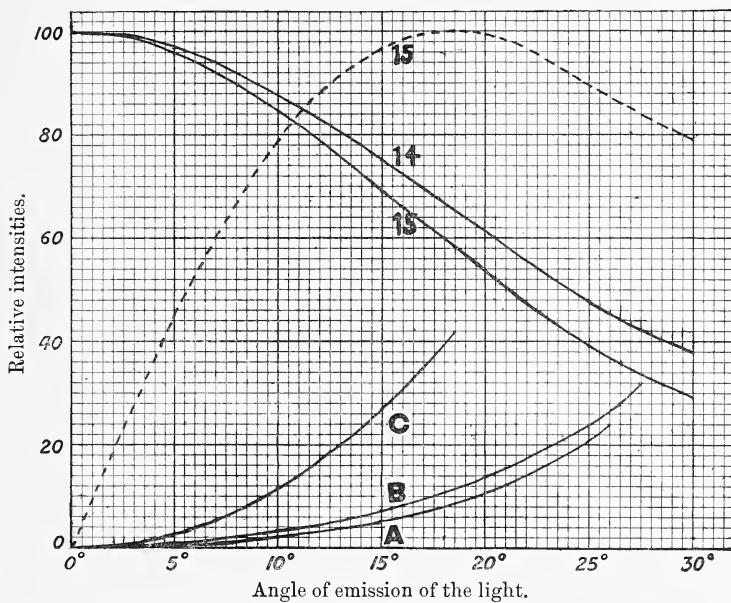


FIG. 6.

- 14. Calculated case of semicylindrical cavity towards light.
- 15. Calculated case of hemispherical cavity towards light.
- A. Cavity equivalent to Case no. 1.
- B. Cavity equivalent to Case no. 8.
- C. Cavity which gives rise to uniform intensity.

— Curves of relative intensity. - - - - Curve of maximum total emission.

"surface flow,"† while grinding does not. In the present connection the application of the fact lies in this, that it follows that all the myriad angles of the *ground* glass are perfectly sharp, and free from any rounding off. Unfortunately, so far as I am aware, no information is available as to the exact shapes of the pits produced by grinding. I hope later on to be

* *Trans. Opt. Soc.*, vol. xi. p. 113, 1909-10.

† Lord Rayleigh could find no evidence of surface flow in glass (*loc. cit.*), but other experimenters are satisfied of its existence (see "discussion" at end of Lord Rayleigh's paper); and it is well established as regards a great many other materials. See especially the Hurter Memorial Lecture given by G. Beilby, on "The Surface Structure of Solids," *Jour. Soc. Chem. Ind.*, p. 1166, 1903.

able to obtain some experimental information, but meanwhile I have contented myself with calculating what may be termed the "equivalent cavity" of the pits for two of the pieces of glass. That is to say, I have calculated, and plotted in fig. 6, the particular shape of pits which would give the same distribution of scattered light as do these two plates, assuming the pit to be a surface of revolution with its axis of rotation normal to the surface of the glass. The calculation is made as follows:—

In fig. 7 is shown a section of the glass plate, the x axis coinciding with the smooth surface and PQR being the equivalent cavity. If a beam of rays parallel to the y axis fall on the cavity, the path of the ray, which emerges from the plate at the angle θ , can be ascertained thus. Let the ray be SQTW. Draw any arbitrary fixed line AB above the cavity,

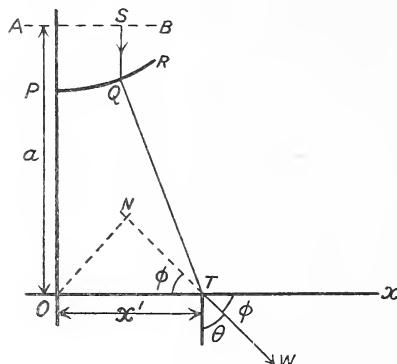


FIG. 7.

parallel to the x axis and at a distance a from it. Then if Q be (x, y) and T $(x', 0)$, and the refractive index of the glass be μ ; and if the ray produced backwards from T meet at N a line through the origin which makes an angle θ with the x axis, the optical length λ of the path from S to the virtual point N is given by the equation,

$$\lambda = a - y + \mu \{y^2 + (x' - x)^2\}^{\frac{1}{2}} - x' \sin \theta.$$

But the positions of the variable points Q and T must be such that λ is a minimum.

Hence

$$\frac{d\lambda}{dx} = 0; \text{ i.e. } \frac{dy}{dx} = \mu \frac{y \frac{dy}{dx} - (x' - x)}{\sqrt{y^2 + (x' - x)^2}}. \quad \quad (1)$$

and

$$\frac{d\lambda}{dx'} = 0; \text{ i.e. } \mu \frac{x' - x}{\sqrt{y^2 + (x' - x)^2}} = \sin \theta \quad \quad (2)$$

(1) and (2) lead to

$$p = \frac{dy}{dx} = \frac{\sin \theta}{\sqrt{(\mu^2 - \sin^2 \theta) - 1}} \quad \quad (3)$$

The table below gives corresponding values of θ and p , calculated from (3), on the assumption that $\mu = 1.5$.

θ	0	$1\frac{1}{4}$	$2\frac{1}{2}$	$3\frac{3}{4}$	5	$7\frac{1}{2}$	10	$12\frac{1}{2}$	15	$17\frac{1}{2}$	20	25	30
p	.0000	.04365	.08736	.1312	.1752	.2641	.3545	.4470	.5421	.6404	.7427	.9622	1.207

In order to find a mathematical expression for the intensity of the light, scattered by an equivalent cavity, as a function of the angle of scatter, it is convenient to regard the matter thus.* Let it be supposed that the rays of the beam of light incident perpendicularly on the plate are separated

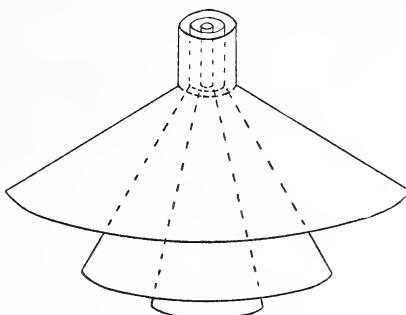


FIG. 8.

into bundles by an infinite number of imaginary tubes forming a system of coaxial cylinders parallel to the direction of the light. If these tubes accompany the rays through the plate, and if after transmission the same bundles as before are to be found within the same tubes, we must further suppose that on the far side of the plate the tubes diverge in the form of a family of hollow coaxial cones, having their common axis perpendicular to the plate, as shown in fig. 8, in which, for greater clearness, only a few of the tubes are drawn. With the help of such an imaginary arrangement it can easily be seen that the light-intensity i , measured at any the same constant distance from the plate, and normally to rays which make an angle θ with the axis, is equal to $kxdx/(\sin \theta d\theta)$, where k is a constant

* We shall ignore the reduction of intensity in the transmitted light due to reflection losses, because in all the cases considered it is so very nearly the same. On making a calculation by means of Fresnel's formula, it appears that when a beam of light falls normally on the interface separating two media of refractive indices 1 and 1.5 respectively, 96.00 per cent. of the light energy is transmitted; and even when the obliquity of the beam has reached 30° to the normal, the percentage of energy transmitted is still as great as 95.85.

depending on the fixed distance from the plate, and on the original (uniform) intensity of the beam, and x measures the distance from the axis of those incident rays which after transmission emerge at the angle θ .

$$i = k \frac{x dx}{\sin \theta d\theta} \dots \dots \dots \quad (4)$$

Hence,

$$\int_{\theta}^{\theta} i \sin \theta d\theta = \frac{k}{2} \left[x^2 \right]_0^x,$$

and

$$\sqrt{k.x} = \sqrt{\left\{ 2 \int_0^{\theta} i \sin \theta d\theta \right\}}.$$

The analytical form of the function $i \sin \theta$ is unknown, only certain pairs of corresponding values of i and θ being given by the experiments. Therefore, in order to obtain the values of x it is necessary to plot the values of $i \sin \theta$ against θ , to draw a fair curve through the points obtained, and to integrate graphically from 0 up to each value of θ . But equation (3) gives the values of dy/dx corresponding to the values of θ . If, therefore, we plot dy/dx against $\sqrt{k.x}$, and integrate graphically, we obtain $\sqrt{k.y}$ as a function of $\sqrt{k.x}$. This completely determines the *shape* of the cavity; the only effect of the unknown constant \sqrt{k} is to leave its actual size uncertain. Now it is evident that "similar" cavities will give identical relative light-distributions; and hence to find the size of cavity in any given case we must know not only the relative distribution, but also the ratio of the actual intensity of the incident light to that of the emergent light, for some one value of θ . As this was not determined in the foregoing experiments, the drawings in fig. 6 show only the *shape* of the cavities equivalent to the plates used in experiments 1 and 8. On comparing the relative intensity curves of these two plates, it will be seen that the intensity falls off more rapidly with increasing angle in the case of no. 1 than in the case of no. 8, and hence we should expect that A, the Equivalent Cavity of the former, would be flatter near the origin than B, the Equivalent Cavity of the latter, but would turn upwards more steeply afterwards, as indeed the figure shows to be the case. This point is further brought out by a comparison of the curvatures of the two curves at the origin, the radius of curvature of A being calculated to be 23.35 and of B 15.2, the unit of length being a side of one of the large squares of the paper.

It is of interest to compare with these the particular Equivalent Cavity which corresponds to *uniform* distribution of the light at all angles. Its equation is readily obtained as follows.

In equation (4) let i the light-intensity equal $\frac{1}{2}a^2k$, where a is any constant. Combining this with (3), we have

$$\frac{dy}{dx} = \frac{\sqrt{\left[2\left(\frac{x}{a}\right)^2 - \left(\frac{x}{a}\right)^4 \right]}}{\sqrt{\left[\mu^2 - 2\left(\frac{x}{a}\right)^2 + \left(\frac{x}{a}\right)^4 \right]} - 1}.$$

Disregarding powers of x/a greater than the sixth, and putting $\mu=3/2$, we have

$$\frac{dy}{dx} = 2.828\left(\frac{x}{a}\right) + 3.06\left(\frac{x}{a}\right)^3 + 2.96\left(\frac{x}{a}\right)^5.$$

$$\therefore \frac{y}{a} = 1.414\left(\frac{x}{a}\right)^2 + 0.765\left(\frac{x}{a}\right)^4 + 0.493\left(\frac{x}{a}\right)^6.$$

The radius of curvature at the origin ρ_0 is given by

$$\rho_0 = \frac{1}{\left| \frac{d^2y}{dx^2} \right|_0} = 3.53a.$$

It is evident that the constant a merely determines the scale and not the shape of the cavity. Therefore, putting it equal to unity for convenience, we obtain $\rho_0=3.53$, and

x	0	.1	.2	.25	.3	.37
y	.0000	.01421	.05708	.09150	.1338	.2094

This is plotted as C, fig. 6, the side of a large square being taken as equal to 1. The comparison of this curve with the others is instructive, representing as it does a sort of upper limit for all such cavity curves. The difference of scale must, of course, be kept in mind; for example, if C were drawn in the same way as A and B, with the side of a large square as the unit of length, then its radius of curvature at the origin would only be a 43rd of that of B, and a 65th of that of A.

In the case of experimental curves such as A and B, it is not easy to find the curvatures at points other than the origin with accuracy; but that the curvature changes rapidly, or in other words that there is a marked departure from the circular form, can be seen by considering the light-intensity distribution that would be given by a hemispherical cavity. In such a case we have, putting $\mu=3/2$,

$$(a) \quad \frac{dy}{dx} = \frac{x}{\sqrt{(r^2 - x^2)}}$$

$$(b) \quad \frac{dy}{dx} = \frac{\sin \theta}{\sqrt{(2.25 - \sin^2 \theta)} - 1}$$

$$(c) \quad i = \frac{xdx}{\sin \theta d\theta}.$$

These lead to

$$i = r^2 \cos \theta \frac{3.25 \sqrt{(2.25 - \sin^2 \theta)} - 4.5 + \sin^2 \theta}{\sqrt{(2.25 - \sin^2 \theta)} \{3.25 - 2 \sqrt{(2.25 - \sin^2 \theta)}\}^2}$$

or, if we put $\sin \phi = \frac{2}{3} \sin \theta$, we have the equation

$$i = (\text{constant}) \cos \theta \frac{(1.5 - \cos \phi)(\cos \phi - .667)}{\cos \phi (1.083 - \cos \phi)^2},$$

which is adapted for logarithmic computation.

This leads to

Angle = θ	0	2.5	5	10	15	20	25	30
Relative intensity in that direction	100.00	99.12	95.71	84.27	69.18	53.80	39.28	29.21

As before, the relative total emission is obtained by multiplying the above function by $\sin \theta$; this leads to

Angle = θ	0	2.5	5	10	15	20	25	30
Relative total emission	100.00	23.34	45.02	78.98	96.64	99.30	89.60	78.81

and it can be shown by a method of successive approximation that the maximum total emission occurs at the angle $\theta = 18^\circ 4$. The dissimilarity of these results—given by a hemispherical cavity—from all the experimental ones can be seen at a glance by comparing its graphs, nos. 15 in fig. 6,* with the earlier ones. It is at once apparent that the actual cavities must depart from the shape of true spheres by having flatter bottoms and steeper sides. And, indeed, on consideration this is what we should expect, because, were we to have a plate the surface of which was covered with small hemispherical cavities packed together as closely as possible, there would still be a large area of the plate in its original flat condition; and assuming, as seems reasonable, that this is not likely to be the case in practice, if we

* Curve no. 14 in fig. 6 shows the light intensity distribution that would be given by an infinitely long semicylindrical cavity, the axis of the circular cylinder being in the surface of the plate. Evidently this case can be calculated from the same equations (a) (b) (c) as before, if (c) be changed to $i = dx/d\theta$. The results of the calculation are given in Table II. no. 14.



FIG. 1.—Coarse ground glass. $\times 800$.

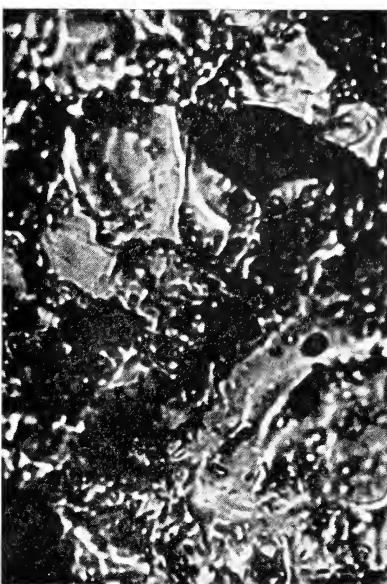


FIG. 2.—Ordinary ground glass. $\times 800$.



FIG. 3.—Extra-fine ground glass. $\times 800$.

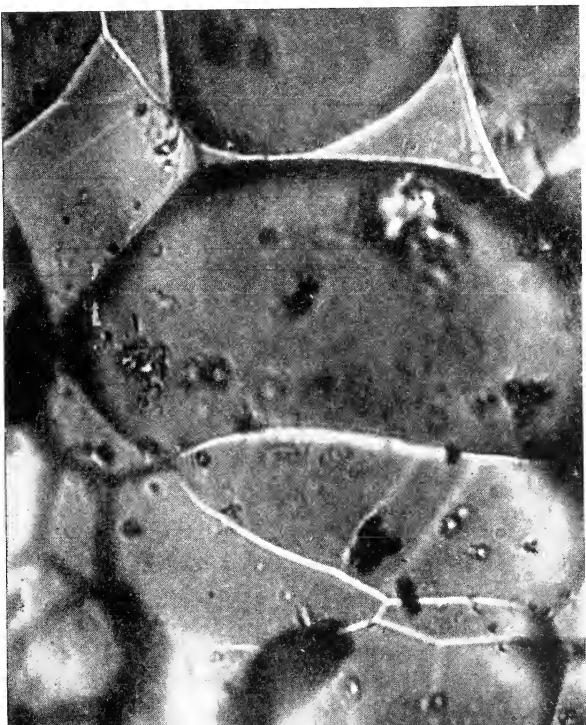


FIG. 4.—Acid-etched ground glass. $\times 800$.



FIG. 5.—Acid-etched ground glass. $\times 100$.

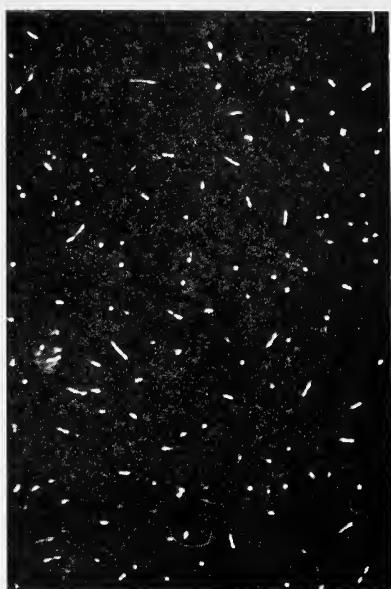


FIG. 6.—Images of a luminous point formed by the pits acting as lenses. $\times 100$.

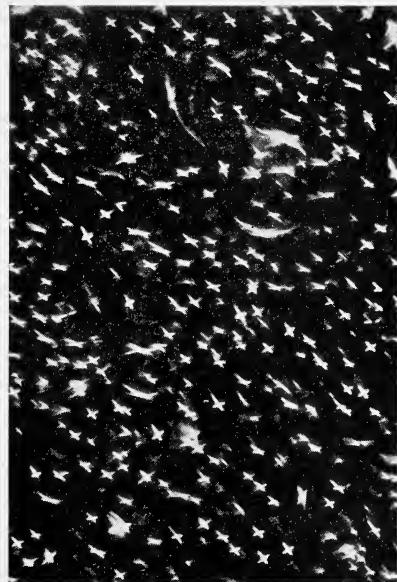


FIG. 7.—Images of a luminous cross formed by the pits acting as lenses. $\times 100$.

imagine all the flat places sloped off into one or other of the adjoining cavities, then we should produce an excess of slightly sloping area. Further discussion of this point, however, had better be postponed until the publication of the succeeding paper.

§ 4. SUMMARY.

The present paper describes a number of experiments made in connection with Christiansen's experiment in which a beam of light is passed through a transparent insoluble powder immersed in a liquid, with the result that light of the particular colour for which the indices of powder and liquid are the same passes unaffected, while light of all other colours is scattered. In this communication only the simplest case of the above is dealt with, that, namely, in which a flat piece of glass, ground on one side, takes the place of the powder, the rugosities of the ground surface representing a single layer of grains, and air takes the place of the liquid. In such a case there is, of course, no colour of light for which the indices of solid and liquid are alike, and indeed it was found that the colour of the light made very little difference to the results. On the other hand, however, it soon became apparent that different ways of grinding the glass surface led to very different effects. Accordingly, as no previous work appears to have been done in this field, and as it seemed a promising one, it was decided to make a systematic study of the various cases. Each specimen of glass employed was photomicrographed, and had its polar light distribution measured by a photometer. Two methods of characterising the particular scattering power of a screen soon suggested themselves and have been formally defined—the one connected with the Angle of Maximum Total Emission, and the other with the Equivalent Cavity.

It is hoped to continue the investigation not only on the above lines, but also in the direction of ascertaining the effect of a number of plates, *i.e.* of successive layers of light-scattering particles; and in investigating the polarisation effects, which some rough preliminary experiments have shown to be marked.

I am glad of this opportunity of acknowledging the help I have received from the Trustees of the Carnegie Trust in the form of grants for the construction of the special apparatus necessary; and desire to tender my grateful thanks to Professor MacGregor for the many facilities for carrying on the work which he kindly placed at my disposal.

DEPARTMENT OF PHYSICS,
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XXV.—On the Compressibilities of Dilute Solutions of certain Inorganic Salts. By W. Watson, M.A., B.Sc., Carnegie Research Fellow in Physics, 1910–1912. *Communicated by* The General Secretary.

(Read May 5, 1913. MS. received May 6, 1913.)

THE experiments to be described in this communication were undertaken with the purpose of obtaining values of the compressibility of certain salts, the volume of dilute solutions of which is less than the volume of the constituent water. The densities of such solutions, chiefly hydroxides, sulphates, and carbonates, are discussed by MacGregor * in a series of papers published in the *Transactions of the Royal Society of Canada*; and Kohlrausch † includes several of them in his paper on the density of dilute solutions.

We have indirect information as regards the behaviour of such solutions under pressure, in the list of compressibilities given by Röntgen and Schneider.‡ From that list, which I shall refer to later, we see that the compressibilities of the hydroxides, sulphates, and carbonates are the smallest of those investigated. Such solutions, then, are furthest removed as regards compressibility from the solvent water, and it was thought interesting to choose examples of these solutions for a more comprehensive investigation of compressibility.

The connection between compressibility and internal pressure and other physical properties is very important because of the light it throws upon chemical constitution, and from this point of view Richards and Stull § carried out a series of researches on compressibility, chiefly devoting themselves to the compressibility of the elements. Tammann, again, from another point of view, has drawn up a table of internal pressures of various solutions calculated from the thermal expansion, and he tests his results by the measurements, among others, of Tait, Amagat, and Röntgen and Schneider. The researches of the first two were mainly on water. The work of the latter has already been mentioned, but it and similar researches on the same lines by Pohl || and Schumann ¶ were all made at comparatively low pressures, a few atmospheres at most, and, in Röntgen's case at any rate, for only two different concentrations. My object, then, was, instead of taking a large number of substances and one or two concentrations of each,

* *Trans. R.S. Canada*, 1889, etc.

† *Annalen der Physik und Chemie*, 1894.

‡ *Ibid.*, 1886.

§ *Zeitschrift für physikalischen Chemie*, 1902.

|| Inaugural Dissertation, Bonn, 1906.

¶ *Annalen der Physik und Chemie*, 1887.

to take a few salts and examine a number of dilute solutions of these at pressures at least up to 1000 atmospheres. These would allow me, in general, to obtain an idea of the variation of internal pressure with concentration and pressure, and in particular to test whether the formula from which Tammann calculates the internal pressures is satisfied in these particular cases.

The substances chosen were Na_2CO_3 , NaOH , KOH , MgSO_4 , and ZnSO_4 . I measured the densities and compressibilities at 15° C . of five solutions of NaOH , and four solutions each of Na_2CO_3 , KOH , ZnSO_4 , and MgSO_4 , making twenty-one solutions in all, all of normal concentration or under. A series for water was also taken.

The substances were the purest obtainable from Kahlbaum. They were in each case assumed to be chemically pure without further chemical examination. The sodium carbonate was prepared in the usual way from sodium bicarbonate. The water was commercial distilled water, redistilled in Jena glass vessels, the middle portion being chosen. The flasks used for holding and preparing the solutions were also of Jena glass, and had been previously calibrated. The general procedure was to make in each case an approximately normal solution and to dilute this as required. The whole process of solution and dilution was carried out in a constant-temperature bath. The only solutions which stood for any length of time (of course in well-stoppered flasks) were the normal solutions. The other solutions were prepared just before use.

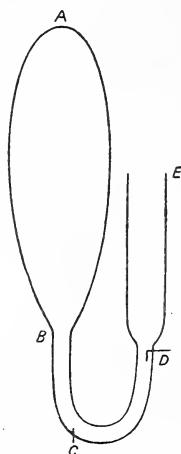
The density was measured by means of an ordinary pyknometer kept for hours in a constant-temperature bath at 15° C . This method, though perhaps not so reliable for very dilute solutions as the upthrust method of Kohlrausch, gave me sufficiently accurate results for my purpose. A mean of three observations was in general taken. The weights used had been previously calibrated, and care was taken that the balance temperature did not vary much from 15° C , and that otherwise evaporation did not affect the results.

The compression cylinder used was of the same pattern as that described by me in a previous communication,* the difference being that I did not require the arrangement for high-temperature work. The cylinder itself was made under my directions in Leipzig, and the plug arrangement for introducing the wires was completed by Herr Dornfeld, Mechaniker in Professor Des Coudres' laboratory. The pump, compression screw, and gauges were obtained from Schaeffer & Budenberg. The gauges were two in number, one reading by kgs. up to 400 kgs./cm.², the other reading by 10 kgs. up to 3000 kgs./cm.². In general one could interpolate to $\frac{1}{10}$ of a

* *Proc. Roy. Soc. Edin.*, 1911.

scale division. The compression screw, in addition to being the means of attaining high pressures, is useful as a fine adjustment. The cylinder was kept immersed in a tank of water of about 9 cubic feet capacity. The water was kept stirred by a system of propellers driven by a hot-air engine, and the temperature maintained constant at 15° C. by incandescent lamps worked by a regulator and relay.

The electrical method of measuring compressibility, first used by Tait and Amagat, and modified by Richards, was employed. The piezometers had the shape shown in the figure, *i.e.* with no stopcock. AB is a bulb about 15 cm. long and of 45 c.c. capacity. The portion BD was made thicker in the walls, as otherwise it was found liable to break under pressure. It became



narrower at D (about 1 mm. diameter), where one platinum wire was fused in, and widened out again in DE. Another platinum wire was inserted at C. The piezometer was completely filled under the receiver of an air-pump with the solution in question, then placed in the thermostat, and after it had attained the temperature of 15° C., liquid was removed from DE until the surface stood at D. The piezometer was then carefully dried and weighed. Mercury was then added to fill the space BD, the adjustment of the level of the mercury at D at 15° being made with the aid of a battery and galvanoscope circuit. The piezometer and contents were again weighed. Knowing the weight of the empty piezometer, the volume of the solution can now be obtained. A small quantity of mercury was then weighed and added in DE; above it was placed pure distilled water; a cap of paper was tied round the top of the tube at E, and the piezometer was then ready to go into the compression cylinder. A metal framework was permanently attached to the lower end of the plug, to which the piezometer was bound. The wires

attached at C and D were connected to the binding screws on the lower end of the plug. The plug having been inserted in the cylinder, the corresponding binding screws on the upper end of the plug were connected through a resistance with a battery and galvanoscope, and pressure was applied. Whenever the galvanoscope shows that the circuit is broken, it means, in normal circumstances, that the mercury surface has fallen below D. The heat of compression took some time to disappear. Readings of the pressure necessary to cause break were taken every $\frac{1}{4}$ hour till they became steady, which occurred after a period of $1-1\frac{1}{2}$ hours. By careful manipulation of the regulating screw we can find the pressure at which the circuit is just broken, and the pressure at which it is just made. These under the best conditions did not differ by as much as 1 kg./cm.; sometimes, however, especially at high pressures, owing to oil getting in and fouling the mercury surface, discrepancies occurred; these had to be remedied by removing the piezometer and cleaning the mercury surface. The above procedure was repeated by adding several more known weights of mercury and obtaining the corresponding pressures.

As an example of the method of calculation, let us take the case of the Na_2CO_3 solution of concentration $c = .91$ (see table on p. 287). After buoyancy corrections had been made, the original volume (V_0) of the solution in the piezometer was in this case found to be 46.10 c.c., and the original volume (V'_0) of the mercury 1.27 c.c. The data required for this, in addition to the observed weights, are the density of the solution and of mercury at 15° . The former is obtained from the density measurements, and the latter was taken to be 13.558. The added weights of mercury were also corrected for buoyancy, but in addition these have also to be corrected for the compressibility of mercury, before we can obtain the apparent compressibility of the solution. Let v be the volume of mercury (obtained from its weight by dividing by 13.558) added in any particular case; then the change of volume experienced by the solution is not v but $v - \{(V'_0 + v) \times \gamma \times P\}$, where P is the excess of pressure at which contact is broken, and γ is the compressibility of mercury. γ was taken to be 3.9×10^{-6} . P , which was observed in kgs./cm.², was then reduced to atmospheres. P was also corrected according to a certificate obtained from the National Physical Laboratory, where the gauges were sent to be calibrated after the experiments were completed. Finally, then, we get the following values for corrected volume decrements and corresponding corrected pressures:

P (in atmospheres)	72	279	390	706	1043
v (in c.c.)144	.544	.750	1.278	1.803
$V_0 = 46.10$ c.c.					

To get the figures denoted by β_F in the table on pp. 287 and 288 we proceed as follows:—

Take the values of P , 279 and 390, with the corresponding values of v , .544 and .750; then we have a change of pressure of 111 giving a change of volume of .206. Hence

$$\frac{\Delta v}{\Delta p} = \frac{.206}{111} \quad \text{and} \quad \frac{1}{V_0} \cdot \frac{\Delta v}{\Delta p} = \frac{1}{46.10} \cdot \frac{.206}{111} = .0000402.$$

Hence $\beta_F = .0000402 + .0000022 = .0000424$, where 22×10^{-7} is the compressibility of glass, and this is the number given in column 3 in the tables as corresponding to the pressure 335, viz. $\frac{1}{2}(279+390)$.

It will be clear from what follows how the value .0000022 for the compressibility of glass was obtained.

According to Tait,* for water, compressibility and pressure are connected by the following formula:

$$\frac{1}{v_0} \frac{dv}{dp} = \frac{A}{B+p} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

where A and B are constants for a given temperature. As Tammann † states, this formula is satisfied by the results of Amagat's ‡ observations on the compressibility of water from 0 to 3000 atmospheres. He gives the values of A and B for various temperatures. Interpolating, we get for 15°

$$A = .1238.$$

$$B = 2540.$$

I took a series of observations on the compressibility of water with the same piezometer as was used for the solutions. I then used equation (1) with the values for A and B just mentioned, and found that it was satisfied if the compressibility of glass was taken to have the mean value 22×10^{-7} . This value for the compressibility of glass was used in calculating the compressibilities of the various solutions.

Further, as indicated by Tait, and developed theoretically by Tammann, when a solution of a salt is made in water, the compressibility of the solution can be expressed by the formula

$$\frac{1}{v_0} \frac{dv}{dp} = \frac{1}{v_{\Delta K}} \cdot \frac{A}{B + \Delta K + p} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (2)$$

In formula (2), A and B are the same constants as before; ΔK has the dimensions of a pressure, and may be taken as a measure of the increase of internal pressure due to the solute; $v_{\Delta K}$ is the volume which unit

* Proc. Roy. Soc. Edin., 1893.

† Ueber die Beziehungen zwischen den inneren Kräften und Eigenschaften der Lösungen (1907).

‡ Comptes Rendus, 1893.

volume of the solution at atmospheric pressure occupies under an external pressure ΔK .

Both ΔK and $v_{\Delta K}$ depend on the concentration; $v_{\Delta K}$ being obtained from the equation

$$v_{\Delta K} = 1 - A \log_e \frac{B + \Delta K}{B}.$$

It is clear that when there is no salt in solution

$$\Delta K = 0 \quad \text{and} \quad v_{\Delta K} = 1.$$

Hence formula (2) becomes

$$\frac{1}{v_0} \frac{dv}{dp} = \frac{A}{B + p},$$

which agrees with (1).

Taking, then, equation (2), I substituted the observed values of the compressibility on the left-hand side, and found by a process of trial and error the values for ΔK which gave most consistent results. I found that in general it was possible to find values for ΔK which satisfied equation (2) within certain limits. I have included in the following table the values of ΔK found in this way for each solution, and also side by side the observed and the calculated value of the compressibility. The symbols have the following meaning:—

A is the equivalent weight of the substance.

c is the concentration (grams in 100 grams water).

S is density relative to water at same temperature.

p is the pressure in atmospheres.

β_F is the value of $\frac{1}{v_0} \frac{dv}{dp}$ found for pressure p .

β_c is the value of same calculated from formula (2).

Na_2CO_3 : $A_1 = 53.06$.

Particulars concerning Solution.	p	$\beta_F \times 10^7$	$\beta_c \times 10^7$	ΔK	$\frac{\Delta K}{c}$	Particulars concerning Solution.	p	$\beta_F \times 10^7$	$\beta_c \times 10^7$	ΔK	$\frac{\Delta K}{c}$
NaOH. $A_1 = 40.06$.											
$c = 4.08$	92	376	383			$c = 1.98$	73	450	441		
$v_0 = 50.00$	214	365	369			$v_0 = 48.04$	262	407	413	230	116
$S = 1.04665$	364	355	354	700	172	$S = 1.01834$	500	388	355		
	533	344	338			$c = 1.20$	70	451	451		
	728	320	321			$v_0 = 48.22$	270	414	420	160	133
	954	313	305			$S = 1.00977$	534	380	386		
	192	408	401				816	358	365		
$c = 2.08$	326	380	390			MgSO ₄ . $A_1 = 60.21$.					
$v_0 = 50.00$	478	374	373	360	173	$c = 2.90$	76	445	443		
$S = 1.02606$	656	349	354			$v_0 = 48.20$	280	404	413	200	68.9
	843	339	337			$S = 1.02965$	544	381	380		
	230	396	407			$c = 1.47$	74	450	457		
$c = 1.77$	403	391	386			$v_0 = 48.20$	264	432	427	110	74.8
$v_0 = 50.00$	562	360	368	310	175	$S = 1.01503$	669	379	375		
$S = 1.02019$	718	353	352			$c = .99$	78	453	462		
	887	348	336			$v_0 = 48.20$	370	429	417	75	76
	180	412	421			$S = 1.01006$	718	367	373		
$c = 1.42$	300	398	404			$c = .59$	78	460	465		
$v_0 = 50.00$	540	378	375	250	176	$v_0 = 48.20$	227	452	441	45	76
$S = 1.01624$	708	357	357			$S = 1.00621$	421	412	412		
	924	345	336				663	383	382		
$c = .71$	114	448	448			ZnSO ₄ . $A_1 = 80.73$.					
$v_0 = 50.00$	239	425	428			$c = 3.82$	40	440	441		
$S = 1.00903$	361	417	411	130	183	$v_0 = 48.20$	292	404	405	260	68.1
	500	398	393			$S = 1.04016$	652	364	363		
	654	375	375			$c = 1.95$	55	458	457		
KOH. $A_1 = 56.00$.						$v_0 = 48.20$	223	425	430	130	66.6
$c = 5.75$	114	385	385			$S = 1.02038$	472	394	396		
$v_0 = 48.17$	345	354	359				744	376	364		
$S = 1.05258$	624	330	333	660	115	$c = 1.31$	75	461	458		
	858	314	314			$v_0 = 48.20$	277	417	426	90	68.7
	76	441	426†			$S = 1.01377$	579	388	386		
$c = 2.94$	281	386	398			$c = .79$	70	472	466		
$v_0 = 48.11$	524	367	369	340	115	$v_0 = 48.20$	245	435	437	55	70
$S = 1.02729$	815	354	340†			$S = 1.00814$	477	400	404		
							745	394	372†		

It will be seen that there is very fair agreement between β_F and β_c —in fact, that observed and calculated values agree within $2\frac{1}{2}$ –3 per cent. The four cases marked with a cross fall outside this limit. That two of these should occur in the case of the same solution, viz. KOH of concentration 2·94, seems to point to some undetected source of error in this case. The volumes are subject to an error of about .001 c.c., while the pressures are subject to an error of about .5 per cent.; this corresponds to a possible error of about 3 per cent. in the compressibilities. We see then that, within the limits of error of observation, calculated and observed values agree; in other words, it is possible, for all the solutions examined, to find a value of ΔK to satisfy the formula

$$\frac{1}{v_0} \frac{dv}{dp} = \frac{1}{v_{\Delta K}} \cdot \frac{A}{B + \Delta K + p}.$$

This value ΔK is a measure of the increase of internal pressure due to the added salt. It will be seen from the numbers in the last column of the table how ΔK varies with the concentration; we see that in general $\frac{\Delta K}{c}$ increases with increasing dilution. For the sake of comparison, I quote in the following table some of Tammann's values for $\frac{\Delta K}{c}$ calculated from the thermal expansion of the solutions, and my values obtained from observations on compressibility:—

	Results from Expansibility.			Results from Compressibility.			
	c	7·1		3·59	.910	.371	.186
Na_2CO_3	ΔK	674		380	120	80	70
	$\frac{\Delta K}{c}$	95		106	132	215	376
	c	4·18	3·19	1·59	4·08	2·08	1·77
NaOH	ΔK	763	540	286	700	360	310
	$\frac{\Delta K}{c}$	182	169	180	172	173	175
	c	5·60	5·31	2·66	5·75	2·94	1·98
KOH	ΔK	661	641	340	660	340	230
	$\frac{\Delta K}{c}$	118	121	128	115	115	116
	c	2·95	1·88		2·90	1·47	.99
MgSO_4	ΔK	236	146		200	110	75
	$\frac{\Delta K}{c}$	80	77		68·9	74·8	76
	c	4·06	1·62		3·82	1·95	1·31
ZnSO_4	ΔK	251	111		260	130	90
	$\frac{\Delta K}{c}$	62	68		68·1	66·6	68·7
	c						70

From this comparison it will be seen that there is good agreement between the two sets of values. The accuracy of Tammann's figures depends largely on the accuracy of the density observations which he used; as these latter were taken from various sources they differ considerably. This can be seen, for example, in the case of NaOH above, where the values separated by the dotted line are from different sources. Again, it should be mentioned that the results quoted are in general mean values for four or five different temperatures ranging from 5° C. to 30° C.

I referred at the outset of this paper to a list of compressibilities by Röntgen and Schneider, and also to the fact that the pressures they worked with never exceeded a few atmospheres. Tammann tested his formula by their results, and showed that calculation and observation agreed to within about 3 per cent.; this is the same degree of accuracy as I find to hold at higher pressures also.

Röntgen and Schneider's table of compressibilities is interesting also from another point of view. It is indicated in their paper how, from their observations on compressibility, it is possible to calculate what they call the relative molecular compressibility. The numbers representing the relative molecular compressibility tell us how much the volume of a certain mass of water changes when a given number of water molecules are replaced by the same number of molecules of a dissolved substance. Collecting their values, we have the following table:—

RELATIVE MOLECULAR COMPRESSIBILITY.

	H.	NH ₄	K.	Li.	Na.
I960	.913	.918	.892
NO ₃	.980	.953	.901	.893	.878
Br	.972	.951	.894	.887	.870
Cl	.954	.933	.872	.868	.849
OH	1.000	1.009	.779	.782	.761
SO ₄	.942	.808682	...
CO ₃669	.660	.644

If, then, it is a general law that the compressibility is inversely proportional to the internal pressure, we would expect to get a similar table for the contraction on solution, since the greater the contraction the greater the internal pressure. To test this I have chosen the quantity which Kohlrausch calls the molecular volume of the body in solution. The greater this molecular volume is, the smaller is the contraction on solution. The molecular volume is given by $\frac{A_1}{Q} - 1000 \frac{S-1}{m}$, where A_1 is the equivalent

weight of the body, S is the specific gravity of the solution, Q is the density of water at the temperature of the solution, and m is the concentration expressed in gram equivalents per litre. I have calculated this quantity for the salts given in the table just quoted, using Röntgen and Schneider's densities. The result is given in the table below:—

MOLECULAR VOLUME.

	NH ₄	K.	H.	Li.	Na.
I	52	47	...	36	36
NO ₃	48	40	32	30	30
Br	44	37	27	28	24
Cl	38	30	20	19	18
SO ₄	30	21	19	12	10
CO ₃	...	10	...	8	1
OH	35	7	0	-2	-4

A comparison of the two tables shows that in general the order is the same in both, the exception being that the terms H and OH occupy a different position in the two cases. With this exception, however, we see that, for these substances at least, small molecular volume or large contraction corresponds to small compressibility. It is to be noted that the terms H and OH are the terms occurring in the formation of the solvent, viz. water. It is also noteworthy that, with the exception of the ammonium salt, the combinations involving H and OH have all a relatively large conductivity, and it is possible that this high conductivity tends to make these combinations more compressible than we would expect from their molecular volumes.

The observations referred to in this paper were made in the Physical Laboratory of the University of Edinburgh. The general expenses of the research were defrayed by grants from the Carnegie Trust, Sir John Jackson's Tait Memorial Fund, and the Tait Memorial Laboratory Fund.

(Issued separately July 17, 1913.)

XXVI.—Plane Strain in a Wedge, with Applications to Masonry Dams. By S. D. Carothers, A.R.C.Sc.I. *Communicated by The General Secretary.*

(MS. received March 6, 1913. Read July 7, 1913.)

(1) It has been shown * that in the case of plane strain the stresses in a body can be obtained from a single stress function χ satisfying

$$\nabla_1^4 \chi = 0, \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (1)$$

where

$$\nabla_1^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}.$$

Solutions of equation (1) will be applied to obtain the stresses in an infinitely long wedge-shaped cylinder, having normal force varying accord-

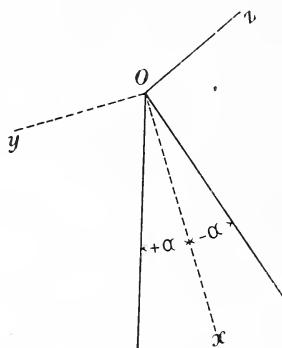


FIG. I.

ing to some integral power of the distance from the angle applied to the plane faces, and the same at all corresponding points along the length of the cylinder. The conditions which will be supposed to exist at the curved boundary subtended by the angle will be taken to be similar to those which exist at points nearer to the angle.

The case of a triangular dam having its free edge in the surface of water and subjected to water pressure on one face will be dealt with in some detail, and a solution for a truncated wedge partially subjected to water pressure will be given.

The notation adopted will be that of Love's *Elasticity*, 2nd edition.

The axes of reference will be selected as shown in fig. 1, the axis of z being the line of intersection of the plane faces.

* A. E. H. Love, *Elasticity*, 2nd edition, p. 201; or J. H. Michell, *Proc. Lond. Math. Soc.*, vol. xxxi. p. 100.

(2) It will at once be seen that, for the type of loading considered, the shear on the plane of xy vanishes everywhere and we have

$$\epsilon_{yz} = \epsilon_{zx} = \epsilon_{zz} = 0;$$

and thus

$$\widehat{y}\widehat{z} = \widehat{z}\widehat{x} = 0.$$

The general stress equations of equilibrium are

$$\left. \begin{aligned} \frac{\partial}{\partial x} \widehat{x}\widehat{x} + \frac{\partial}{\partial y} \widehat{x}\widehat{y} + \rho X &= 0, \\ \frac{\partial}{\partial x} \widehat{x}\widehat{y} + \frac{\partial}{\partial y} \widehat{y}\widehat{y} + \rho Y &= 0, \end{aligned} \right\} \quad \quad (2)$$

which, if the body forces are omitted, are identically satisfied if we put

$$\widehat{x}\widehat{x} = \frac{\partial^2 \chi}{\partial y^2}, \quad \widehat{y}\widehat{y} = \frac{\partial^2 \chi}{\partial x^2}, \quad \widehat{x}\widehat{y} = -\frac{\partial^2 \chi}{\partial x \partial y}; \quad \quad (3)$$

where χ is some function of x and y only.

The stress equations in cylindrical co-ordinates are

$$\left. \begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} (r \widehat{r}\widehat{r}) + \frac{1}{r} \frac{\partial}{\partial \theta} (r \widehat{\theta}\widehat{\theta}) - \frac{\widehat{\theta}\widehat{\theta}}{r} &= 0, \\ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 r \widehat{\theta}\widehat{\theta}) + \frac{1}{r} \frac{\partial}{\partial \theta} (\widehat{\theta}\widehat{\theta}) &= 0, \end{aligned} \right\} \quad \quad (4)$$

and it has been shown* that the stress components $\widehat{r}\widehat{r}$, $\widehat{\theta}\widehat{\theta}$, and $r\widehat{\theta}\widehat{\theta}$, when expressed in terms of a single stress function χ , are

$$\left. \begin{aligned} \widehat{r}\widehat{r} &= \frac{1}{r} \frac{\partial \chi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \chi}{\partial \theta^2}, \\ \widehat{\theta}\widehat{\theta} &= \frac{\partial \chi}{\partial r^2}, \\ r\widehat{\theta}\widehat{\theta} &= -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \chi}{\partial \theta} \right). \end{aligned} \right\} \quad \quad (5)$$

(3) Solutions of equation (1) which will be used are as follows:—

$$\left. \begin{aligned} \chi &= A \sin 2\theta + B \cos 2\theta + C\theta + D, \\ \chi &= r[(A + B\theta) \sin \theta + (C + D\theta) \cos \theta], \\ \chi &= r^2[A \sin 2\theta + B \cos 2\theta + C\theta + D], \\ \chi &= r^n[A \sin n\theta + B \cos n\theta + C \sin(n-2)\theta + D \cos(n-2)\theta]. \end{aligned} \right\} \quad \quad (6)$$

(4) To represent a moment of $+M$ (see fig. 2) at the angle of the wedge put

$$\left. \begin{aligned} \chi &= -M(\sin 2\theta - 2 \cos 2a\theta)/2(\sin 2a - 2a \cos 2a) \\ &= -M(\sin 2\theta - 2 \cos 2a\theta)/2m, \text{ say,} \end{aligned} \right\} \quad \quad (7)$$

giving stresses

$$\left. \begin{aligned} \widehat{r}\widehat{r} &= +2Mr^{-2} \sin 2\theta/m, \\ \widehat{\theta}\widehat{\theta} &= 0 \text{ everywhere,} \\ \widehat{r}\widehat{\theta} &= +Mr^{-2}(\cos 2\theta - \cos 2a)/m. \end{aligned} \right\} \quad \quad (8)$$

* J. H. Michell, *Proc. Lond. Math. Soc.*, vol. xxxi. p. 111.

These stresses represent a moment M per unit length of z axis, and satisfy the conditions $\widehat{\theta\theta} = \widehat{r\theta} = 0$ at $\theta = \pm \alpha$.

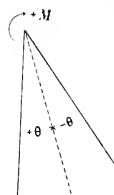


FIG 2.

(5) For a force F^* per unit length of z axis directed along the positive direction of y put

$$\chi = Fr\theta \cos \theta / (2\alpha - \sin 2\alpha), \quad \quad (9)$$

giving stresses

$$\begin{cases} \widehat{rr} = -2Fr^{-1} \sin \theta / (2\alpha - \sin 2\alpha), \\ \widehat{\theta\theta} = \widehat{r\theta} = 0 \text{ everywhere.} \end{cases} \quad \quad (10)$$

For a force F per unit length of z axis directed along the positive direction of the axis of x put

$$\chi = -Fr\theta \sin \theta / (2\alpha + \sin 2\alpha), \quad \quad (11)$$

giving stresses

$$\begin{cases} \widehat{rr} = -2Fr^{-1} \cos \theta / (2\alpha + \sin 2\alpha), \\ \widehat{\theta\theta} = \widehat{r\theta} = 0 \text{ everywhere.} \end{cases} \quad \quad (12)$$

(6) Uniform Pressure.

For uniform pressure P on $\theta = \alpha$ and uniform tension P on $\theta = -\alpha$ put

$$\begin{aligned} \chi &= -Pr^2(\sin 2\theta - 2 \cos 2\alpha\theta) / 2(\sin 2\alpha - 2\alpha \cos 2\alpha) \\ &= -Pr^2(\sin 2\theta - 2 \cos 2\alpha\theta) / 2m, \text{ say,} \end{aligned} \quad \quad (13)$$

giving stresses

$$\begin{cases} \widehat{rr} = P(\sin 2\theta + 2 \cos 2\alpha\theta) / m, \\ \widehat{\theta\theta} = -P(\sin 2\theta - 2 \cos 2\alpha\theta) / m, \\ \widehat{r\theta} = P(\cos 2\theta - \cos 2\alpha) / m. \end{cases} \quad \quad (14)$$

For uniform pressure on both sides of the wedge put

$$\chi = -Pr^2/2,$$

giving

$$\widehat{rr} = \theta\theta = -P, \quad \text{and} \quad \widehat{r\theta} = 0 \text{ everywhere.}$$

The stress function for an unlimited solid with a plane upper boundary, when one half of this is loaded with a pressure P per unit of area, is

$$\chi = -Pr^2(\sin 2\theta + 2\theta + \pi) / 4\pi.$$

* Force at an angle has been treated by Professor J. H. Michell, *Proc. Lond. Math. Soc.*, vol. xxxiv. See also Love's *Elasticity*, p. 208.

(7) For pressure P_r on $\theta = \alpha$ and tension P_r on $\theta = -\alpha$ put

$$\begin{aligned}\chi &= -P_r^3(\cos \alpha \sin 3\theta - 3 \cos 3\alpha \sin \theta)/6(\cos \alpha \sin 3\alpha - 3 \cos 3\alpha \sin \alpha) \\ &= -P_r^3(\cos \alpha \sin 3\theta - 3 \cos 3\alpha \sin \theta)/6m, \text{ say,} \quad (15)\end{aligned}$$

giving stresses

$$\left. \begin{aligned}\widehat{rr} &= P_r(\cos \alpha \sin 3\theta + \cos 3\alpha \sin \theta)/m, \\ \widehat{\theta\theta} &= -P_r(\cos \alpha \sin 3\theta - 3 \cos 3\alpha \sin \theta)/m, \\ \widehat{r\theta} &= P_r(\cos \alpha \cos 3\theta - \cos 3\alpha \cos \theta)/m.\end{aligned}\right\} \quad \quad (16)$$

For pressure P_r on both sides of the wedge put

$$\begin{aligned}\chi &= -P_r^3(\sin \alpha \cos 3\theta - 3 \sin 3\alpha \cos \theta)/6(\sin \alpha \cos 3\alpha - 3 \sin 3\alpha \cos \alpha), \\ &= -P_r^3(\sin \alpha \cos 3\theta - 3 \sin 3\alpha \cos \theta)/6m, \quad \quad (17)\end{aligned}$$

giving stresses

$$\left. \begin{aligned}\widehat{rr} &= P_r(\sin \alpha \cos 3\theta + \sin 3\alpha \cos \theta)/m, \\ \widehat{\theta\theta} &= -P_r(\sin \alpha \cos 3\theta - 3 \sin 3\alpha \cos \theta)/m, \\ \widehat{r\theta} &= -P_r(\sin \alpha \sin 3\theta - \sin 3\alpha \sin \theta)/m.\end{aligned}\right\} \quad \quad (18)$$

To obtain the case of pressure P_r on $\theta = \alpha$, the face $\theta = -\alpha$ being free from stress, take half the sum of χ as given by equations (15) and (17). After some reduction there is obtained

$$\chi = -P_r^3[(3 \sin^2 \alpha - \sin^2 \theta) \sin \theta + \tan^3 \alpha (3 \cos^2 \alpha - \cos^2 \theta) \cos \theta]/24 \sin^3 \alpha, \quad . \quad (19)$$

giving stresses

$$\left. \begin{aligned}\widehat{rr} &= P_r[(\cos^2 \alpha - \sin^2 \theta) \sin \theta - \tan^3 \alpha (\cos^2 \theta - \sin^2 \alpha) \cos \theta]/4 \sin^3 \alpha, \\ \widehat{\theta\theta} &= -P_r[(3 \sin^2 \alpha - \sin^2 \theta) \sin \theta + \tan^3 \alpha (3 \cos^2 \alpha - \cos^2 \theta) \cos \theta]/4 \sin^3 \alpha, \\ \widehat{r\theta} &= P_r[(\sin^2 \alpha - \sin^2 \theta) \cos \theta + \tan^3 \alpha (\cos^2 \theta - \cos^2 \alpha) \sin \theta]/4 \sin^3 \alpha.\end{aligned}\right\} \quad \quad (20)$$

(8) Weight of Wedge.

If the weight of the wedge is taken into consideration, its central plane being placed at an angle β with the vertical, we have for the stress equations

$$\left. \begin{aligned}\frac{1}{r} \frac{\partial}{\partial r}(r \widehat{rr}) + \frac{1}{r} \frac{\partial}{\partial \theta} \widehat{r\theta} - \frac{\widehat{\theta\theta}}{r} + \rho g \cos(\beta - \theta) &= 0 \\ \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \widehat{\theta\theta}) + \frac{1}{r} \frac{\partial}{\partial \theta} \widehat{\theta\theta} + \rho g \sin(\beta - \theta) &= 0.\end{aligned}\right\} \quad \quad (21)$$

and

If we assume

$$\widehat{rr} = \frac{1}{r} \frac{d\chi}{dr} + \frac{1}{r^2} \frac{d^2 \chi}{d\theta^2} + A', \quad \widehat{\theta\theta} = \frac{\partial^2 \chi}{\partial r^2} + B', \quad \text{and} \quad \widehat{r\theta} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \chi}{\partial \theta} \right) \quad . \quad (22)$$

and substitute these values in equation (21) and differentiate out the values of A' and B' can be obtained. A solution is

$$A' = B' = -\rho g r \cos(\beta - \theta). \quad \quad (23)$$

Again, if we assume

$$\chi = \rho g r^3(A \sin 3\theta + B \cos 3\theta + C \sin \theta + D \cos \theta)$$

and differentiate by the aid of equations (22) and (23), and remember that, for the weight alone, $\widehat{\theta\theta}$ and $\widehat{r\theta}$ are each equal to zero at $\theta=\pm\alpha$, four equations are obtained to determine the four unknowns A, B, C, and D; these give finally

$$\chi = \rho gr^3[s(3\sin^2\alpha\sin\theta - \sin^3\theta) + c(3\cos^2\alpha\cos\theta - \cos^3\theta)]/12, \quad . \quad (24)$$

and

$$\left. \begin{aligned} \widehat{rr} &= -\rho gr[s(\cos^2\alpha - \sin^2\theta)\sin\theta - c(\cos^2\theta - \sin^2\alpha)\cos\theta + 2\cos(\beta - \theta)]/2, \\ \widehat{\theta\theta} &= \rho gr[s(3\sin^2\alpha - \sin^2\theta)\sin\theta + c(3\cos^2\alpha - \cos^2\theta)\cos\theta - 2\cos(\beta - \theta)]/2, \\ \widehat{r\theta} &= -\rho gr[s(\sin^2\alpha - \sin^2\theta)\cos\theta + c(\cos^2\theta - \cos^2\alpha)\sin\theta]/2, \end{aligned} \right\} \quad . \quad (25)$$

where

$$s = \sin\beta/\sin^2\alpha, \quad \text{and} \quad c = \cos\beta/\cos^2\alpha.$$

If rectangular co-ordinates are used it can be shown, by a process analogous to the above, that the stresses due to the weight are

$$\left. \begin{aligned} \widehat{xx} &= -\rho g(\sin\beta\cot^2\alpha y + \cos\beta.x)/2, \\ \widehat{yy} &= -\rho g(\cos\beta\tan^2\alpha x + \sin\beta.y)/2, \\ \widehat{xy} &= -\rho g(\sin\beta.x + \cos\beta.y)/2. \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad (26)$$

Equation (19) admits of easy transformation to rectangular co-ordinates and gives for pressure Pr on $\theta=\alpha$

$$\chi = -P\left[\frac{3}{\sin\alpha}(y^3 + x^2y) + \frac{3}{\cos\alpha}(x^3 + xy^2) - \frac{x^3}{\cos^3\alpha} - \frac{y^3}{\sin^3\alpha}\right]/24. \quad . \quad (27)$$

The stresses due to the weight and pressure Pr on $\theta=\alpha$ are

$$\left. \begin{aligned} \widehat{xx} &= -\frac{P}{4}\left[\frac{3\sin^2\alpha - 1}{\sin^3\alpha}y + \frac{x}{\cos\alpha}\right] - \frac{\rho g}{2}[\sin\beta\cot^2\alpha y + \cos\beta.x],^* \\ \widehat{yy} &= -\frac{P}{4}\left[\frac{y}{\sin\alpha} + \frac{3\cos^2\alpha - 1}{\cos^3\alpha}x\right] - \frac{\rho g}{2}[\cos\beta\tan^2\alpha x + \sin\beta.y], \\ \widehat{xy} &= \frac{P}{4}\left[\frac{x}{\sin\alpha} + \frac{y}{\cos\alpha}\right] - \frac{\rho g}{2}[\sin\beta.x + \cos\beta.y] \end{aligned} \right\} \quad . \quad (28)$$

(9) The principal stresses are, by a well-known theorem, given by

$$p = \frac{\widehat{xx} + \widehat{yy}}{2} \pm \frac{\sqrt{(\widehat{xx} - \widehat{yy})^2 + 4\widehat{xy}^2}}{2},$$

and the direction of the principal stresses with reference to the fixed axes is given by

$$\tan 2\phi = 2\widehat{xy}/(\widehat{xx} - \widehat{yy}),$$

and from equations (28) the values $(\widehat{xx} + \widehat{yy})/2$, $(\widehat{xx} - \widehat{yy})$, and $2\widehat{xy}$ can be written out.

For example, if $\alpha = \beta = 18^\circ$ and $\rho = 2.30$ and the pressure Pr is due to

* For an equivalent solution by Professor Levy, see *Comptes Rendus*, vol. cxxvii. pp. 10-15, Paris, 1908.

water pressure; taking the weight of a cubic foot of water as the unit, we readily obtain

$$(\widehat{xx} + \widehat{yy})/2 = -0.9852x + 0.7570y.$$

$$\widehat{xx} - \widehat{yy} = -0.7430x + 3.8435y.$$

$$2\widehat{xy} = 0.9072x - 1.6615y.$$

If it is found what these become at the points, say, $y=0$, $y=\pm x/10$, $y=\pm 2x/10$, $y=\pm 3x/10$, and $y=\pm x \tan \alpha$, we shall then have the stresses at nine points across the section of the wedge. The results are as follows :—

$y.$	$x \tan \alpha.$	$3x/10.$	$2x/10.$	$x/10.$	0.	$-x/10.$	$-2x/10.$	$-3x/10.$	$-x \tan \alpha.$
p_1/x	-1.0512	-1.0471	-1.1218	-1.3255	-1.5712	-1.8389	-2.1136	-2.3923	-2.4622
p_2/x	-0.4272	-0.4691	-0.5458	-0.4935	-0.3992	-0.2829	-0.1596	-0.0323	-0.0002
ϕ	18° 00'	22° 28'	43° 42'	32° 14'	25° 19'	21° 48'	19° 40'	18° 16'	18° 00'

Those values of ϕ under the line are measured from the axis of x , the others from the axis of y , the axis being determined by the change of sign of $2\widehat{xy}/(\widehat{xx} - \widehat{yy})$.

The maximum shear in the wedge occurs at the outer face, and is easily seen to be $r\widehat{r}/2$ at that face.

$$\text{Maximum shear} = -\frac{1}{2}[\text{Pr} \cos 2\alpha / \sin^2 2\alpha + \rho gr \sin (\alpha - \beta) / \sin 2\alpha]. \quad . \quad (29)$$

This shear acts on those planes parallel to the z axis which cut the back face at 45°.

(10) From the values of ϕ just obtained the curves along which the shear vanishes can be drawn. Each curve cuts one face normally, bends round and becomes practically parallel to the other face. If the stress functions are transformed to new axes having the axis of x in the face $\theta=\alpha$, the axis of y being drawn into the wedge, then the nature of the curve in the region of this face becomes amenable to algebra. The stress functions for pressure Pr on $\theta=\alpha$ and the weight respectively are

$$\left. \begin{aligned} {}_{\alpha}\chi_{\text{P}} &= -\frac{\text{P}}{6}[x^3 - 3xy^2 \cot^2 2\alpha + 2y^3 \cot^3 2\alpha], \\ {}_{\alpha}\chi_g &= \frac{\rho g}{12}[2x^3 \cos(\alpha - \beta) + 6xy^2 \cot^2 2\alpha \sin(\alpha - \beta) \\ &\quad + y^3 \{\cos \beta \sin \alpha (3 - \tan^2 \alpha) - \sin \beta \cos \alpha (3 - \cot^2 \alpha)\}]. \end{aligned} \right\} \quad (30)$$

Similarly, if the plane $y=0$ is taken as lying in the face $\theta=-\alpha$ we have

$$\left. \begin{aligned} {}_{-\alpha}\chi_{\text{P}} &= -\text{P}[3xy^2 \sin 2\alpha \cos 2\alpha - y^3(3 \cos^2 2\alpha - 1)]/6 \sin^2 2\alpha, \\ {}_{-\alpha}\chi_g &= \frac{\rho g}{12}[2x^3 \cos(\alpha + \beta) + 6xy^2 \sin(\alpha + \beta) \\ &\quad + y^3 \{\sin \beta \cos \alpha (3 - \cot^2 \alpha) + \cos \beta \sin \alpha (3 - \tan^2 \alpha)\}]. \end{aligned} \right\} \quad (31)$$

In writing out the stresses from the above it must be remembered that the equivalent of the term $-\rho gr \cos(\beta - \theta)$ must be added to \widehat{xx} and \widehat{yy} .

When y is small $\tan 2\phi$ is small, and $\tan^2\phi$ is negligible in comparison to unity, and we get

$$dy/dx = \widehat{xy}/(\widehat{xx} - \widehat{yy}),$$

and this is of the form

$$dy/dx = -y/(a_1x - a_2y)$$

at either face. The primitive of this equation is

$$y^{\alpha} \cdot [(a_1 + 1)x - a_2y] = \text{constant.}$$

Assuming $\beta = \alpha = 18^\circ$ and $\rho g/P = 2.30$, the curves are as follows :—

$$\text{Near front face } xy^{0.3136} - 3.368y^{1.3136} = \text{constant.}$$

$$\text{Near back face } xy^{4.865} - 1.510y^{5.865} = \text{constant.}$$

The curves have been drawn for the foregoing case, and are shown in fig. 3 below.

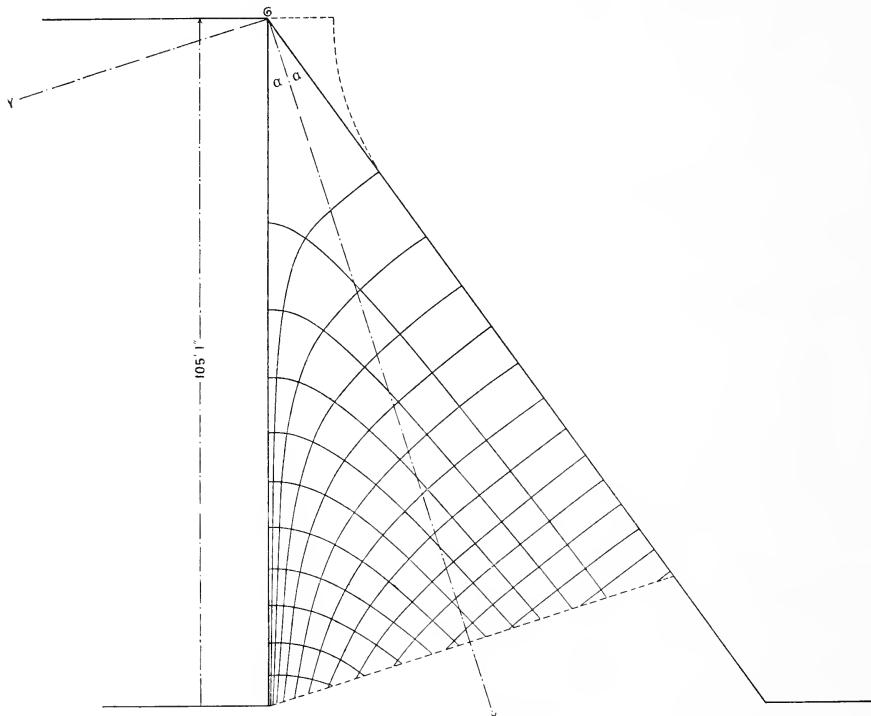


FIG. 3.

DIAGRAM SHEWING LINES ON WHICH NORMAL PRESSURE EXISTS DUE TO THE WEIGHT OF THE WEDGE ($\text{SPG}:2.30$) AND WATER PRESSURE ON THE VERTICAL FACE ($2\alpha - 36^\circ$)

(11) From the foregoing the solution for a truncated wedge can be obtained to a high degree of approximation, provided the part cut off or the part added, as the case may be, is not a large part of the whole.

Suppose ABCD (fig. 4) to represent a dam subjected to water pressure to a height ED, O being the point of intersection of AD and BC, and AB taken as horizontal.

Let $OA = a$, $OE = b$.

The stresses can be written down for pressure P_r over OD and uniform tension P_b over OD, and also for the weight of ODC. We have thus on the face OD pressure $P(r-b)$ over ED and tension $P(b-r)$ over OE.

Introduce at point X, where $OX = b/3$, a force $= Pb^2/2$ acting normal to

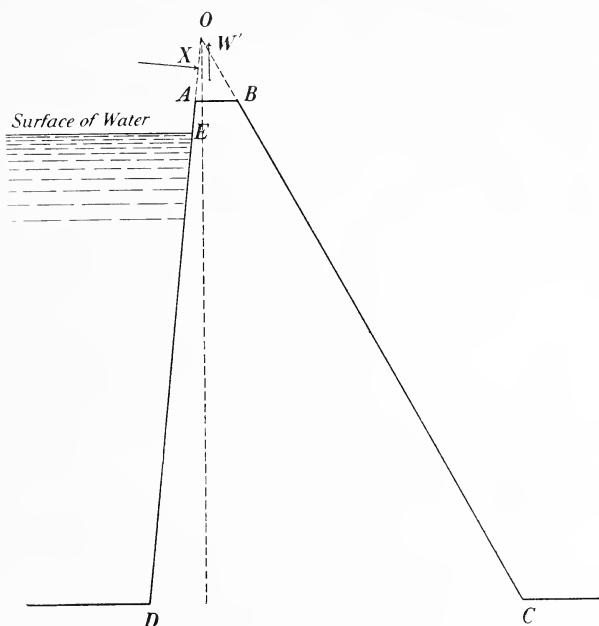


FIG. 4.

OD and into the wedge, and at the centre of gravity of OAB introduce a force equal to the weight of OAB reversed, say W' . The force $Pb^2/2$ at X and W' will form a set of forces in equilibrium with the tension on OE and the actual weight of OAB, and by the principle of equipollent loads the stresses due to this system are negligible at all points well outside area OAB, and thus the solution for the whole system of forces will be equivalent to the solution for pressure $P(r-b)$ over ED and the weight of ABCD only.

The force $Pb^2/2$ and W' can be transferred to O and suitable moments introduced, and the stresses due to these forces and moments can be written down by the aid of equations (8), (10), and (12).

The components of these forces along the axis of z and y and the moment about O are for the general case stated above as follows:—

$$\left. \begin{aligned} X &= Pb^2 \sin \alpha/2 - \rho g a^2 \cos(\alpha - \beta) \sin 2\alpha \cos \beta/2 \cos(\alpha + \beta), \\ Y &= -Pb^2 \cos \alpha/2 - \rho g a^2 \cos(\alpha - \beta) \sin 2\alpha \sin \beta/2 \cos(\alpha + \beta), \\ M &= -Pb^3/6 - \rho g a^3 \cos^2(\alpha - \beta) \sin 2\alpha \tan \beta/3 \cos(\alpha + \beta). \end{aligned} \right\} . \quad (32)$$

The general solution in plane polars is as follows:—

$$\left. \begin{aligned} \widehat{r}r &= Pr[(\cos^2 \alpha - \sin^2 \theta) \sin \theta - \tan^3 \alpha (\cos^2 \theta - \sin^2 \alpha) \cos \theta]/4 \sin^3 \alpha \\ &\quad - \frac{\rho g r}{2} [s(\cos^2 \alpha - \sin^2 \theta) \sin \theta - c(\cos^2 \theta - \sin^2 \alpha) \cos \theta + 2 \cos(\beta - \theta)] \\ &\quad + Pb[\sin 2\alpha - \sin 2\theta - 2 \cos 2\alpha(\alpha + \theta)]/2(\sin 2\alpha - 2\alpha \cos 2\alpha) \\ &\quad - 2Xr^{-1} \cos \theta/(2\alpha + \sin 2\alpha) - 2Yr^{-1} \sin \theta/(2\alpha - \sin 2\alpha) \\ &\quad + 2Mr^{-2} \sin 2\theta/(\sin 2\alpha - 2\alpha \cos 2\alpha), \\ \widehat{\theta} &= -Pr[(3 \sin^2 \alpha - \sin^2 \theta) \sin \theta + \tan^3 \alpha (3 \cos^2 \alpha - \cos^2 \theta) \cos \theta]/4 \sin^3 \alpha \\ &\quad + \frac{\rho g r}{2} [s(3 \sin^2 \alpha - \sin^2 \theta) + c(3 \cos^2 \alpha - \cos^2 \theta) \cos \theta - 2 \cos(\beta - \theta)] \\ &\quad + Pb[\sin 2\alpha + \sin 2\theta - 2 \cos 2\alpha(\alpha + \theta)]/2(\sin 2\alpha - 2\alpha \cos 2\alpha), \\ \widehat{\theta} &= Pr[\sin^2 \alpha - \sin^2 \theta) \cos \theta + \tan^3 \alpha (\cos^2 \theta - \cos^2 \alpha) \sin \theta]/4 \sin^3 \alpha \\ &\quad - \frac{\rho g r}{2} [s(\sin^2 \alpha - \sin^2 \theta) \cos \theta + c(\cos^2 \theta - \cos^2 \alpha) \sin \theta] \\ &\quad + Pb[\cos 2\alpha - \cos 2\theta]/2(\sin 2\alpha - 2\alpha \cos 2\alpha) \\ &\quad + Mr^{-2} [\cos 2\theta - \cos 2\alpha]/(\sin 2\alpha - 2\alpha \cos 2\alpha), \end{aligned} \right\} . \quad (33)$$

where

$$s = \sin \beta / \sin^2 \alpha, \quad \text{and} \quad c = \cos \beta / \cos^2 \alpha.$$

The solution in rectangular co-ordinates is:—

$$\left. \begin{aligned} \widehat{x}x &= -\frac{P}{4} [(3 \sin^2 \alpha - 1)y / \sin^3 \alpha + x / \cos \alpha] - \frac{\rho g}{2} [\sin \beta \cdot \cot^2 \alpha y + \cos \beta \cdot x] \\ &\quad - \frac{Pb}{2} [\sin 2\alpha - 2\alpha \cos 2\alpha - 2 \cos 2\alpha(\theta + xy/r^2)]/(\sin 2\alpha - 2\alpha \cos 2\alpha) \\ &\quad - 2Xx^3/r^4(2\alpha + \sin 2\alpha) - 2Yx^2y/r^4(2\alpha - \sin 2\alpha) \\ &\quad - 2M[\cos 2\alpha xy(x^2 + y^2) - 3xy^3 + xy^3]/r^6(\sin 2\alpha - 2\alpha \cos 2\alpha), \\ \widehat{y}y &= -\frac{P}{4} [y / \sin \alpha + (3 \cos^2 \alpha - 1)x / \cos^3 \alpha] - \frac{\rho g}{2} [\cos \beta \tan^2 \alpha x + \sin \beta \cdot y] \\ &\quad - \frac{Pb}{2} [\sin 2\alpha - 2\alpha \cos 2\alpha - 2 \cos 2\alpha(\theta - xy/r^2)]/(\sin 2\alpha - 2\alpha \cos 2\alpha) \\ &\quad - 2Xxy^2/r^4(2\alpha + \sin 2\alpha) - 2Yy^3/r^4(2\alpha - \sin 2\alpha) \\ &\quad + 2M[\cos 2\alpha xy(x^2 + y^2) + 3xy^3 - x^3y]/r^6(\sin 2\alpha - 2\alpha \cos 2\alpha), \\ \widehat{x}y &= \frac{P}{4} [x / \sin \alpha + y / \cos \alpha] - \frac{\rho g}{2} [x \sin \beta + y \cos \beta] \\ &\quad + \frac{Pb}{2} [\cos 2\alpha(x^2 - y^2)/r^2 - 1]/(\sin 2\alpha - 2\alpha \cos 2\alpha) \\ &\quad - 2Xx^2y/r^4(2\alpha + \sin 2\alpha) - 2Yxy^2/r^4(2\alpha - \sin 2\alpha) \\ &\quad + M[\cos 2\alpha(x^4 - y^4) + 6x^2y^2 - x^4 - y^4]/r^6(\sin 2\alpha - 2\alpha \cos 2\alpha). \end{aligned} \right\} . \quad (34)$$

(12) It has been recently shown * that the voids in various building-stones and cements form a considerable percentage of the total volume in many cases. In view of this it has been thought desirable to consider the effect of porosity on the stresses in a masonry dam. To deal with the question in any form it will be necessary to make some assumption as to the pressure of the water in the pores. It will be assumed in what follows that this pressure falls off uniformly along any plane $x = \text{constant}$, from full water pressure at front face to zero at free face, the axis of x being taken in the central plane of the dam.

A point of importance in dealing with the question is that of the relation between volume porosity and sectional porosity. By sectional porosity is meant the ratio which the area of air-space cut through in any section bears to the total area of the section. Under certain circumstances volume porosity and sectional porosity are equal, but in what follows only sectional porosity will be used. It will be assumed that the pores are so small as to cause no disturbance of stress at their boundaries.

Let p = pressure of water in the pores,

m = area of solid per unit of area of any section,

\bar{m} = area of pores;

then $m + \bar{m}$ = unity.

The above assumption as to p gives

$$p = Pr \sin(\alpha + \theta) / \sin 2\alpha = wr \sin(\alpha + \theta), \text{ say,}$$

where Pr = pressure of water on face $\theta = \alpha$,

or

$$P = \bar{\rho}g \cos(\alpha - \beta),$$

where $\bar{\rho}$ = density of water.

If $\widehat{r\sigma}$, $\widehat{\theta\sigma}$, and $\widehat{r\theta\sigma}$ denote the actual stresses in the elastic material of the dam (not the stresses exerted across unit area of any section at the point), then the stress equations of equilibrium are

$$\frac{\partial}{\partial r} \widehat{r\sigma} + \frac{1}{r} \frac{\partial}{\partial \theta} \widehat{r\theta\sigma} + \frac{\widehat{r\theta} - \widehat{\theta\theta}}{r} - \frac{\bar{m}}{m} w \sin(\alpha + \theta) + \frac{\rho g}{m} \cos(\beta - \theta) = 0,$$

$$\frac{\partial}{\partial r} \widehat{\theta\sigma} + \frac{1}{r} \frac{\partial}{\partial \theta} \widehat{\theta\theta\sigma} + 2 \frac{\widehat{r\theta}}{r} - \frac{\bar{m}}{m} w \cos(\alpha + \theta) + \frac{\rho g}{m} \sin(\beta - \theta) = 0.$$

If we make the same assumption as in equations (22), then

$$A' = B' = \frac{\bar{m}}{m} wr \sin(\alpha + \theta) - \frac{\rho}{m} gr \cos(\beta - \theta),$$

* Messrs Baldwin-Wiseman and Griffith, *Proc. Inst. Civil Eng.*, vol. clxxix. p. 306.

and the stresses are

$$\left. \begin{aligned} \widehat{rr} &= Pr[(\cos^2\alpha - \sin^2\theta) \sin \theta - \tan^3\alpha(\cos^2\theta - \sin^2\alpha) \cos \theta]/4m \sin^3\alpha \\ &\quad - \rho gr[s(\cos^2\alpha - \sin^2\theta) \sin \theta - c(\cos^2\theta - \sin^2\alpha) \cos \theta + 2 \cos(\beta - \theta)]/2m \\ &\quad + \bar{m}Pr \sin(\alpha + \theta)/m \sin 2\alpha, \\ \widehat{\theta\theta} &= -Pr[(3 \sin^2\alpha - \sin^2\theta) \sin \theta + \tan^3\alpha(3 \cos^2\alpha - \cos^2\theta) \cos \theta]/4m \sin^3\alpha \\ &\quad + \rho gr[s(3 \sin^2\alpha - \sin^2\theta) \sin \theta + c(3 \cos^2\alpha - \cos^2\theta) \cos \theta - 2 \cos(\beta - \theta)]/2m \\ &\quad + \bar{m}Pr \sin(\alpha + \theta)/m \sin 2\alpha, \\ \widehat{r\theta} &= Pr[(\sin^2\alpha - \sin^2\theta) \cos \theta + \tan^3\alpha(\cos^2\theta - \cos^2\alpha) \sin \theta]/4m \sin^3\alpha \\ &\quad - \rho gr[s(\sin^2\alpha - \sin^2\theta) \cos \theta + c(\cos^2\theta - \cos^2\alpha) \sin \theta]/2m, \end{aligned} \right\} \quad (35)$$

where

$$s = \sin \beta / \sin^2\alpha, \quad c = \cos \beta / \cos^2\alpha.$$

The stresses exerted across unit area of any section are

$$\left. \begin{aligned} \widehat{rr} &= m \widehat{rr} - \bar{m} p, \\ \widehat{\theta\theta} &= m \widehat{\theta\theta} - \bar{m} p, \\ \widehat{r\theta} &= m \widehat{r\theta}. \end{aligned} \right\} \quad \quad (36)$$

and These give exactly the same resultant stresses across any plane as in the case of a non-porous dam. The interpretation of this result appears to be clear.

(13) If it is assumed that the front face is vertical, or $\beta = \alpha$, the conditions that \widehat{rr} should vanish at the front face when the dam is full of water are as follows:—

$$\text{For a porous dam} \quad . . . \quad \cot^2 2\alpha = \rho/\bar{\rho} - \bar{m}.$$

$$\text{For a non-porous dam} \quad . . . \quad \cot^2 2\alpha = \rho/\bar{\rho}.$$

It should perhaps be remarked that the value of ρ in the porous dam would in general be different from ρ for the non-porous dam.

(14) Displacements.

The movements for a wedge of isotropic material acted on by forces have been calculated for three cases:—

Case I. Bending due to pressure $Pr/2$ on $\theta = \alpha$ and tension $Pr/2$ on $\theta = -\alpha$.

Case II. Compression due to pressure $Pr/2$ on both faces.

Case III. Displacements due to the weight of the wedge.

The displacements are in each case calculated on the assumption that at $x=h$ and $y=0$ the wedge is held fixed, so that

$$u = v = du/dy = 0.$$

In Case I.

$$\left. \begin{aligned} 2\mu u &= -\frac{P}{2m} \left[c_1 + \frac{\lambda + 2\mu}{2(\lambda + \mu)} (c_2 - c_1) \right] (h - x) y, \\ 2\mu v &= \frac{P}{4m} \left[\frac{\lambda + 2\mu}{2(\lambda + \mu)} (c_2 - c_1) (x^2 + h^2 - y^2 - 2hx) + c_2 y^2 + c_1 (3h^2 - 2hx - x^2) \right], \end{aligned} \right\} \quad (37)$$

where

$$m = 8 \cos \alpha \sin^3 \alpha, \quad c_1 = 2 \sin \alpha \sin 2\alpha, \quad c_2 = \cos \alpha + 3 \cos 3\alpha.$$

In Case II.

$$\left. \begin{aligned} 2\mu u &= - \frac{P}{4m_1} \left[\frac{\lambda + 2\mu}{2(\lambda + \mu)} (s_1 + s_2)(h^2 - x^2 + y^2) - s_1(h^2 - x^2) + s_2 y^2 \right], \\ 2\mu v &= \frac{P}{2m_1} \left[\frac{\lambda + 2\mu}{2(\lambda + \mu)} (s_1 + s_2) - s_2 \right] xy, \end{aligned} \right\} \quad (38)$$

where

$$m_1 = -8 \sin \alpha \cos^3 \alpha, \quad s_1 = 3 \sin 3\alpha - \sin \alpha, \quad s_2 = \sin \alpha + \sin 3\alpha.$$

If the results of the above two cases are added, the displacements due to pressure P on $\theta = \alpha$ will be obtained.

When $\lambda = \mu$ the results are as follows:—

$$\left. \begin{aligned} 2\mu u &= - \frac{P}{2m} (\cos \alpha + 2 \cos 3\alpha)(h - x)y - \frac{P}{4m_1} [\sin \alpha (h^2 - x^2 + y^2) + 4 \sin 3\alpha y^2], \\ 2\mu v &= - \frac{P}{4m} [\cos \alpha (3h^2 - 2hx - x^2 + y^2) - 4 \cos 3\alpha (h - x)x] \\ &\quad + \frac{P}{2m_1} (2 \sin 3\alpha - \sin \alpha) xy. \end{aligned} \right\} \quad (39)$$

In Case III.

$$\left. \begin{aligned} u &= \frac{\rho g}{4\mu} \left[\left(\cos^2 \alpha - \frac{\lambda}{2(\lambda + \mu)} \right) \left(\frac{\cos \beta}{\cos^2 \alpha} \frac{h^2 - x^2}{2} + \frac{\sin \beta}{\sin^2 \alpha} (h - x)y \right) \right. \\ &\quad \left. + \left(\frac{\lambda + 2\mu}{2(\lambda + \mu)} - 3 \cos^2 \alpha \right) \frac{\cos \beta}{\cos^2 \alpha} \frac{y^2}{2}, \right] \\ v &= \frac{\rho g}{4\mu} \left[\left(\frac{\lambda}{2(\lambda + \mu)} - \sin^2 \alpha \right) \left(\frac{\cos \beta}{\cos^2 \alpha} xy + \frac{\sin \beta}{\sin^2 \alpha} \frac{y^2}{2} \right) \right. \\ &\quad \left. - \left(\frac{\lambda + 2\mu}{2(\lambda + \mu)} - 3 \sin^2 \alpha \right) \frac{\sin \beta}{\sin^2 \alpha} \frac{h^2 - x^2}{2} + \left(\cos^2 \alpha - \frac{\lambda}{2(\lambda + \mu)} \right) \frac{\sin \beta}{\sin^2 \alpha} (h - x)h \right]. \end{aligned} \right\} \quad (40)$$

(15) It is of considerable interest in the present case to have some definite notion as to the extent of the motion at any point of the base. It is at once obvious that, as the base forms a considerable portion of the whole boundary, any serious movements of the base would somewhat discount the value of the foregoing results in their application to masonry dams resting on rock. The movements for the case of a wedge of $2\alpha = 36^\circ$, having the front face vertical, have been calculated for the extreme points of the base, which have been taken to be $x = 100$ feet and $y = \pm 32.49$ feet, the dam being thus 105.15 feet high.

The movements have been calculated for the three cases already stated in § (14).

	At Front Face.		At Back Face.	
	u (in feet).	v (in feet).	u (in feet).	v (in feet).
Case I.	0·000,0000	- 0·000,2425	0·000,0000	- 0·000,2425
" II.	+ 0·000,0946	- 0·000,2170	+ 0·000,0946	+ 0·000,2170
" III.	- 0·000,2719	+ 0·000,1976	- 0·000,2719	- 0·000,0695
Net movement	- 0·000,1763	- 0·000,2619	- 0·000,1763	- 0·000,0550

The movements at the axis of z^* due to the same forces are as follows:—

$$\begin{aligned} \text{Case I. } u &= 0\cdot000,0000 \text{ feet, } v = - 0\cdot006,895 \text{ feet.} \\ \text{" II. } u &= + 0\cdot000,1577 \text{ " } v = 0\cdot000,000 \text{ " } \\ \text{" III. } u &= + 0\cdot000,8586 \text{ " } v = + 0\cdot001,294 \text{ " } \end{aligned}$$

The data used in the calculation of the above were

$$\rho g/2\cdot3 = P = 62\cdot5 \text{ lbs. weight.}$$

and

$$\lambda = \mu = 2E/5 = 1,000,000 \text{ lbs. weight per square inch.}$$

(16) The complete solution of the problem of the dam would require the determination of the stresses not only in the dam itself but also in the foundation, with the necessary condition of continuity of displacement throughout the whole mass, and for the simplest form of dam would require the satisfaction of the boundary conditions along four separate boundaries, and to obtain this seems practically hopeless.

The calculation of the stresses by the great writers Messrs Graeff and Delocre,[†] Rankine,[‡] Bouvier,[§] etc., has been worked out in the case of practical masonry dams on the well-known theory of the line of resistance, or what may be called the point load and bending moment method, and it has usually been assumed that at least the normal stresses on any section (generally taken horizontal) vary uniformly. (At the same time, it should be stated that some at least of the above writers were well aware that uniform variation of stress might not have been true.) Further, it has generally been taken for granted, tacitly or otherwise, that what applies to a section above the base also applies at the base.

The usual methods of calculation of stresses, while giving the total shear on a section, do not indicate how it is distributed, and this has led to

* For actual movements in the Vyrnwy dam, see *Proc. Inst. Civil Eng.*, vol. cxv. p. 117.

† *Annales des ponts et chaussées*, 1866, pp. 186, 212.

‡ Rankine's *Miscellaneous Scientific Papers*, p. 550.

§ *Annales des ponts et chaussées*, vol. x., 1875.

almost endless discussion amongst engineers, and many theories of stress, generally different from the elastic theory, have been propounded. A point which has caused a great deal of recent discussion is the distribution of stress on the base of the dam.

(17) Messrs Pearson and Atcherley* have obtained the stresses in a dam by assuming the normal pressure to be linear with a parabolic distribution of shear vanishing at front and flank on a horizontal basal section—*i.e.* such a distribution as obtains in a transverse section of a beam subjected to a bending moment and thrust. They have also taken a linear uniform distribution of shear as another case. They have shown that the former assumption leads to large tension on vertical sections near the tip of the tail in a certain form of dam examined, and the latter to sensible tension; but, as the writer interprets their results, the solution given does not satisfy the conditions at either the front or flank of the dam.

The late Sir Benjamin Baker, in discussing† the stresses in a masonry dam, has thrown doubt on the existence of such large tensions as were found by Messrs Pearson and Atcherley. He refers to the great difficulty of making proper assumptions as to the stresses on the base, owing to the many disturbing influences, such as temperature and workmanship. The remarks‡ of Dr Brightmore, M.Inst.C.E., on the question of large tensions on vertical sections near the tip of the tail of a dam seem to the writer very much to the point. He considers that the parabolic distribution of shear at the base is not justified, at least for dams with a sloping back, and he puts forward the plea that the old assumptions should not be abandoned until it has been shown that some weakness existed in designs which had been elaborated on their basis.

(18) Considerable light is thrown on the question of shear by considering the stress functions of the present paper. For example, on the plane $x = \text{const.}$ the stress function of—

$$\text{Equation (9)} \text{ gives } \widehat{x}y = -2Py^6/x^3r^4m.$$

$$\text{,, (13) ,, } \widehat{x}y = P(r^2 - x^2 \cos 2a)/r^2m.$$

$$\text{,, (15) ,, } \widehat{x}y = Px/2 \sin a.$$

The final of these equations gives uniform shear on the section, and it may easily be shown that if the loading is still further concentrated towards the section, up to a limit which depends on a , the shear still remains nearly uniform. Again, if the section $x = \text{const.}$ is taken perpendicular to

* *Some Disregarded Points in the Stability of Masonry Dams*, London, 1904 (Dulau & Co.).

† *Proc. Inst. Civil Eng.*, vol. clxii. p. 120.

‡ *Ibid.*, p. 112.

the loaded face $\theta = a$, and there is a total load P between $x=0$ and $x=a$, the face $\theta = -a$ being free, then on the section $x=a$

$$\begin{aligned} \text{For uniform loading . . . } & \widehat{x\dot{y}} = -Py^2/r^2a(\tan 2a - 2a), \\ \text{For load varying as } x . . . & \widehat{x\dot{y}} = -2Py/a^2 \tan^2 a, \end{aligned}$$

and here again concentration of the load to the section tends to produce uniformity of shear, although at $y=0$ the shear must vanish in every case. The above examples serve to show that the manner of loading has an all-important bearing on the question of shear, whatever modifying influence the foundations may produce near the base in the case of an actual dam, and the same remark applies to normal pressure.

(19) The question still remains what are the stresses at the base of the dam, and to this the writer can give no real answer; and this raises the further question as to whether the movements at the base, as found in this paper, are inconsistent with what may obtain in the case of an actual dam resting on a good sound rock foundation. According to the elastic theory the stresses will be accurate only if the movements at the base are consistent with those found by equations (39) and (40).

Suppose, however, the complete solution of the dam were obtained in the general case; the question of stresses due to shrinkage of the material in actual dams while setting and drying, and the stresses due to changes of temperature, still remain untouched, and it seems clear that the stresses set up by either of these causes must attain their maximum value at the base, where the abrupt change of section takes place. It is thought the stresses produced by either of the above causes may far outweigh those due to the super-rigidity or the lack of rigidity in a good sound rock foundation, and this contention seems to be accentuated when it is observed that the movements at the extreme points of the base are less than those which would be produced by a change of temperature of 2° F. in a wedge of concrete. In view of the elastic movements being small in comparison with the probable temperature movements, it is held that the stresses as found will in all probability be reasonably correct over a considerable period of the year, depending on the depth of the line of contact of the foundation below the surface of the ground and the range of temperature at that depth.

(Issued separately September 5, 1913.)

XXVII.—On some Pseudo-hermaphrodite Examples of *Daphnia pulex*.

By J. H. Ashworth, D.Sc., Lecturer in Invertebrate Zoology in the University of Edinburgh.

(MS. received July 7, 1913 Read July 7, 1913.)

AMONG about a hundred specimens of *Daphnia pulex* (De Geer) obtained for class purposes, at the end of May, 1912, I found two females * in each of which the antennule of one side resembled that of a male. These cases seem worthy of notice in view of the paucity of existing records of similar abnormalities in *Daphnia*.

Previous Records of Abnormalities of the Antennules in Cladocera.

Kurz † found, in October, 1873, an example of *D. pulex* in which the right antennule was short, as in the female, while the left approached in form that of the male, though it was not quite as large as the latter. The first trunk-limb on the right side was like that of the female, while the left one possessed a prehensile hook and flagellum, as in the male. The right gonad was an ovary, the left a testis. The head, carapace, and abdomen more nearly resembled those of a female, except that the anterior margin of the carapace was not evenly rounded but humped—not, however, as strongly as in the male,—and fringed with long hairs. Early in August, 1873, Kurz observed a specimen of *D. schaefferi* Baird (a variety of *D. magna* Straus), which presented a right antennule like that of the female; but the left, though similar in form to that of the male, was considerably less than the latter, and lacked the seta present on the anterior margin of the antennule in males. The trunk-limbs, the abdomen, and the carapace were of the female type. The reproductive organs of this specimen were not examined.

The third abnormal Cladoceran recorded by Kurz was an example of *Alona quadrangularis* (O. F. Müller), taken on November 1, 1873, which had the general appearance of a male. The right and left antennules were equally long (as long as the rostrum) and provided with olfactory filaments, but the left one bore also a flagellum, like that of the male. The right first trunk-limb was of the form usual in the female; the left one was hook-like

* The entire collection was carefully examined; males were not present.

† "Über androgyne Missbildung bei Cladoceren," *Sitzb. K. Akad. Wiss. Wien, Math.-naturw. Klasse*, Bd. lxix., i Abt. (1874), pp. 40-46.

but much smaller than the corresponding appendage of an adult male. The right gonad was an ovary and contained nearly ripe eggs, the left one was a functional testis. A second specimen of the same species, found in May, 1873, had the male "habitus," but both its antennules and the abdomen were of the female type. The right first trunk-limb was similar to that of the male, and on this side a vas deferens was present; the left side of the specimen and the reproductive organs were not examined.

Of the four specimens recorded by Kurz, two were stated by him to be true hermaphrodites, each having an ovary and a testis; the other two were examined only externally, so that whether they were true hermaphrodites or only pseudo-hermaphrodites was not determined. Kurz put forward a curious explanation of the occurrence of these abnormal examples. He stated that they occurred about the time when the first males began to appear, but while they were still rare, and he regarded these abnormal specimens as representing forms transitional between females and males.

Grochowski * has recorded an example of *Leptodora hyalina* Lilljeborg (= *L. kindtii* (Focke)), in which one antennule was short, as in the female, and the other long, as in the male, and a female specimen of the Chydorid *Rhypophilus personatus* Schödler (= *Pleuroxus uncinatus* Baird), in which the rostrum was said to be of the form usual in males. Both these specimens, regarding the gonads of which no information was given, were found among batches of females of their respective species collected towards the end of summer.

Woltereck † noticed, among the structural irregularities in some of his specimens of *Daphnia*, the occurrence of "Zwitterbildung," but has not described these hermaphrodite examples.

Kuttner ‡ has recorded observations on a series of abnormal *Daphnia pulex*, which appeared in her extensive culture-material. The starting-point of the series was a female which differed from the normal only in having rather longer antennules and a shorter rostrum. Among its first brood appeared a female in which the antennules were as long, and exhibited the same structure, as those of a male. § The second to the twelfth broods (except the fifth, which died before it was examined) were

* "Über Hermaphroditismus bei Cladoceren," *Kosmos*, Bd. xxi. (1896), pp. 301-310. Abstract in *Zool. Centralbl.*, iv Jahrg. (1897), p. 414.

† "Über natürliche und künstliche Varietätenbildung bei Daphniden," *Verhandl. D. Zool. Ges.*, 1908, p. 239.

‡ "Unters. über Fortpflanzungsverhältnisse u. Vererbung bei Cladoceren," *Internat. Revue Hydrobiol.*, Bd. ii. (1909), pp. 658-667.

§ Except that, judging from the figure (*op. cit.*, fig. 1), a seta was not present on the anterior margin of the antennule.

inspected, and in all except one some abnormal examples were found. In some specimens the only abnormal feature was a slight enlargement of one or both antennules; in others, one or both antennules were larger but still of the female form, and the first trunk-limbs were either normal (*i.e.* as in the female) or exhibited a small hook; in eleven other cases one or both antennules resembled those of the male, and in five of these the first trunk-limb of one or both sides possessed a hook. Accurate records were kept of the sixth to the twelfth broods, comprising altogether 104 specimens. Of these 79 were normal females, 1 a normal male, and 24 exhibited one or other of the abnormal features above mentioned. The third generation presented abnormalities; among the 350 specimens comprising this generation there were 253 normal females, 85 pseudo-hermaphrodite females, 3 true hermaphrodites, 3 pseudo-hermaphrodite males, and 6 normal males. In the fourth generation, 17 pseudo-hermaphrodites occurred among 80 individuals. Kuttner pointed out that the proportion of normal females to specimens exhibiting male characters was nearly 3:1, and, while not interpreting this as an indication of the operation of inheritance on Mendelian lines, she remarked that the constant proportions indicated that the explanation of their occurrence was to be sought in the domain of "Vererbungsmechanik," but she concluded that the problem was of so complex a character that its solution was at present impossible.

Description of Abnormal Specimens of Daphnia pulex.

I. The first specimen now to be described is remarkable for the form of its left antennule. This appendage (figs. 1 and 2) is well developed, and has the general structure and proportions of the corresponding antennule of a normal male (fig. 4), and was probably movable. It bears on its slightly concave distal end seven olfactory filaments, and on its anterior margin, a little beyond the middle of its length, carries a single pointed seta, like that present in a corresponding position in the male. This antennule differs in one respect from that of a male, in that instead of having at its distal end a prehensile flagellum about .12 mm. in length, there is a seta only .024 mm. long. This seta is bulbous at its base, and its distal portion is short and finger-like, whereas the corresponding flagellum of the male has a long cylindrical basal portion and a tapering distal portion of about equal length.

The right antennule of this specimen (fig. 3) agrees exactly in form and size with that of a female, and its olfactory filaments exhibit the usual arrangement and structure.

The specimen is undoubtedly a female, for it possesses a spacious brood-pouch, containing eleven parthenogenetic eggs, so far developed that in each the young Daphnia, with several pairs of appendages, is clearly recognisable.

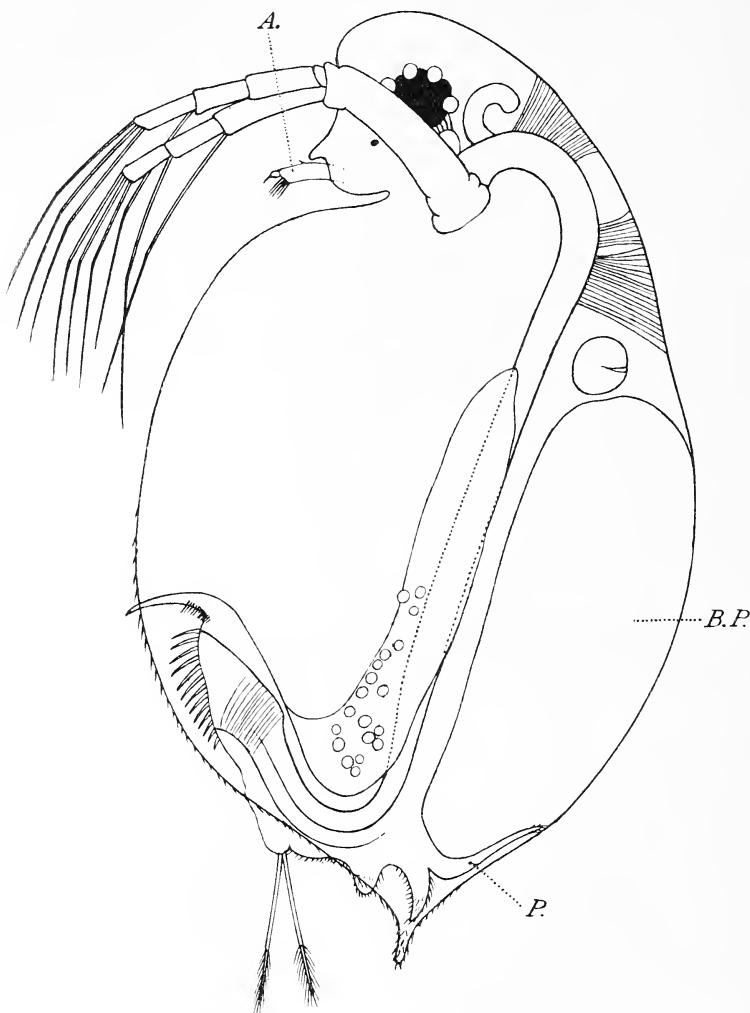


FIG. 1.—Specimen I., from the left side. Note the aberrant left antennule (A.).
B.P., brood-pouch; P., one of the three dorsal processes of the abdomen.
 $\times 60$.

The carapace has the configuration of that of a typical female (see fig. 1); the dorsum is strongly arched over the region of the brood-pouch, and the ventral margin exhibits a regular curvature. It will be noted that there is no trace in this specimen of the projection, or of the plumose hairs, which are present on the anterior margin of the carapace in males.

The rostrum is rather smaller than that of an ordinary female. The first and second trunk-limbs of both sides were dissected out and found to correspond exactly, in form and size, with those of an ordinary female. The processes projecting dorsally from the abdomen agree in general form with those of a normal female, but they are not quite so large.

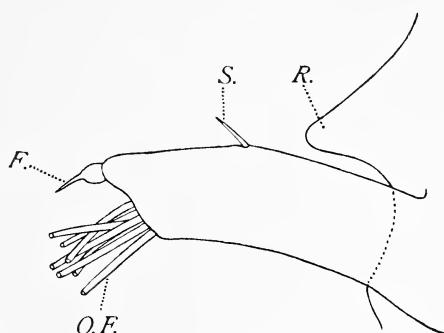


FIG. 2.—The aberrant left antennule of specimen I., left aspect. The rostrum (*R.*) is also indicated.
× 300.

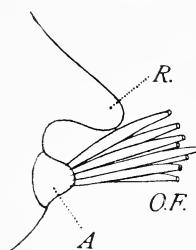


FIG. 3.—The normal right antennule (*A.*) of specimen I., right aspect.
× 300.

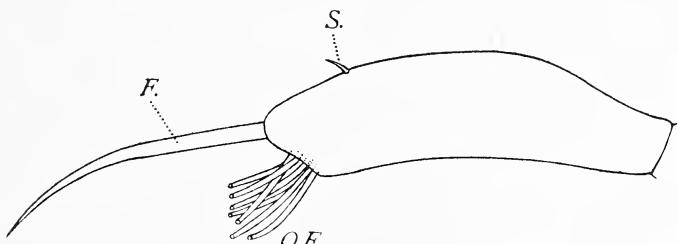


FIG. 4.—Left antennule of a male, left aspect, for comparison with fig. 2. × 300.
A., antennule; *F.*, flagellum of antennule; *O.F.*, olfactory filaments;
R., rostrum; *S.*, seta.

The specimen is 1·95 mm. long, from the front of the head to the base of the spine at the posterior end of the carapace, and the maximum depth of the carapace is 1·25 mm. It is therefore rather a small example of its species.*

The trunk was cut into serial sections, an examination of which shows

* Females, bearing young in the brood-pouch, and collected in the same locality early in June, were found to range in length from 1·9 to 2·5 mm., and most of them were 2·2 to 2·5 mm. long; the average length of ten specimens, taken at random, was 2·3 mm., and the average of their maximum depths 1·4 mm. The average dimensions of six males of the same species were—length, 1·45 mm.; depth, 8 mm.

that the right and left ovaries are equally developed, that a testis is not present, and that the specimen was not parasitised.

Examination of the external and internal features shows that this specimen is a female, which departs from the normal only in the possession of a left antennule similar (except in the length of its flagellum) to that of a male.

II. The second specimen was, unfortunately, dead when it came into my

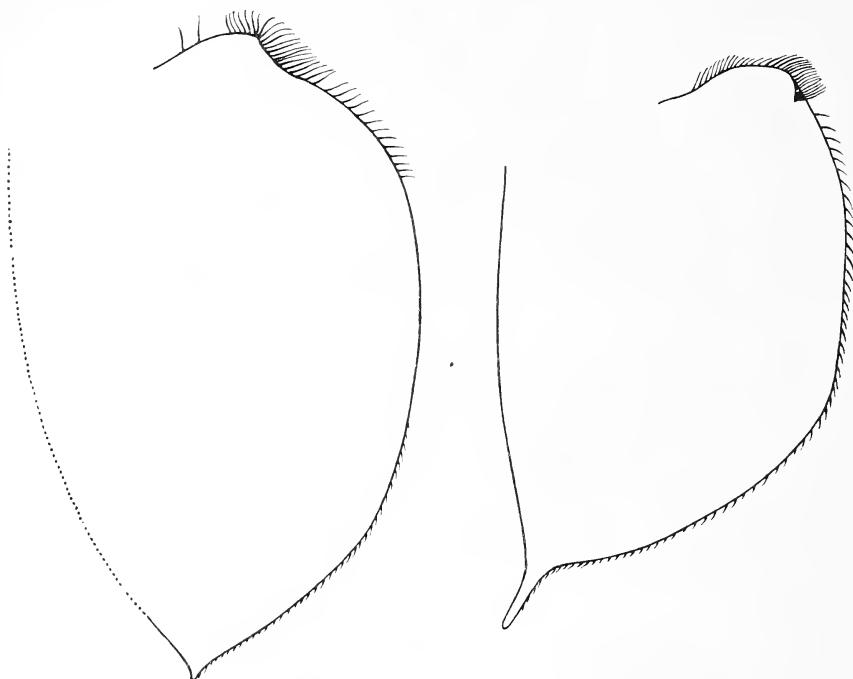


FIG. 5.—Outline of the right valve of the carapace of specimen II. Note the anterior projection bearing hairs. The contour of the dorsal region of the carapace, indicated by the dotted line, was not well preserved. $\times 60$.

FIG. 6.—Outline of the right valve of the carapace of a male, for comparison with fig. 5. $\times 60$.

hands, but, although its soft parts were not in good condition, its appendages were perfect. This example is less than the preceding, its length being only 1·6 mm., and the maximum depth of its carapace about 9 mm. The carapace presents the general contour of that of a female; it is arched dorsally, though not so markedly as in the previous example, and a brood-pouch is present, but it is not large and does not contain any young. The ventral margin of the carapace forms a fairly even convex curve (fig. 5), as in females. (The male carapace (fig. 6) has a more rhomboidal form.)

The anterior margin of the right valve of the carapace is prolonged into a blunt process on which, and on the neighbouring ventral edge of the carapace, a fringe of plumose hairs is present. The process is not as large as that of a male, and its hairs are not so closely set (*cf.* figs. 5 and 6).

The left antennule corresponds exactly with that of a typical female, but on the right side, arising from a deep pit on the side of the rostrum, there is an antennule of very different form (fig. 7). The basal portion of this right antennule is almost pear-shaped, its broad, proximal end is lodged in the pit above mentioned, and the appearances suggest that some movement of the antennule was possible. Attached to the narrow end of the basal segment is a flagellum similar to, but larger than, that of the male antennule (*cf.* figs. 4 and 7). The length of the antennule, from its base to

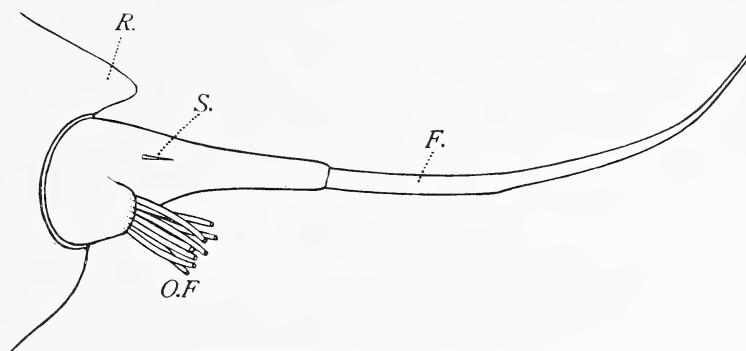


FIG. 7.—The aberrant right antennule of specimen II., right aspect. $\times 300$.
F., flagellum; O.F., olfactory filaments; R., rostrum; S., seta.

the tip of the flagellum, is .33 mm. Near the ventral margin of its right side, this antennule bears a small elevation, from the flat summit of which arise seven olfactory filaments. This elevation is about the same width at its base as the antennule of a female (see fig. 3), but is not quite the same shape.

The first and second trunk-limbs of both sides, when dissected free, prove to be identical in form with those of a normal female. The processes on the dorsal side of the abdomen are imperfectly preserved, but they have the same general form as those of females.

The gonads are ovaries, containing the usual oil-globules.

Analysis of the characters of this specimen shows that, though undoubtedly a female, it exhibits on the right side two of the male secondary sexual characters, viz. (1) the process and hairs on the anterior margin of the carapace, and (2) an antennular flagellum of similar form to, but larger than, that of a male. The basal part of the abnormal antennule does

not, however, agree in form with that of the male of *D. pulex*, but more nearly approaches that of males of the genus *Ceriodaphnia*, e.g. *C. affinis* Lilljeborg.*

When the foregoing account had been completed, I received from Dr W. E. Agar, of the University of Glasgow, the heads of two aberrant specimens of *Daphnia pulex*, which, together with some notes and serial sections of the trunks, he has kindly placed at my disposal.

III. The first of these specimens was one of a brood of fifteen females, the other fourteen of which were normal, examined in May 1910. The right antennule (fig. 8) of this individual approaches in form that of a male. Its basal joint is exactly the same length as in the male, but is

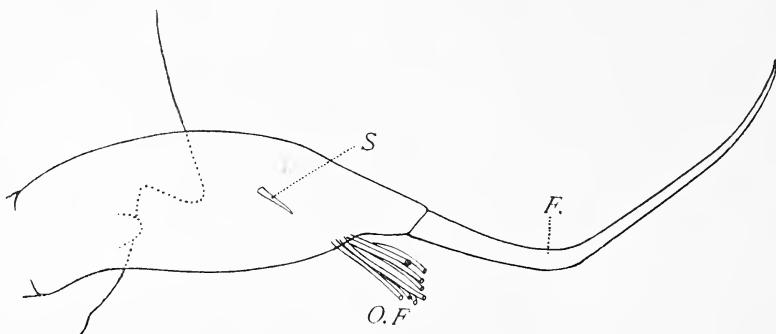


FIG. 8.—The aberrant right antennule of specimen III., right aspect. The outlines of the rostrum and of the left normal antennule, as seen on focussing through the right antennule, are indicated by dotted lines. $\times 300$.

F., flagellum; *O.F.*, olfactory filaments; *S.*, seta.

considerably stouter in its proximal and middle portions, narrowing, however, distally until its diameter is equal to that of the base of the flagellum. The olfactory filaments are of the usual form and number, but the flagellum is considerably larger than that of a normal male. The basal joint of this antennule bears a single seta, similar in form to, but larger than, that present on the antennule of a normal male. The rostrum of this specimen is much smaller than is usual in females, but the remaining characters of the specimen are those of a normal female. Before this *Daphnia* was preserved it produced two broods, which together comprised thirty specimens, all normal females. Dr Agar kept these young individuals until they became mature and produced broods, altogether 281 specimens, of which 275 were normal females and 6 normal males.

* Lilljeborg, W., "Cladocera Sueciae," *Nova Acta R. Soc. Sci. Upsala*, Ser. 3, vol. xix. (1900). See pl. xxviii. figs. 25, 26.

The parent of the abnormal specimen above described came under observation, in one of Dr Agar's cultures, in May, 1910. It was normal, and produced two broods, all the members of which were normal females. The third brood consisted of fifteen females, fourteen normal ones and the specimen just described. It is worthy of record that the fifty-seven young produced by the fourteen sisters of this aberrant specimen were all normal females. Thus, among all the available relatives of the aberrant specimen, including its parent, sisters, nieces, children, and grandchildren, altogether 383 individuals, not one was abnormal.

Investigation of the abnormal specimen proves it to be a female in which only one character—the right antennule—exhibits any departure from the normal. This appendage is clearly of the same type as that of a

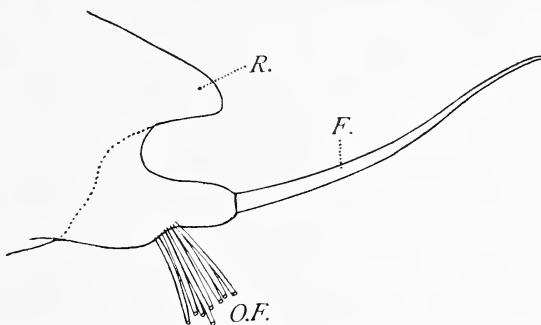


FIG. 9.—The aberrant right antennule of specimen IV., right aspect. $\times 300$.
F., flagellum; O.F., olfactory filaments; S., seta.

male *Daphnia pulex*, but actually surpasses the latter in the grade of development of its parts. All the known relatives (383 were examined) of the specimen were found to be entirely normal.

IV. Dr Agar's second specimen was found among a large number taken near Glasgow on September 12, 1909. All its characters agree with those of a female, except the right antennule (fig. 9), which is much larger than the normal female antennule present on the left side of the same specimen, and bears distally a large flagellum, movable in life, as well as olfactory filaments of the usual form and number. The basal part of this appendage is considerably shorter than that of a male *D. pulex*, is of different shape, and a seta is not present on its anterior margin. The nearest counterpart of this aberrant antennule appears to be the antennule of a female *Ceriodaphnia reticulata* (Jurine), but the former is considerably larger than the latter in all its parts. Dr Agar tells me that the trunk-appendages of both sides were those of a female, and that both ovaries were well developed and

functional. The specimen was isolated, and the two broods of young—each consisting of four females—which it produced, were examined and found to be normal.

It is noteworthy that, of the four abnormal antennules of *Daphnia pulex* recorded in this communication, two (those of specimens II. and IV.) present resemblances to the antennules of species of *Ceriodaphnia*, a fact which suggests the close genetic affinity of this genus with *Daphnia*.

The occurrence in female *Daphnia pulex* of an antennule of male form may be regarded as evidence of the presence in the female of male characters, which, in normal cases, are in a completely latent state, and is not inconsistent with the conception that the female is a heterozygote. These and other cases of pseudo-hermaphroditism in the Crustacea would appear to indicate that the secondary sexual characters are not indissolubly linked with the primary ones. In each of the four cases here recorded it will be noted that the antennule—the first of the series of appendages to appear in development—is the only aberrant appendage; it is evident, therefore, that after the antennule had been formed the normal mode of growth of the appendages was at once restored.* Further, two of the examples (III. and IV.) above described,† the young of which were available for examination, differ from the aberrant forms recorded by Kuttner (see pp. 308, 309) in that they produced normal offspring, the structural peculiarities of the antennule not being transmitted.

* The only other departure from the normal mode of growth was in regard to the antero-ventral edge of the right valve of the carapace in specimen II., which presented almost the configuration of that of a male (see p. 313).

† The young of the first specimen (I.) had been for a considerable time under a cover-glass, in a limited amount of water, before the abnormality of their parent was noticed, and, unfortunately, they proved to be no longer viable.

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OF THE

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SESSION 1912-13.

Part IV.]

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XXVIII.—Atmospheric Electric Potential Results at Edinburgh during 1912. By G. A. Carse, M.A., D.Sc., and G. Shearer, M.A., B.Sc.

(MS. received June 2, 1913. Read November 3, 1913.)

§ 1. In this paper are given the results of calculations made from continuous records of atmospheric electric potential obtained during the year 1912 at the Physical Laboratory of the University of Edinburgh.

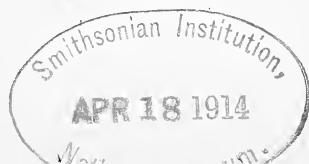
During the summer of 1909 intermittent observations of the atmospheric electric potential and earth-air current were made at various points in Edinburgh, the results of which have already been communicated to this Society,* and since October 1911 there has been an electrograph in operation at the laboratory.

The apparatus used for recording the electric potential of the atmosphere consisted of a water-dropper of the well-known Kelvin type, which was connected to an electrometer of the Dolezalek pattern. As is usual in work of this kind, the water-dropper was connected to the needle of the electrometer, the quadrants being kept at a constant difference of potential by means of cells, the potential of the cells being so adjusted that the instrument could record potential gradients from $+400 \text{ v/m}$ to -200 v/m . The tank containing the water was supported on sulphur blocks, the insulating power of which was tested from time to time and was found satisfactory.

A daily record was made by the trace produced by a beam of light which was reflected from the mirror of the electrometer needle on to a drum carrying bromide paper and revolving at such a rate that one hour was represented on the record by 1.34 cms. approximately. A clock was installed which earthed the tank for five minutes every hour, the clock being adjusted so that it unearthing exactly at the hour. The object of this was to obtain a series of points which gave the reading of the electrometer corresponding to zero potential on the tank, and it also had the additional advantage of dividing the record into hour intervals, thus saving a considerable amount of time in the measurement of the curves. The position of these points showed at a glance whether or not the electrometer was maintaining a constant zero.

At first a little difficulty was experienced in obtaining the zero points in

* Carse and MacOwan, *Proc. Roy. Soc. Edin.*, vol. xxx. pt. vi. p. 460.



a straight line, but after some adjustments of the damping and earthing arrangements, this difficulty was overcome.

The position of the water-dropper is on the south wall of the laboratory, the tube projecting a distance of 6 feet from the wall at a height of 34 feet above the ground-level. As the laboratory itself is 245 feet above sea-level, the height of the collector is 279 feet measured from sea-level.

From October to the end of December 1911 the water-dropper was in continuous operation, but it has been decided not to discuss any of the observations obtained during that period as the details of the apparatus were frequently altered in order to overcome various difficulties. At first it was found difficult to get a point image of light on the drum sufficiently intense to give a good trace and yet not so large as to produce a broad line. An incandescent electric lamp was placed outside the box containing the clockwork and electrometer, and the light from this passed down a tube through a lens and was reflected from the mirror of the electrometer on to the drum. The lamp and lens were adjusted until the image of one of the filaments appeared sharp on the drum, and then this line was cut down to a point by placing immediately in front of the drum a narrow slit. This has been found to work quite satisfactorily since its installation.

A Dolezalek paper needle was found unsuitable for the measurement of high atmospheric potentials. When the potential of the needle reaches a certain limit, the needle moves towards the quadrants and gets discharged. This has already been noticed by G. W. Walker,* who overcame the difficulty by prolonging the vertical axis of the needle and loading it with a small nut. A specially heavy needle, made of aluminium, was used in this work. A platinum wire attached to the vertical axis of the needle dipped in an insulated dashpot containing sulphuric acid, which acted as an efficient means of damping.

Since the beginning of the year 1912 the whole apparatus has worked satisfactorily. There have been occasional breakdowns of insulation, and on several days the water in the tube leading from the tank became frozen. It is comparatively easy to decide by inspection of the record whether the instrument and insulation are working properly or not, and the records of any days on which there was ground for suspecting that something was wrong have been neglected.

§ 2. The electrograph, of course, records merely the actual electric potential at the point where the water breaks up into drops. For purposes of comparison it is desirable that the results should be reduced to ground-

* *Proc. Roy. Soc., lxxxiv. p. 585, 1910-1911.*

level, and instead of discussing the variation of the potential at the point of observation, it is usual to discuss the variation of the ground value of the atmospheric potential gradient.

With a view to finding the factor by which the potential on the tank must be multiplied in order to get the potential at one metre from the ground, observations were frequently taken on the grass in front of the laboratory. In making these observations of the absolute magnitude of the potential gradient, a Wilson* Universal electrometer was used. This instrument measures the charge induced on a plate, and from this the potential gradient in volts per metre can be calculated. The observations were usually made at intervals of a minute for about an hour. These were averaged, and the mean compared with the mean derived from the electrograph record, and thus the factor was obtained which reduced the potential readings at the water-dropper to the corresponding ground value of potential gradient. A flame collector and a Kelvin multicellular electrostatic voltmeter were also used. At first the flame collector was a spirit lamp on a glass stand, while later a Lutz† flame collector with an ebonite stand has been used and has the advantage of being serviceable even in a fairly high wind.

A slightly different method has also been employed to obtain the factor. Simultaneous observations of the potential on the tank and at a known height above the ground have been made in order to see how closely the one curve follows the other. Two observers took simultaneous readings of an Exner electroscope which was connected to the Lutz flame collector placed out in the open and of the Kelvin voltmeter connected to the electrograph tank. In figs. 1A and 1B the values of the readings of the flame collector electroscope and of the voltmeter are plotted against the corresponding times; the first pair of curves was obtained on a day when there was comparatively little wind, although there was considerable variation in the value of the potential during the time of observation, while the second pair was obtained on a day on which there was a strong wind and rapid changes in the value of the potential. A comparison of the curves shows that the variation of potential on the tank follows very closely that given by the flame collector: it will be noticed that the tank shows a tendency to lag behind the flame collector. This lag is chiefly due to the difference in electrical capacity of the two systems and the relative efficiency of a flame and a water-dropper as collectors. The closeness of the agreement between the curves shows that the use of a factor to deduce from the value of the

* *Proc. Camb. Phil. Soc.*, XIII. iv. p. 184, 1905; also XIII. vi. p. 363, 1906.

† *Münch. Ber.*, xxxvi. p. 507, 1906.

potential of the tank the value of the potential gradient at ground-level in the open is justified.

In the matter of obtaining the reduction factor it may be here remarked that in conjunction with the flame collector a Kelvin electrostatic voltmeter supplied by Kelvin & White was used. This voltmeter was specially insulated with ebonite and had a scale with a range from 80 to 300 volts, and has proved very convenient, being very much less trouble to read than an electroscope, while giving results in no way inferior to those obtained by means of an electroscope.

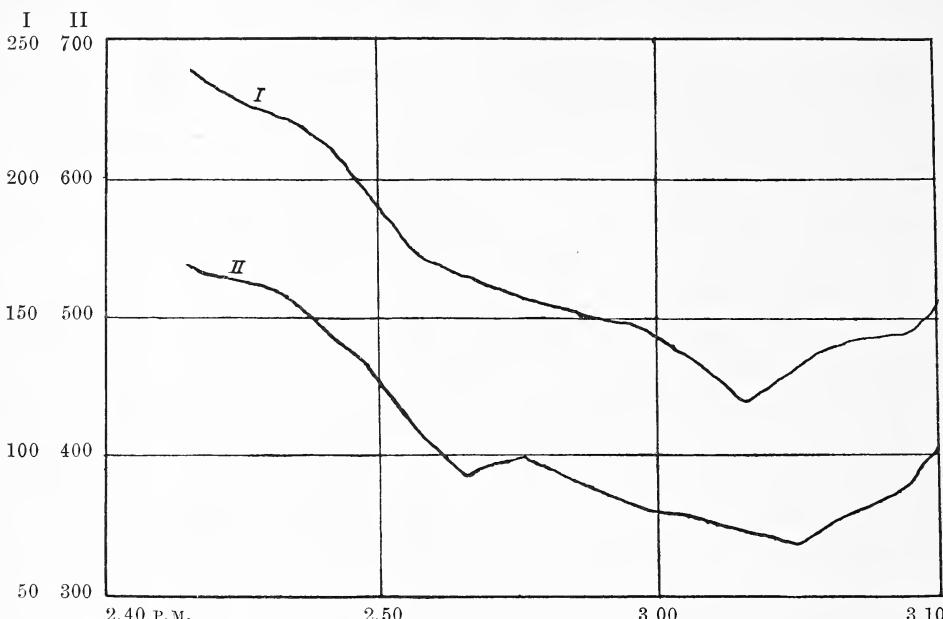


FIG. 1A.—I. Flame Collector on Grass in Front of Laboratory. II. Tank of Electrograph.

§ 3. Instead of adopting the method of measuring ordinates, the curves were divided up into hour intervals; this was easily done as the timing and earthing device mentioned earlier gave a point at each hour on the line of zeros. Ordinates having been drawn at each hour, the area included between the zero line, the ordinates, and the curve was measured with an Amsler's planimeter reading to .05 sq. cm., and thus the mean for the hour was obtained. After measuring a few curves, such facility was attained that the time required to measure by this method was not much longer than that required to measure ordinates, while a more accurate value of the mean is thus obtained.

Owing to errors in measurement of areas, the mean value of the potential gradient for any hour of any one day may be out by about five

volts, so that the mean value for that hour for the month may be in error by about one volt. Owing to difficulties caused by the unevenness of the ground and the proximity of buildings, there may be a fairly large error

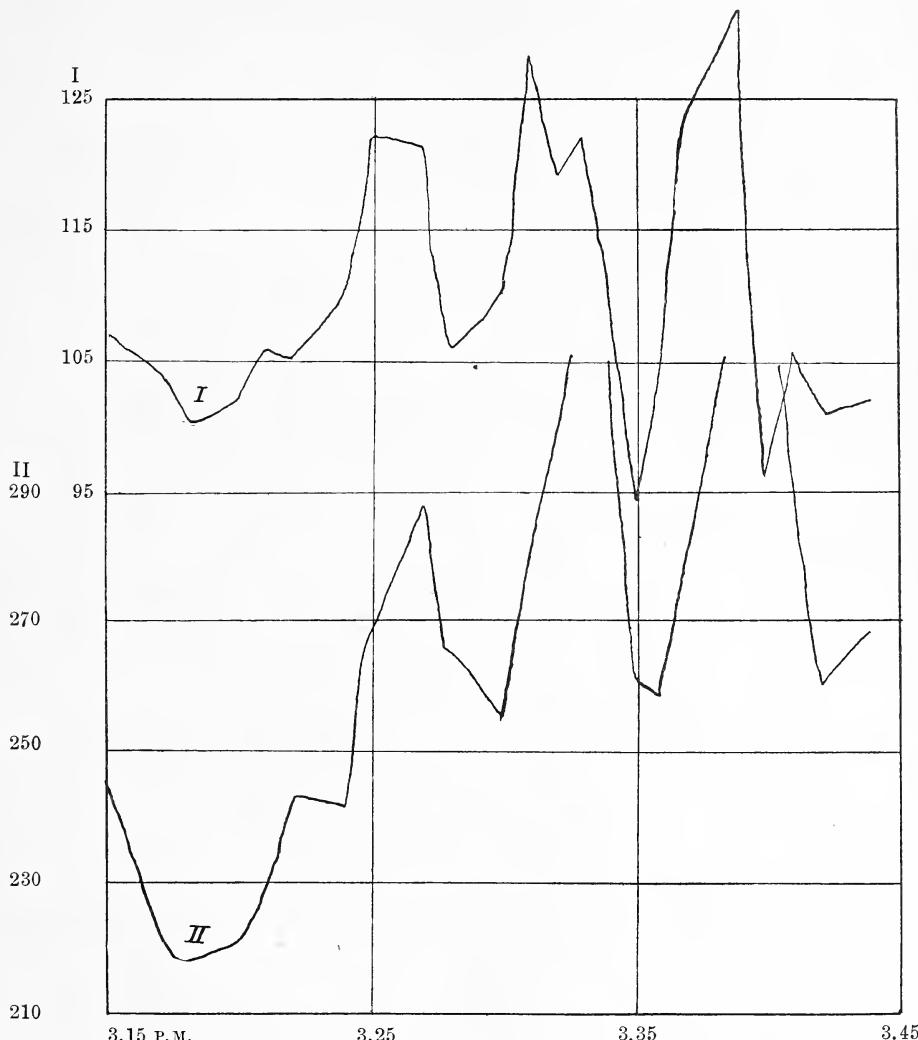


FIG. 1B.—I. Flame Collector on Grass. II. Tank of Electrograph.

in the reduction factor, which, however, does not exceed 10 per cent. Any error in this factor does not affect the relative values of the numbers given for the potential gradient, but affects the absolute values as expressed in volts per metre. From the observations taken outside, a factor was obtained by which these areas had to be multiplied to give the potential gradient in volts per metre.

All curves were measured except those which had to be rejected on account of the freezing of the water-spout or breakdowns of the insulation or of the mechanism of the apparatus. In deducing mean values all the measurements made were used.

§ 4. Table I. gives the mean value of the potential gradient in volts per metre for each hour of the day, for the various months of the year. The last three columns give the diurnal variation for the whole year and for the winter and summer months. By winter is meant the six months October to March, while April to September are included under summer—a division of the year which will be discussed more fully later.

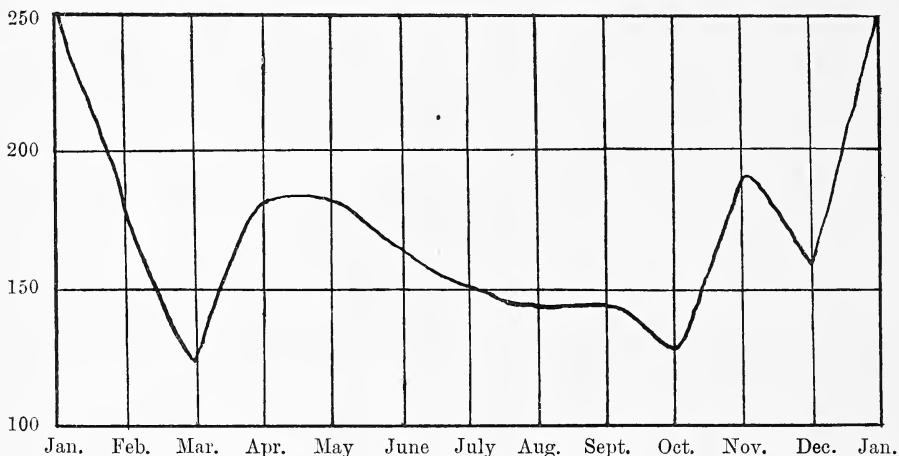


FIG. 2.—Annual Variation.

In the last row of Table I. will be found the mean monthly values of the potential gradient, and fig. 2 shows graphically the variation of the mean monthly value during the year. It will be noticed that there is a well-marked maximum in January, and a minimum during the summer months. Of course it is not to be expected that such a curve deduced from one year's observations would be a smooth one; nevertheless it is clear that the mean value undergoes a periodic change during the year with a minimum in the summer and a maximum in mid-winter. The results obtained above are in agreement with the annual variation got by other observers, *e.g.* at Kew,* Karasjok,† Potsdam,‡ and Kremsmünster,§ etc.

Table II. gives the diurnal inequality for the various months and seasons

* Chree, *Phil. Trans.*, 206, 299, 1907.

† Simpson, *Phil. Trans.*, 205, 61, 1906.

‡ Lüdeling, *Meteor. Zeit.*, 23, 114, 1906.

§ Blumenschein, *Wien. Ber.*, 121, 25, 1912.

TABLE I.

1912.		Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.	Summer.	1912.
a.m.	p.m.														a.m.	
12-1	226	165	106	168	155	138	119	151	137	114	164	142	149	153	145	12-1
1-2	203	144	103	160	149	130	110	115	126	108	164	128	137	142	132	1-2
2-3	173	145	92	140	146	123	99	123	116	101	159	118	128	131	125	2-3
3-4	186	115	97	130	154	129	100	128	119	97	145	106	126	124	127	3-4
4-5	163	113	90	137	154	147	111	115	125	109	160	106	128	124	132	4-5
5-6	174	113	102	160	165	158	110	128	134	102	170	102	135	127	143	5-6
6-7	201	123	142	200	218	190	142	136	140	108	172	141	159	148	171	6-7
7-8	226	135	153	227	233	191	164	191	161	119	179	157	178	162	195	7-8
8-9	250	186	152	223	212	188	183	240	170	120	164	177	189	175	203	8-9
9-10	260	171	161	226	214	189	181	204	167	135	200	182	191	185	197	9-10
10-11	246	214	159	244	205	192	188	122	164	139	212	182	189	192	186	10-11
11-12	300	282	167	176	182	171	155	173	156	137	197	152	187	206	169	11-12
															p.m.	
12-1	317	244	129	151	177	169	162	136	145	134	182	186	178	199	157	12-1
1-2	325	223	119	165	161	163	157	74	133	136	188	194	170	198	142	1-2
2-3	339	210	122	173	176	155	168	143	130	146	208	157	177	197	158	2-3
3-4	328	218	107	182	192	167	163	144	142	133	237	178	183	200	165	3-4
4-5	322	211	128	176	173	161	174	129	147	131	248	174	181	202	160	4-5
5-6	304	233	129	168	180	158	174	118	137	146	232	167	179	202	156	5-6
6-7	288	210	130	181	207	161	157	105	140	146	228	192	179	199	159	6-7
7-8	248	197	129	210	197	191	146	164	153	144	229	211	185	193	177	7-8
8-9	291	186	118	186	176	172	167	182	161	139	228	182	182	191	174	8-9
9-10	286	163	129	198	186	169	151	158	154	136	208	172	176	182	169	9-10
10-11	226	198	109	196	176	161	161	145	146	128	150	166	168	164	164	10-11
11-12	240	192	108	192	170	140	154	135	142	120	178	147	160	164	156	11-12
Mean	255	183	124	182	182	163	150	144	144	126	194	158	167	174	161	Mean

TABLE II.—DIURNAL INEQUALITY.

1912.		1912.												1912.		
	a.m.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.	Summer.	Winter.
12-1	-29	-18	-18	-14	-27	-25	-31	+ 7	- 7	-12	-13	-16	-18	-21	-16	-16
1-2	-52	-39	-21	-22	-33	-40	-40	-29	-18	-18	-30	-30	-30	-32	-29	-1-2
2-3	-82	-38	-32	-42	-36	-40	-51	-21	-28	-25	-35	-40	-39	-43	-36	2-3
3-4	-69	-68	-27	-52	-28	-34	-50	-16	-25	-29	-49	-49	-41	-50	-34	3-4
4-5	-92	-70	-34	-45	-28	-16	-39	-29	-19	-17	-34	-52	-39	-50	-29	4-5
5-6	-81	-70	-22	-22	-17	-5	-40	-16	-10	-24	-24	-56	-32	-47	-18	5-6
6-7	-54	-60	+18	+18	+36	+27	-8	-8	-4	-18	-22	-17	-8	-26	+10	6-7
7-8	-29	-48	+29	+45	+51	+28	+14	+47	+17	-7	-15	-1	+11	-12	+34	7-8
8-9	-5	+3	+28	+41	+30	+25	+33	+96	+26	-6	-30	+19	+22	+1	+42	8-9
9-10	+5	-12	+37	+44	+32	+26	+31	+60	+23	+ 9	+ 6	+24	+24	+11	+36	9-10
10-11	-9	+31	+35	+62	+23	+29	+38	+22	+20	+13	+18	+24	+22	+18	+25	10-11
11-12	+45	+99	+43	-6	0	+ 8	+ 5	+29	+12	+11	+ 3	- 6	+20	+32	+ 8	11-12
p.m.																
12-1	+62	+61	+ 5	-31	-5	+ 6	+12	- 8	+ 1	+ 8	-12	+28	+11	+25	- 4	12-1
1-2	+70	+40	- 5	-17	-21	+ 0	+ 7	-70	-11	+10	- 6	+36	+ 3	+24	-19	1-2
2-3	+84	+27	- 2	- 9	- 6	- 8	+18	- 1	-14	+20	+14	- 1	+10	+23	- 3	2-3
3-4	+73	+35	-17	0	+10	+ 4	+13	0	- 2	+ 7	+43	+20	+16	+26	+ 4	3-4
4-5	+67	+28	+ 4	- 6	- 9	- 2	+24	-15	+ 3	+ 5	+54	+16	+14	+28	- 5	4-5
5-6	+49	+50	+ 5	-14	- 2	- 5	+24	-26	- 7	+20	+38	+ 9	+12	+28	- 5	5-6
6-7	+33	+27	+ 6	- 1	+25	- 2	+ 7	-39	- 4	+20	+34	+34	+12	+25	- 2	6-7
7-8	-7	+14	+ 5	+28	+15	+28	- 4	+20	+ 9	+18	+35	+53	+18	+16	+13	7-8
8-9	+36	+3	- 6	+ 4	- 6	+ 9	+17	+38	+17	+13	+34	+24	+15	+17	+13	8-9
9-10	+31	-20	+ 5	+16	+ 4	+ 6	+ 1	+14	+10	+10	+14	+14	+ 9	+ 8	+ 8	9-10
10-11	-29	+15	-15	+14	- 6	- 2	+11	+ 1	+ 2	+ 2	+ 1	- 8	- 1	- 6	+ 3	10-11
11-12	-15	+ 9	-16	+10	-12	-23	+ 4	- 9	- 2	- 6	-11	-11	- 7	-10	- 5	11-12
Mean .	225	183	124	182	182	163	150	144	144	126	194	158	167	174	161	Mean
Total range of variation	176	169	77	114	87	69	89	166	54	49	103	109	65	78	78	Total range of variation
No. of curves measured.	23	19	30	30	26	31	8	30	29	22	24	302	147	155	No. of curves measured.	

of the year, and also the total range of the variation, while the last row shows the number of curves measured each month. It is to be noted that during the last three weeks of August the apparatus was not in operation owing to the vacation, and the mean values for that month have been deduced from a few days at the beginning of the month, several of which were far from typical of the season of the year. In fig. 3 a graphical representation of this variation is shown for each month; and for the whole year and the two half-years October to March and April to September, in fig. 4.

An examination of these curves shows that they all have a minimum in the early morning about 4 a.m., whereas the other part of the curve varies considerably throughout the year. In mid-winter the curve approximates roughly to a single sine curve with a maximum in the afternoon; as the year advances, the time of occur-

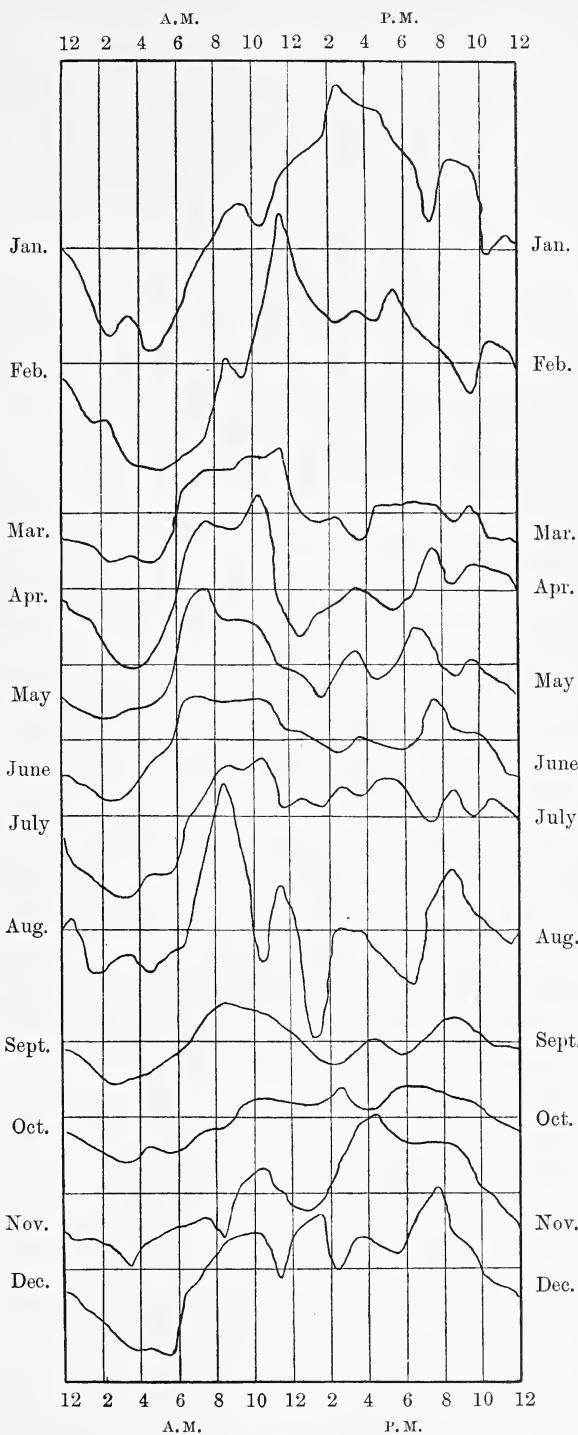


FIG. 3.

rence of this maximum is earlier and a secondary minimum and maximum appear, although these are not nearly so well defined as the first.

The early morning minimum also undergoes a change during the year, occurring somewhat earlier in the summer than in the winter. The curve for the year is in close agreement as far as form is concerned with that obtained at Kew,* Greenwich,† and Kremsmünster,‡ although in the case of Kew the afternoon minimum is much more pronounced. The curves for summer and winter show that there is a decided difference in the type of variation in the months included under these classes. For the months October to March a well-defined minimum occurs at 4 a.m. and a maximum

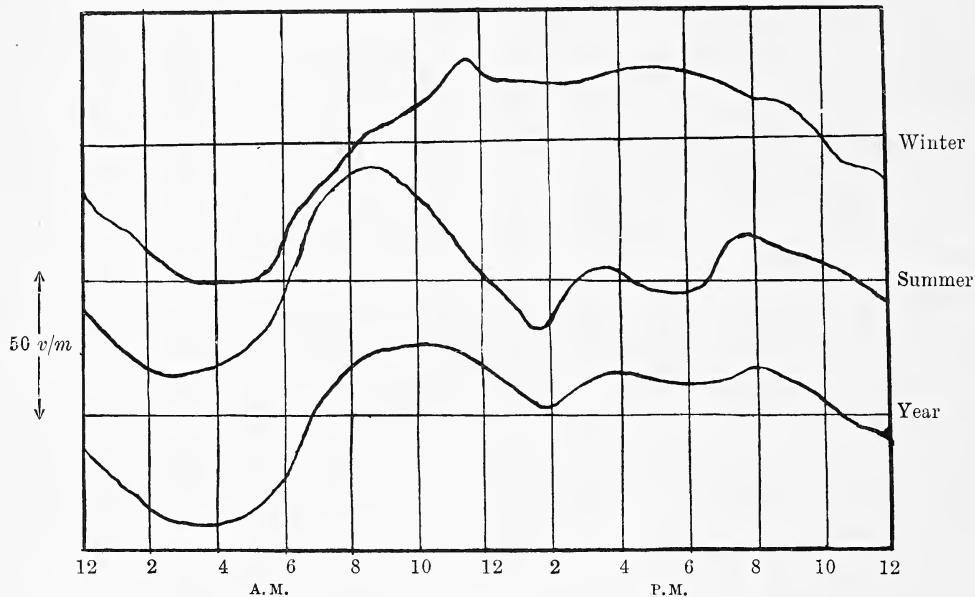


FIG. 4.

in the afternoon. In the summer a well-marked minimum occurs at 3 a.m. and a maximum at 9 a.m., while there seem to be two other maxima and minima much less prominent.

As the Physical Laboratory is situated in the middle of the town, the atmosphere, especially during the day, is likely to be loaded with smoke and dust particles, and it is hardly to be expected that the curves showing the diurnal variation should be as smooth as those obtained at stations where the air is clearer. The disturbance due to soot, etc., would be most marked in the latter part of the day, while the effect would be much diminished in the early morning. It has, however, been observed at most other stations that the smoothest part of the curves occurs in the early

* Loc. cit.

† Greenwich Mag. and Met. Observations.

‡ Loc. cit.

hours of the day, while irregularities appear later, so that it would seem as if the effect of soot, etc., was by no means the sole or even the most important cause of the irregularities.

The daily and yearly variation of the potential gradient at Edinburgh is also shown graphically by fig. 5, in which the isopleths corresponding to potential gradients of 300, 250, 200, 170, 150, 130 volts per metre have been

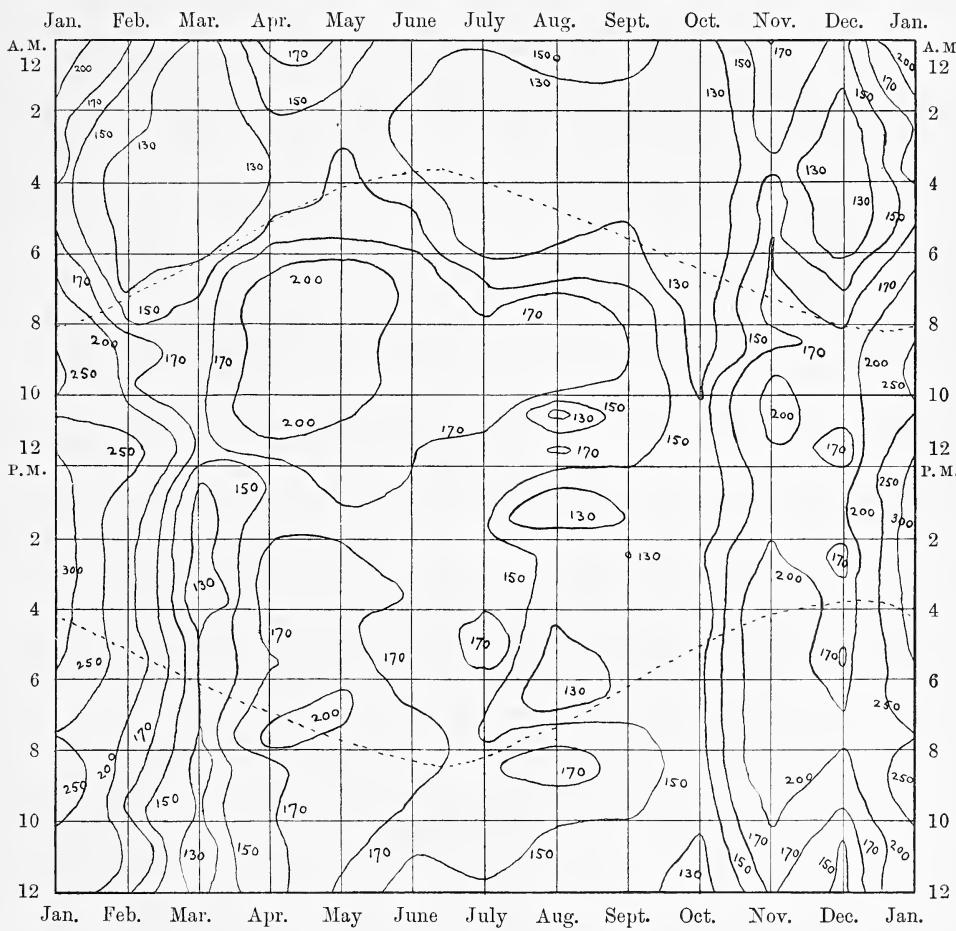


FIG. 5.

drawn. After the examples of Lutz * and Blumenschein, † curves showing the times of sunrise and sunset have been inserted in this diagram; they are indicated by the dotted lines.

§ 5. The curves for the various months and seasons of the year have been analysed into Fourier series of the form—

$$y = a_0 + a_1 \sin(t + a_1) + a_2 \sin(2t + a_2) + \dots$$

* *Münich. Ber.*, p. 305, 1911.

† *Loc. cit.*

by the 24-equidistant ordinate method. The values of the first four amplitudes and phases are given in Table III. In the calculation of these series the time was measured from midnight mean local time, the other times used in the paper being Greenwich mean time. The mean local time at Edinburgh is thirteen minutes behind Greenwich mean time.

TABLE III.

	a_0 .	a_1 .	a_2 .	a_3 .	a_4 .	α_1 .	α_2 .	α_3 .	α_4 .	a_1/a_2 .	a_1/a_0 .	a_2/a_0 .
January .	255	71.3	11.9	10.6	6.1	219.7	102.6	140.8	324.6	5.99	.280	.047
February .	183	52.6	24.2	9.2	12.6	221.9	110.4	356.7	119.6	2.17	.287	.132
March .	124	20.6	19.6	7.1	2.8	275.8	184.9	33.6	78.2	1.05	.166	.158
April .	182	12.0	32.9	15.1	4.6	270.1	187.5	91.0	288.0	0.36	.066	.181
May .	182	15.8	24.6	8.6	3.7	277.1	212.8	96.0	10.2	0.64	.087	.135
June .	163	17.1	21.5	3.8	1.8	270.4	205.3	172.8	354.9	0.80	.105	.132
July .	150	28.1	19.5	11.3	5.2	238.2	182.0	100.0	226.4	1.44	.187	.130
August .	144	15.0	31.5	12.3	17.7	328.9	188.4	86.4	296.1	0.48	.104	.219
September	144	8.4	17.7	3.5	2.2	260.6	184.3	80.2	248.2	0.47	.058	.123
October .	126	20.3	6.8	1.5	0.8	213.3	178.6	343.6	198.5	2.99	.161	.054
November	194	16.3	12.6	4.5	7.3	202.1	237.1	42.4	208.4	1.29	.084	.065
December	158	33.1	19.0	4.4	10.6	217.8	180.2	25.6	357.0	1.74	.209	.120
Year .	167	23.7	17.0	5.5	2.0	233.1	183.0	80.2	301.9	1.39	.142	.102
Winter .	174	37.3	11.3	3.4	1.3	221.7	160.7	42.9	59.8	3.30	.214	.065
Summer .	161	13.8	24.1	8.5	4.5	267.8	193.4	94.9	292.4	0.57	.086	.150

It is interesting to compare the relative amplitudes of the 24-hour and 12-hour waves, and for this purpose the ratio a_1/a_2 has been calculated, and is also given in the table. If a_1/a_2 is greater than unity, the 24-hour term is of a greater importance than the 12-hour term, and a study of the values of a_1/a_2 during the year shows that for the six months October to March the ratio exceeds unity, while for the six months April to September, with the single exception of July, the ratio is less than unity. This implies that at Edinburgh during the winter the 24-hour wave is of most importance, while during the summer the 12-hour wave is predominant. It was this consideration which led to the division of the year into the two portions, October to March and April to September.

A comparison of the values of a_1 and a_2 for the different months shows that a_1 is a maximum in January and decreases to a minimum in September, its range of values being extremely wide—from 8.4 in September to 71.3 in January. On the other hand, a_2 reaches a maximum in April, and a minimum in October. These variations are shown graphically in fig. 6.

As, however, the mean value of the potential gradient for the month varies with the month, it is also advisable to examine the values of the ratios a_1/a_0 and a_2/a_0 , and these are also tabulated in Table III. It will be seen that a_1/a_0 is high in the winter, having a maximum in February, a low value

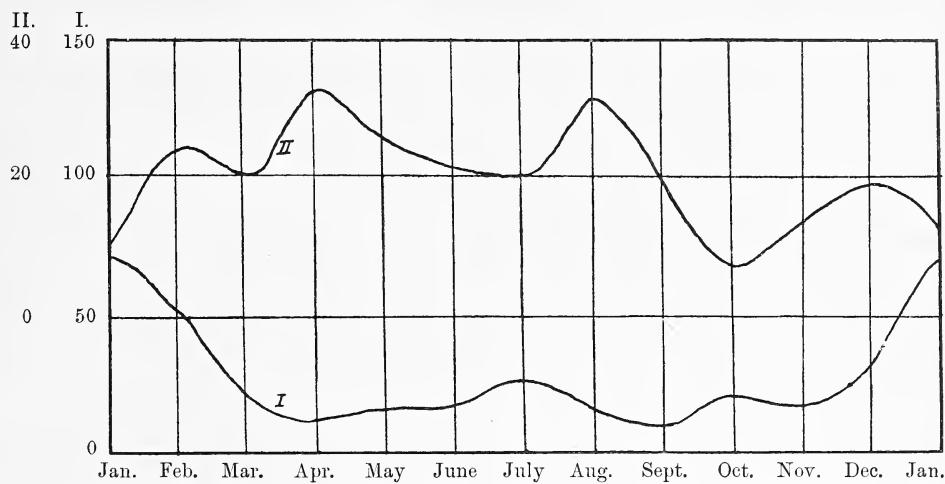


FIG. 6.—I. Annual Variation of a_1 . II. Annual Variation of a_2 .

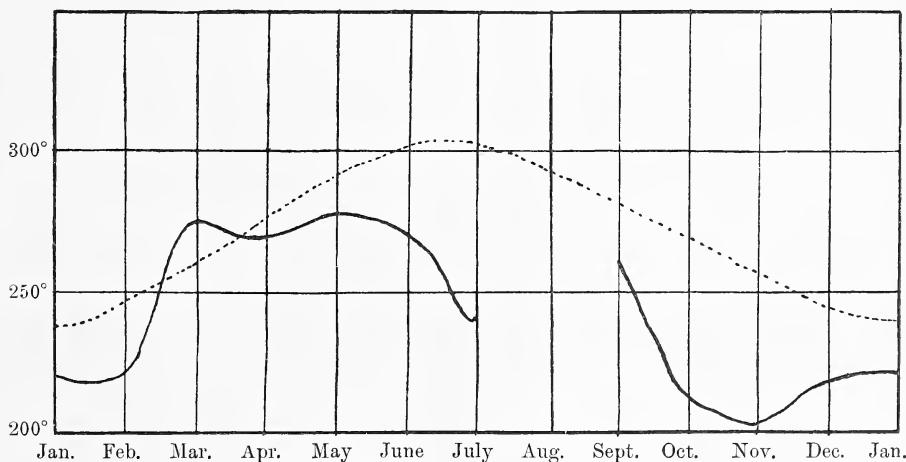


FIG. 7.—Annual Variation of α_1 Time of Sunrise.

in the summer, and a minimum in September, while a_2/a_0 has a maximum in August and a minimum in January.

The annual variation of the phase of the 24-hour wave is shown in fig. 7; it is a minimum in winter and a maximum in summer. The extremely high value of α_1 in August must be regarded as untrustworthy, as the number of curves measured for August was, as mentioned before,

very small. Along with this curve is plotted the time of sunrise for the fifteenth day of each month. It will be seen that if a smooth curve be drawn through the points representing the values of a_1 for the various months, this curve is approximately parallel to that for sunrise, the difference being on an average about 20° – 30° , or about two hours, so that we may say that roughly the first harmonic lags about two hours behind the sun.

The phase of the second harmonic does not seem to undergo any very marked annual variation. Exceptionally low values are obtained in January and February, while the value for November is high compared with the others. For all the other months the value of a_2 varies comparatively little, while the value of a_2 for the year works out at 183° —a value which agrees closely with that obtained at other stations, as will be shown later.

§ 6. The harmonic analysis of the curves gives a more convenient method of comparing results at different stations than merely examining the numbers and the curves themselves.

TABLE IV (a).

Station.	Period.	Year.				Winter.			Summer.		
		a_0 .	a_1/a_0 .	a_2/a_0 .	a_1/a_2 .	a_1/a_0 .	a_2/a_0 .	a_1/a_2 .	a_1/a_0 .	a_2/a_0 .	a_1/a_2 .
Kew . .	1898–1904	159	.053	.156	0·34	.102	.103	0·99	.079	.213	0·37
Paris (B.C.) .	1894–1898220	.104	2·12	.130	.200	0·65
Paris (E.T.) .	1896–1898133	.085	1·56
Karasjok .	1903–1904	139	.281	.165	1·70	.356	.144	2·46	.165	.093	1·78
Potsdam .	1904269	.101	2·66	.096	.152	0·63
Kremsmünster	1902–1910	106	.243	.136	1·79	.298	.128	2·33	.195	.152	1·28
Edinburgh .	1912	167	.142	.102	1·39	.214	.065	3·30	.086	.150	0·57

Tables IV (a) and IV (b) give the values of the different amplitudes and phases obtained at various stations. We adopt the scheme given by Chree,* which is to express the amplitudes as fractions of the mean value of the potential gradient for the corresponding season of the year. In some cases the data are for all days, in others for quiet days, and the terms “winter” and “summer” refer to periods of four or six months. The third column of this table gives the mean values of the potential gradient in volts per metre obtained at the various stations. The mean value for 1912 at Edinburgh is 167 volts per metre.

* Art. “Atmos. Elect.” *Ency. Brit.*, 11th ed., 1910.

The third and fourth harmonics are of comparatively small importance, and it is difficult to say whether they represent real periodic changes in the potential gradient or whether they would get less important if a larger amount of material were available for discussion. Chree * discusses

TABLE IV (b).

Station.	Period.	Year.		Winter.		Summer.	
		$\alpha_1.$	$\alpha_2.$	$\alpha_1.$	$\alpha_2.$	$\alpha_1.$	$\alpha_2.$
Kew . . .	1898-1904	166	188	206	180	87	186
Paris (B.C.) . . .	1894-1898	223	206	95	197
Paris (E.T.) . . .	1896-1898	216	171
Karasjok . . .	1093-1904	177	158	189	155	141	144
Potsdam . . .	1904	194	185	343	185
Kremsmünster . . .	1902-1910	221	188	212	172	231	196
Edinburgh . . .	1912	233	183	222	161	268	193

the question of the importance that should be attached to the 8- and 6-hour terms. He points out that "with the exception of the values of α_4 in winter—when the amplitude at Kew was so small that considerable uncertainty must exist—the Kew and Bureau Central, Paris, values of α_3 and α_4 are sufficiently close to raise a strong presumption that the

TABLE V.

	Year.				Winter.				Summer.			
	$\alpha_1.$	$\alpha_2.$	$\alpha_3.$	$\alpha_4.$	$\alpha_1.$	$\alpha_2.$	$\alpha_3.$	$\alpha_4.$	$\alpha_1.$	$\alpha_2.$	$\alpha_3.$	$\alpha_4.$
Kew . . .	165	186	38	291	206	180	23	239	87	186	116	304
Edinburgh . . .	233	183	80	302	222	161	43	60	268	193	95	292

8- and 6-hour terms represent a true atmospheric electricity effect. The comparative agreement between the phase angles from the earlier and later series of observations at the Bureau Central and the Eiffel Tower favour the same conclusion." The values for α_3 and α_4 obtained at Edinburgh seem to lead to a similar conclusion, as will be seen from Table V., which gives the values of α_1 to α_4 for winter, summer, and the

* *Phil. Trans.*, 206, p. 299, 1906.

year at Kew and Edinburgh. It will be noticed that the agreement between Kew and Edinburgh is very good (again with the exception of α_4 in winter) and is closer than that between Kew and Bureau Central, Paris.

§ 7. The comparison between the values of the potential gradient and its diurnal and seasonal variations obtained at Edinburgh and other stations should give an idea of the effect of town air on these phenomena, since the Physical Laboratory is situated in the centre of Edinburgh and is surrounded by buildings. The above observations seem to show that this effect is smaller than one might expect, as was also indicated by a comparison of electrograph records obtained at the Royal Observatory, Blackford Hill, with those obtained at the Physical Laboratory.* A few isolated observations of the electrical conductivity of the air in the vicinity of the laboratory have shown that this quantity is of the order 5×10^{-5} E.S.U.

The conductivity was measured by means of a Gerdien apparatus, and its value agrees with values obtained by means of a Wilson Universal electrometer at the Physical Laboratory in 1909 (Carse and MacOwan, *loc. cit.*), while Wilson and Gerdien obtained values of the order 5×10^{-4} E.S.U. for country air.

In view of the fact that a high conductivity is generally associated with a low value of the potential gradient and *vice versa*, the comparatively low value of the conductivity at Edinburgh would naturally be accompanied by a high potential gradient. The mean value of the potential gradient for the year (167 volts per metre) is somewhat higher than that obtained at many stations in Germany.

The apparatus used in this investigation was supplied by the Carnegie Trust and the Tait Memorial Fund, and we are indebted to the late Professor J. G. MacGregor, F.R.S., for the kindly interest he took in the work.

* Carse, Shearer, Jameson, *Proc. Roy. Soc. Edin.*, vol. xxxiii. pt. ii. p. 194, 1913.

PHYSICAL LABORATORY,
UNIVERSITY OF EDINBURGH,
June 1913.

(Issued separately December 8, 1913.)

OBITUARY NOTICES.

John M'Arthur, F.C.S.

(Read January 20, 1913.)

JOHN M'ARTHUR, F.C.S., received his chemical training at Anderson's College, Glasgow (now the Royal Technical College), under Dr Wm. Dittimar, F.R.S., to whom he acted for several years as chief assistant. In recognition of their joint researches on the Atomic Weight of Platinum, on the Determination of Potash in Sea Water, and elaborate investigations of samples brought back from the voyages of H.M.S. *Challenger*, he was elected to the Fellowship of the Royal Society of Edinburgh in 1888.

Entering the Battersea works of Price's Patent Candle Co., Ltd., in 1887, under the late John Calderwood, F.R.S., he remained as chief chemist, general technical manager, and technical adviser for over twenty-five years.

Although he had been in less robust health for the past five years, his sudden decease on 19th December 1912 was quite unexpected.

Professor Andrew Jamieson, M.Inst.C.E. By Mr J. Wilson.

(Read January 20, 1913.)

PROFESSOR JAMIESON, M.Inst.C.E., whose death took place on the 4th December 1912, at his residence, 16 Rosslyn Terrace, Kelvinside, Glasgow, W., was well known throughout the country as a consulting engineer and as the author of a large number of standard text-books on electrical and engineering subjects. He was elected to the Fellowship of the Royal Society of Edinburgh in 1882.

Professor Jamieson, who was the eldest son of the late Rev. George Jamieson, D.D., of Old Machar Cathedral, Aberdeen, was born at Grange, Banffshire, in 1849, and received his education at the Gymnasium, Old Aberdeen, and at Aberdeen University. His apprenticeship was served with Messrs Hall, Russel & Co., marine engineers and shipbuilders, Aberdeen Ironwork. He afterwards entered the service of the Great North of Scotland Railway Company, ultimately attaining the position of chief draughtsman in the loco. and carriage department. When only twenty-three years of age he was appointed assistant to Sir William Thomson (Lord Kelvin) and Professor Fleeming Jenkin, who placed him in charge of the testing staff, to supervise the manufacture of the submarine cables at the Woolwich works of Messrs Siemens Bros.

Then followed several years during which Mr Jamieson was mostly abroad. For two years he acted as chief electrical assistant to Thomson and Jenkin, along the coast of Brazil, for the Western Brazilian, and Platino-Brazilian Companies. On returning to this country, the late Sir John Pender and Sir James Anderson appointed him chief electrician to the Eastern Telegraph Company, and he was sent to the Mediterranean to superintend the laying of the Marseilles-Malta cable. During the next three years, from 1877 to 1880, Mr Jamieson obtained great experience and played a prominent part in extending the Eastern Telegraph Company's sphere of operations in the Mediterranean and along the East African and Indian coasts. In 1878 he acted as electrician to the expedition which was entrusted with the work of maintaining communication between the Admiralty in London and the Black Sea Fleet during the Russo-Turkish war.

In 1880 Mr Jamieson resigned his position with the Eastern Telegraph Company to become Principal of the Glasgow College of Science and Art. In 1887, when this institution amalgamated with the Mechanics' Institute and Anderson's College, forming the Glasgow and West of Scotland Technical

College—now the Royal Technical College,—Mr Jamieson was elected Professor of Engineering. After a time he devoted himself entirely to electrical engineering and resigned his professorship to engage in consulting work, his services being in great requisition amongst the engineering and shipbuilding firms on the Clyde. He was often engaged as consulting engineer for municipal and other engineering undertakings.

Latterly Professor Jamieson organised on a large scale a system of correspondence tuition to prepare students for various engineering examinations. His well-known text-books on engineering subjects, now passed through many editions, serve as a fitting memorial to the great energy and persevering ability he displayed in educational matters.

Professor Jamieson was a member of several engineering societies, the "Proceedings" and "Transactions" of which have been greatly enriched by his various contributions.

The Institution of Civil Engineers awarded him a "Telford Premium," and the Institution of Engineers and Shipbuilders in Scotland its gold medal.

Professor Jamieson was the patentee of several inventions, notably a submarine signalling key and a special grapnel for hooking and raising submarine cables.

R. H. Traquair, M.D., LL.D., F.R.S. By John Horne, LL.D., F.R.S.

(Read March 3, 1913.)

By the death of Dr Traquair this Society has lost one of its most distinguished Fellows, and one of the prominent leaders of his time in fossil ichthyology. The record of his early days may be briefly told. He was born at the Manse, Rhynd, Perthshire, in 1840, and received his school education in Edinburgh, where, as a boy, he followed the impulses of a born naturalist. He set himself to collect shells, butterflies, and moths, and he hammered the Carboniferous rocks in search of fossils. In his later life he often remarked that his keen interest in fossil fishes was first aroused when hammering the ironstone nodules of the Wardie Shales, which revealed to him a fragment of a Palaeoniscid fish. Even at that early stage the ambition seized him of devoting his life to research in the field of natural science. With this object in view he passed through the medical curriculum in Edinburgh University and graduated in medicine. His skill as a dissector attracted the notice of Professor Goodsir and Sir William Turner, who was then Senior Demonstrator in Anatomy, which led to his appointment as one of the demonstrators in that department. At Goodsir's suggestion he studied the asymmetry of the flat fishes and chose this subject for his medical thesis, for which he was awarded a gold medal. This elaborate memoir, which still remains a model of exact description, was subsequently published in the *Transactions* of the Linnean Society.

In 1866 he became Professor of Natural History in the Royal Agricultural College, Cirencester; in 1867, Professor of Zoology in the Royal College of Science, Dublin; and in 1873 he was appointed Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh—a post which he held till his retirement in 1906. The last of these appointments gave him the chance in life which he eagerly desired. He had chosen palæichthyology as his special line of research. He came to a Museum enriched by the Hugh Miller and other collections. The labours of Hibbert, Hugh Miller, Fleming, C. W. Peach, Powrie, and others had proved the abundance of fossil fishes in the Old Red Sandstone and Carboniferous systems in Scotland. The department which he served gave him the means of acquiring many of the best specimens for the Museum. Few men have such opportunities, but it was extremely fortunate for Scottish palæontology that they fell to a man whose methods of studying the

collections under his charge revolutionised our knowledge of palæozoic ichthyology.

In order to form some idea of the thoroughness of his methods we may recall his conception of palæontology as a branch of science. He maintained that scientific palæontology is essentially a part of zoology or botany, and that without a thorough knowledge of recent biology no one can hope to produce work of any value in palæontology. He went further, and contended that, if additional light is to be thrown on the question of evolution, it is through palæontology, working hand in hand with recent morphology and embryology, that the light must come.

In his own sphere of fossil ichthyology his work was of the highest value both to the zoologist and the geologist. As a zoologist he studied the osteology and phylogenetic relationships of the fossil fishes. But his investigations did not rest there. He next tried to ascertain their vertical range and lateral distribution with the view of aiding the geologist in working out the stratigraphy of the rock formations. At the same time he examined closely the relation between the fish remains and the sediments in which they occurred, in order to throw light on the physical conditions that prevailed during their deposition. A scheme of research so comprehensive demanded from Dr Traquair patient, continuous, and minute investigation. He was slow and deliberate in his work, he was frequently accused of being dilatory; but this trait was largely due to his reluctance to commit himself to definite conclusions until he had exhausted all available evidence.

When Dr Traquair began his researches in Edinburgh in 1873, the study of fossil ichthyology had assumed a critical phase. Agassiz had laid its foundations in his classic work, *Recherches sur les poissons fossiles*, which was begun in 1833 and completed in 1843. He therein established a new classification of fishes according to their scales, arranging them in four orders—*Ganoidei*, *Placoidei*, *Ctenoidei*, and *Cycloidei*. In 1844 Müller pointed out the unsatisfactory nature of this system and advanced an independent one which was adopted by some zoologists. But the classification of Agassiz held the field. It was widely accepted by geologists and palæontologists on the ground of its convenience. A few years of detailed study of the osteology of several Carboniferous fishes led Dr Traquair to reject this classification altogether. In 1879 he thus expressed his views. He frankly admitted that the name of Agassiz would go down to posterity as that of one of the greatest naturalists of the nineteenth century. "But," he added, "it is hardly possible for the zoologist of the present day to suppress some feelings of wonder that a man so well versed in general

zoology and anatomy as Agassiz should have based his classification of fishes upon characters so trivial as the mere external aspect of their scales, or that he should have distinguished many of the families into which he divided the order of the Ganoids by characters equally superficial." He maintained that it could not stand the test of anatomical inquiry.

The detailed researches which enabled Dr Traquair to give the death-blow to this classification are embodied in two classic memoirs, viz. his "Monograph on the Ganoid Fishes of the British Carboniferous Formations: Part I., Palæoniscidæ," published by the Palæontographical Society in 1877, and his memoir "On the Structure and Affinities of the Platysomidæ," which appeared in the *Transactions* of this Society in 1879. Until the publication of these researches, the Palæoniscidæ and the Platysomidæ had been compared with the existing North American bony pike, *Lepidosteus*. But Dr Traquair showed that the affinities of *Palæoniscus*, as indicated by the skeleton, point most strongly not to *Lepidosteus*, to which its angular scales and fulcrated fins give it superficial resemblance, but to *Polyodon*. Hence it followed that this sub-family ought to be grouped with the Acipenseroidei. In short, they were closely related to the modern sturgeons. He further showed that the Platysomidæ were merely a specialised offshoot from the Palæoniscidæ. Traquair founded his conclusions on the fundamental characters of the skeleton, which since that time has been recognised as the proper basis of a scientific classification.

Another series of researches of extreme zoological interest, showing Dr Traquair's powers as an original investigator, is associated with the new fish fauna found by the Geological Survey in the Downtonian rocks of the south of Scotland. Previous to this discovery the family of the Cœlolepidæ of Pander, represented by the genus *Thelodus*, was known only by scattered scales in strata of like age in England and other parts of the world. From their shagreen-like scales the Cœlolepidæ were considered to be sharks. But the beautiful specimens of *Thelodus* and *Lanarkia*—two genera of the Cœlolepidæ—in the Survey collection led him to place them with the Ostracodermi and in the order Heterostraci, of which the only family previously recognised was that of the Pteraspidæ. He enlarged the order of the Heterostraci, and included in it four families, the Cœlolepidæ, the Psammosteidæ, the Drepanasidæ, and the Pteraspidæ. He pointed out that the armour plates of the last three of these families had been formed by the fusion of the Cœlolepid scales with each other and with hard tissue developed in a deeper layer of the skin.

The geological aspects of Dr Traquair's researches are well illustrated in his paper on "The Distribution of Fossil Fish Remains in the Carboniferous

Rocks of the Edinburgh District," published in the *Transactions* of this Society in 1903. This paper contains the results of his work in this field extending over a period of thirty years.

By means of the fish remains he arranged the Carboniferous rocks in two divisions—an upper and lower, drawing the boundary line about the horizon of the Millstone Grit. He found that the fish faunas varied in accordance with the type of sedimentation. Thus the fish remains occurring in the limestones of open sea origin are essentially different from those met with in the estuarine beds. The marine limestones of the Lower Carboniferous rocks yield mainly Elasmobranch forms; Dipnoids and Ganoids being rare. On the other hand, the estuarine strata from the bottom to the top of the Carboniferous system are characterised not only by Dipnoids and Ganoids, but by a set of Elasmobranchs differing generically and specifically from those of the marine limestones. Every geologist must recognise the extreme importance of this deduction.

He also discussed the question of the value of fossil fishes as zonal indices of stratigraphical horizons. He admitted that it was hardly possible to establish satisfactory life zones by means of the fishes in the Lower Carboniferous rocks, but he called attention to the remarkable divergence in the characters of the estuarine fish fauna on the south side of the Southern Uplands from that in the Midland Valley; and he speculated as to the probable influence of a land barrier in accounting for this divergence. He showed that in the estuarine beds of the lower division of the system many of the species must have lived for a long lapse of time without any change in their specific characters. But above the Millstone Grit he encountered a new fauna from which nearly all the Lower Carboniferous species and a number of the genera had disappeared. He held that the cause of this remarkable palaeontological break in the fish fauna was a question which specially concerned the geologist.

In 1887 Dr Traquair began the detailed study of the fish fauna of the Old Red Sandstone, which led to an extensive revision of the nomenclature. Following the classification of Murchison and Salter, he arranged the strata of this formation in Scotland in three divisions—a lower, middle, and upper; the sequence being determined by the assemblage of fish remains in each division. He pointed out the resemblance of the fish fauna in the formation south of the Grampians to that of the Lower Old Red Sandstone of the west of England and adjoining part of Wales, and to the Lower Devonian rocks of Canada. On the other hand, he showed that the Orcadian rocks north of the Grampians yield an entirely different fish fauna, which he grouped with the Middle Devonian. The third great fish fauna found in

the Upper Old Red Sandstone, with its characteristic genera, *Asterolepis*, *Psammosteus*, *Bothriolepis*, *Holoptychius*, etc., he correlated with that of the Upper Devonian of the north-west of Russia, the Fammenien of Belgium, and the Catskill of the United States.

But Dr Traquair was not content with this broad classification of the formation in three divisions. He pointed out the existence of three fish faunas in the Middle Old Red Sandstone of Caithness (Achanarras, Thurso, and John o' Groats), each with a more or less distinct assemblage of fishes. He correlated the lowest or Achanarras fauna with that near the base of the middle division at Cromarty and on the south side of the Moray Firth. When Dr Flett subsequently found a similar sequence of fish remains in the Old Red Sandstone of the Orkneys, it seemed to confirm the accuracy of Traquair's conclusions in a remarkable degree. But the recent detailed work of the Geological Survey in Caithness has demonstrated the existence of a fish fauna older than any of those recognised by Dr Traquair, yielding *Thursius macrolepidotus*, *Coccosteus decipiens*, and *Dipterus Valenciennesi*. This discovery profoundly impressed Dr Traquair, for he wondered how this fauna should appear in the lower part of the Caithness succession and yet be absent from the base of the series in the Moray Firth basin and in the Orkneys. This difficulty has been successfully overcome by Mr Carruthers, who has suggested that it points to a later submergence of the land areas in the region of the Moray Firth and the Orkneys.

With the assistance of Mr Taylor, Lhanbryde, Dr Traquair recognised three life-zones in the Upper Old Red Sandstone on the south side of the Moray Firth; the lowest being represented by the Nairn sandstones with *Asterolepis maxima*, the second by the Alves sandstones with *Psammosteus Taylori*, and the highest by the Rosebrae sandstones with *Holoptychius nobilissimus*.

Throughout his long career he published upwards of 130 papers, chiefly on fossil fishes, which have appeared in the monographs of the Palaeontographical Society, the *Transactions* of the Royal Society of Edinburgh, and the *Proceedings* of the Royal Physical Society. His great monographs on the Palaeoniscidae and the Asterolepididae, which appeared in successive instalments in the publications of the Palaeontographical Society between 1877 and 1912, are still incomplete. He had begun for the Geological Survey a synopsis of his researches on the Old Red Sandstone fishes of Scotland. He was anxious to finish it for the guidance of younger men, but the infirmities of age prevented it. In the closing years of his life he prepared a memoir on the Wealden fishes of Bernissart, Belgium, published in 1911 by the Royal Museum of Natural History, Brussels.

One striking feature of Dr Traquair's work was his artistic restorations of the fossil forms, which have been largely reproduced in scientific text-books.

Honours fell to him in recognition of his work. At an early age he was elected a Fellow of the Royal Society of London, largely through the influence of Professor Huxley, who fully appreciated the value of his earliest researches. He received the honorary degree of LL.D. from the University of Edinburgh. He was awarded the Neill Medal and Makdougall-Brisbane Medal by this Society, the Lyell Medal by the Geological Society, and, lastly, the honour which he prized most of all, a Royal Medal by the Royal Society.

Dr Traquair was not a man who carried his heart upon his sleeve. He had certain idiosyncrasies which tended to repel rather than attract. But those who knew him intimately, and who were in a position to discuss with him the bearing of his work, realised that underneath this veneer there was much kindness of heart. Above all, he was a genuine lover of truth, whose great aim was to avoid reckless generalisation and to promote the study of fossil ichthyology by thorough and rigidly accurate methods.

Robert M. Ferguson, Ph.D., LL.D. By Dr Andrew E. Scougal.

(Read December 15, 1913.)

Robert McNair Ferguson was born at Airdrie, in July 1828. But he may be accounted as to all intents and purposes an Edinburgh man; for his father removed to Edinburgh when Robert was only three years old, and it was in Edinburgh that the son was brought up and educated, and there that he had his home during all the rest of his life.

From a very early period in his life the lad—or boy—would seem to have made up his mind to be a teacher. At the close of his primary school course as a pupil at the practising school attached to what was then the Free Church Training College at Moray House, he became a monitor in that school, then an assistant master there, and afterwards one of the lecturers on the Training College staff. During this period of training for the teaching profession—which lasted from 1843 to 1858—he attended classes at the University and at the New College, proving himself to be a first-rate student and gaining the highest honours in philosophy, mathematics, and natural philosophy; he passed the examination of King's College, London, for a lectureship in physics and chemistry; and, perhaps most important of all in view of what his main future work was to be, he studied in Germany under Professor Bunsen and other distinguished teachers, and finally graduated, in 1855, as a Doctor of Philosophy of the University of Heidelberg, with mechanics, physics, and chemistry as his special subjects.

From his residence in Germany he brought back with him not only increased attainments in science, but also an intimate knowledge of German life and literature and such a command of the German language that he could use it really “like a native.” He had also a good knowledge and command of French—an acquirement which doubtless had something to do with his being entrusted later on with the instruction in chemistry of three young Orleanist princes, the great-grandsons of Philippe Égalité.

In 1858, at the age of thirty, he found a favourable opening for entering on what was to be the main work of his life. In that year he and his friend Mr Bickerton, in partnership, took over from Dr Alexander Reid the Edinburgh Institution (8 Queen Street, Edinburgh), a private secondary school which had already made for itself a good and widespread reputation. Under its new headmasters the Institution steadily increased both in efficiency and in numbers, until it gained such a secure position among the

secondary schools of the city that it was one of the very few private schools in Edinburgh that managed to survive the crisis for such schools which was occasioned by the starting of the Merchant Company's new educational scheme in 1870. The Institution, although somewhat shaken for a little while, ere long fully regained its former prestige and success. In 1875 Mr Bickerton retired, owing to the state of his health, and Dr Ferguson continued to manage the school, as its sole head, until 1898. Thus, when he then retired he had been at the head of this important Edinburgh school for no fewer than forty years.

But Dr Ferguson was a man of wide and varied interests and of much energy. During these forty years he did not confine his activities merely to the work of his school, heavy and taxing though that must have been.

In 1875 he had been elected a fellow of the Scottish (now the Royal Scottish) Society of Arts. Of this Society he became one of the leading members, and was its President for two years. Between 1863 and 1893 he contributed to its *Proceedings* eight papers, two of which were of such importance as to be awarded the Society's silver medals.

He became a member of the Edinburgh Mathematical Society on its foundation in 1883, was its President in 1885-86, and for many years took an active share in its proceedings.

It was in 1868 that he was admitted as a Fellow of the Royal Society of Edinburgh. From an early period of his membership down to very shortly before his death—that is, for more than forty years—he took an active share in the work of the Society, with much acceptableness to his fellow-members. Between 1869 and 1880 he contributed three papers, on magneto-electric and physics subjects, that were published in the Society's *Proceedings* (vols. vii., ix., and x.); for three separate terms (1877-80, 1886-89, 1899-1902) he served on the Council; and from 1902 till 1911 he was the Society's representative on the Board of Governors of George Heriot's Trust. His election to this last office Dr Ferguson highly appreciated, not only as in itself an honour, but also as affording him a sphere of thoroughly congenial public work. He took great interest in the affairs of the Trust, and rendered valuable service on its governing body, especially in connection with its educational schemes. It was with great regret that he resigned this post because his growing deafness had made him feel that he should give up all work that involved attendance at public meetings. The Heriot Trust work was the last work of that kind that he did give up.

In 1866 Dr Ferguson published, through Messrs W. & R. Chambers, a text-book on *Electricity*, which was very well received. After having been more than once reprinted, it appeared in 1882 in a new edition, revised by

Professor Blyth of the West of Scotland Technical College. Dr Ferguson was also the author of several articles on natural philosophy subjects in the earlier editions of *Chambers's Encyclopaedia*.

Amid all this varied and strenuous activity he found time to take, with the keenness and energy characteristic of him, a prominent part in political affairs in Edinburgh and the East and North of Scotland. Details as to his labours in this field were given in the full and interesting obituary notice of him which appeared in the *Scotsman* newspaper, and need not be repeated here. But, the politician apart, it may be permissible to quote here from that notice a few lines which reveal personal traits of the man : "Dr Ferguson took a keen interest in politics. . . . He was one of the most indefatigable of members and office-bearers. In spite of his years, no weather was too inclement, and no meeting was too late, for him to attend. . . . Frequently he presided at public meetings, and the well-chosen character of his remarks and his conduct of the business always gave satisfaction. . . . In addition to such services, which he gave freely, he was also a generous subscriber to the . . . party funds."

In 1892 he received from the University of Edinburgh the honorary degree of Doctor of Laws, in recognition, as the Dean of Faculty said in presenting him, of the importance of his educational and scientific work.

As a teacher and headmaster Dr Ferguson stood in the very front rank. He understood boys, and had that sympathetic insight into both their good points and their failings that is one of the best possessions of the "born teacher." We his pupils soon came to feel that it would be a shame—"low" we ourselves would have called it—to be "mean" or "caddish" or "slackers" with "the Doctor" (as we called him), who himself *played the game* so keenly, so straightly, and so fairly. Thus he succeeded in establishing a high and hearty and thoroughly healthy tone in his school. His wide range of attainments stood him in good stead there ; for in staff emergencies he could and did take, with much acceptance among the boys, the higher classes of the school in any subject in the curriculum. But his own special subjects were mathematics and science, and of these he was a supremely good teacher. Many of his pupils who went on to more advanced work in these subjects found reason to be deeply grateful for the thoroughness of the grounding they had received under Dr Ferguson. He was one of the pioneers in the teaching of practical experimental science in schools. Very few secondary schools in the early 'sixties had science laboratories. But Dr Ferguson contrived, in spite of serious limitations as to suitable accommodation, to fit up one at the Edinburgh Institution ; and in it, cramped and ill-equipped though it was in contrast with what is nowadays demanded,

he succeeded, by his resourcefulness and enthusiasm and skill as a teacher, in doing no little genuine and stimulating work in this then much-neglected branch of an all-round school curriculum.

By a strange fatality this same little laboratory was, in the end of 1897, the scene of a sad accident by which Dr Ferguson was physically crippled for the rest of his days. An attendant's blunder in connection with the use of oxy-hydrogen apparatus caused an explosion which dreadfully injured Dr Ferguson's right foot; in spite of all surgical care, the mischief spread; and at last the leg had to be amputated at the knee-joint. It was specially sad that this accident should have happened just at the time when Dr Ferguson was concluding his arrangements for retiring from the Institution, and that it should have befallen one who was so fond of physical exercise. He had been accustomed to spend his autumn holidays, generally in company with his friend Mr Bickerton, in travelling, mostly on foot, through various countries in Europe; he was fond of long rambles; and he was keenly devoted to golf. But he bore what must have been to him such a sore deprivation bravely and uncomplainingly; and, having handed over the Edinburgh Institution to his successor, he cheerfully, and with unimpaired spirit and energy, resumed, as far as was now possible for him, his former pursuits and activities, and continued so working until almost the very close of his long life of over eighty-three years.

Dr Ferguson died in his own house in Edinburgh on the 31st of December 1912, leaving behind him the record of an honourable and honoured career of strenuous, able, and fruitful work for his day and generation. His loss was deeply mourned by all who knew him. He was a most attractive personality—frank, manly, keen and alert in intellect, cheery and buoyant in spirit, generous-hearted, and absolutely straightforward. His well-furnished mind, broadened and quickened by wide reading and much travel, made him a delightful companion. He drew very close to him those who had the privilege of being his intimate friends, and he will long be fondly remembered by them. But his most fitting and most lasting memorial—that, too, which he himself would have most highly valued—will assuredly be the affectionate and grateful remembrance of their old teacher and true friend that will live so long in the minds and hearts of his many—so many—former pupils.

APPENDIX.

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PROCEEDINGS OF THE STATUTORY GENERAL MEETING.
The 130th Session, 1912-1913.

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Hall, 24 George Street, on Monday, 28th October 1912, at 3 p.m.,

Principal Sir Wm. Turner, K.C.B., President, in the Chair,

the Minutes of last Statutory Meeting, 23rd October 1911, were read, approved, and signed.

On the motion of Dr J. S. BLACK, seconded by Dr C. G. KNOTT, Mr G. H. GULLIVER and Mr DAVID GIBB were appointed Scrutineers, and the ballot for the New Council commenced.

The TREASURER's Accounts for the past year, 1911-1912, were submitted. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following Council had been duly elected :—

Principal Sir W.M. TURNER, K.C.B., D.C.L., F.R.S., President.	}	Vice-Presidents.
JOHN HORNE, LL.D., F.R.S., F.G.S.,		
JAMES BURGESS, C.I.E., LL.D., M.R.A.S.,		
Professor T. HUDSON BEARE, M.Inst.C.E.,		
Professor F. O. BOWER, M.A., D.Sc., F.R.S.,		
Professor SIR THOMAS R. FRASER, M.D., LL.D., F.R.C.P.E., F.R.S.,	Secretaries to Ordinary Meetings.	
BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S.		
CARGILL G. KNOTT, D.Sc., General Secretary.		
ROBERT KIDSTON, LL.D., F.R.S., F.G.S., Professor ARTHUR ROBINSON, M.D., M.R.C.S., JAMES CURRIE, M.A., Treasurer.		
JOHN S. BLACK, M.A., LL.D., Curator of Library and Museum.		

ORDINARY MEMBERS OF COUNCIL.

RAMSAY H. TRAQUAIR, M.D., LL.D., F.R.S.	ANDREW WATT, M.A.
Professor JAMES WALKER, D.Sc., Ph.D., LL.D., F.R.S.	JAMES H. ASHWORTH, D.Sc.
Sir W. S. M'CORMICK, M.A., LL.D.	Professor GEORGE A. GIBSON, M.A., LL.D.
Professor CRUM BROWN, M.D., LL.D., F.R.S.	Professor R. A. SAMPSON, M.A., D.Sc., F.R.S.
Professor T. H. BRYCE, M.A., M.D.	Professor D'ARCY W. THOMPSON, C.B., B.A., F.L.S.
WILLIAM ALLAN CARTER, M.Inst.C.E.	Professor E. T. WHITTAKER, Sc.D., F.R.S.

On the motion of Mr JAMES CURRIE, thanks were voted to the Scrutineers.

On the motion of Dr KNOTT, thanks were voted to the Auditors, Messrs LINDSAY, JAMIESON, & HALDANE, and they were reappointed.

On the motion of Dr KIDSTON, thanks were voted to the Treasurer, Mr JAMES CURRIE.

PROCEEDINGS OF THE ORDINARY MEETINGS,
Session 1912-1913.

FIRST ORDINARY MEETING.

Monday, 4th November 1912.

Principal Sir Wm. Turner, K.C.B., President, in the Chair.

At the request of the Council an Address was delivered:—

On Reflex Inhibition as an Element in the Co-ordination of Animal Movements. By Professor CHARLES SCOTT SHERRINGTON, M.A., M.D., LL.D., F.R.S., Holt Professor of Physiology in the University of Liverpool.

Professor CHARLES SCOTT SHERRINGTON signed the Roll of Honorary Fellows.

SECOND ORDINARY MEETING.

Monday, 18th November 1912.

Dr Horne, F.R.S., Vice-President, in the Chair.

The following communications were read:—

1. On the Deviation of the Law of Torsional Oscillation of Metals from Isochronism. By Professor W. PEDDIE. *Proc.*, vol. xxxiii. pp. 225-242.
2. On a fuller Test of the Law of Torsional Oscillation of Wires, and On a continued Investigation on the Behaviour of certain Torsionally Oscillating Wires. By Mr JAMES B. RITCHIE. Communicated by Professor W. PEDDIE. *Proc.*, vol. xxxiii. pp. 177-182.
3. On the Energetics of the Induction Balance. By J. P. DALTON, M.A., B.Sc. Communicated by Professor W. PEDDIE.
4. Periodicity in Plants. By R. A. ROBERTSON, M.A., B.Sc., and Miss ROSALIND CROSSE. *Proc.*, vol. xxxiii. pp. 85-102.
5. Theory of Axisymmetric Determinants from 1857 to 1880. By Dr THOMAS MUIR. *Proc.*, vol. xxxiii. pp. 49-63.

The following Candidates for Fellowship were balloted for, and declared duly elected:—
WILLIAM THOMAS GORDON, M.A., D.Sc., B.A., and GEORGE HENRY MASSON, M.D., D.Sc., M.R.C.P.E.

THIRD ORDINARY MEETING.

Monday, 2nd December 1912.

Principal Sir William Turner, K.C.B., President, in the Chair.

Before beginning the business of the Meeting, the PRESIDENT referred to the loss the Society had sustained through the death of Dr R. H. Traquair, and briefly mentioned the scope of his work, and the many offices he had occupied in the Society. [Obituary Notice, *Proc.*, vol. xxxiii. pp. 336-341.]

The following communications were read:—

1. The Right Whale of the North Atlantic, *Balaena biscayensis*, its Skeleton described and compared with that of the Greenland Right Whale, *Balaena mysticetus*. By the PRESIDENT. *Trans.*, vol. xlviii. pp. 889-922.
2. The Loss of Energy at Oblique Impact of Two Confined Streams of Water. By Professor A. H. GIBSON, D.Sc. *Trans.*, xlviii. pp. 799-811.
3. The Hydrozoan Zoophytes collected by the British Antarctic Expedition of Sir E. Shackleton, 1908. By Dr JAMES RITCHIE. Communicated by Mr WM. EAGLE CLARKE. *Proc.*, vol. xxxiii. pp. 9-34.

4. Scottish National Antarctic Expedition: Observations on the Anatomy of the Weddell Seal (*Leptonychotes Weddellii*). Pt. IV.—The Brain. By Professor DAVID HEPBURN, M.D., C.M. *Trans.*, vol. xlviii, pp. 827-847.

5. Scottish National Antarctic Expedition: The Central Nervous System of the Weddell Seal. By Dr H. A. HAIG. Communicated by Dr W. S. BRUCE. *Trans.*, xlviii. pp. 849-866.

The following Candidates for Fellowship were balloted for, and declared duly elected:—JOSEPH BEARD, F.R.C.S., M.R.C.S., etc., and HARRY RICHARDSON, M.Inst.E.E., etc.

FOURTH ORDINARY MEETING.

Monday, 16th December 1912.

Professor F. O. Bower, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. The Precipitation of Salts by the Corresponding Acids. By IRVINE MASSON, M.Sc. Communicated by Professor JAMES WALKER, F.R.S. *Proc.*, vol. xxxii, pp. 64-68.

2. Jurassic Plants from Cromarty and Sutherland, Scotland. By Professor A. C. SEWARD, F.R.S. Communicated by Dr KIDSTON, F.R.S. *Trans.*, vol. xlviii, pp. 867-888.

3. The Antarctic Fishes of the Scottish National Antarctic Expedition. By C. TATE REGAN, M.A. Communicated by Dr W. S. BRUCE. *Trans.*, vol. xl ix. pp. 229-292.

4. The Porifera of the Scottish National Antarctic Expedition. By Professor EMILE TOPSENT. Communicated by Dr W. S. BRUCE. *Trans.*, vol. xl ix. pp. 579-643.

5. A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine. Part V.—The Anatomy of the Gut and its Appendages. By Professor F. J. COLE. Communicated by Professor W. A. HERDMAN, F.R.S. *Trans.*, vol. xl ix. pp. 293-344.

Dr W. T. GORDON signed the Roll, and was duly admitted a Fellow of the Society.

FIFTH ORDINARY MEETING.

Monday, 6th January 1913.

Professor T. Hudson Beare, B.Sc., M.Inst.C.E., Vice-President, in the Chair.

The following Communications were read:—

1. Obituary Notices of:—

(a) Professor George Chrystal, M.A., LL.D. By Dr J. S. BLACK and Dr C. G. KNOTT. *Proc.*, vol. xxxii, pp. 477-503.

(b) David Harris, F.S.S. By Professor D. FRASER HARRIS. *Proc.*, vol. xxxii, pp. 475-476.

2. The Absorption of Light by Inorganic Salts. No. VII.—Aqueous Solutions of Iron Salts. By JOHN S. ANDERSON, M.A., B.Sc. Communicated by Dr R. A. HOUSTOUN. *Proc.*, vol. xxxiii. pp. 35-43.

3. The Absorption of Light by Inorganic Salts. No. VIII.—Alcoholic Solutions of Copper, Cobalt, and Nickel Salts in the Ultra-Violet. By ALEX. R. BROWN, M.A., B.Sc. Communicated by Dr R. A. HOUSTOUN. *Proc.*, vol. xxxiii, pp. 44-48.

4. On a Method of determining Vapour Densities at High Temperatures and on a New Form of Quartz Manometer. By G. E. GIBSON, B.Sc., Ph.D. Communicated by Professor JOHN GIBSON. *Proc.*, vol. xxxiii, pp. 1-8.

Dr W. S. SYME signed the Roll, and was duly admitted a Fellow of the Society.

SIXTH ORDINARY MEETING.

Monday, 20th January 1913.

Dr B. N. PEACH, F.R.S., Vice-President, in the Chair.

Before the beginning of the public business, the CHAIRMAN announced the death of Dr George A. Gibson, Edinburgh.

The following Communications were read:—

1. Obituary Notice of Professor Andrew Jamieson, M.Inst.C.E., Glasgow, and of John M'Arthur, F.C.S., Sussex. *Proc.*, vol. xxxiii. pp. 334, 335.

2. Studies in Floral Zygomorphy. I.—The Initiation of Staminal Zygomorphy. By JOHN M'LEAN THOMPSON, M.A., B.Sc. Communicated by Professor F. O. BOWER, F.R.S. *Trans.*, vol. xlix. pp. 691–704.

3. Changes of Electrical Resistance accompanying Longitudinal and Transverse Magnetisations in Nickel. By Dr C. G. KNOTT. *Proc.*, vol. xxxiii. pp. 200–224.

The following Candidates for Fellowship were balloted for, and declared duly elected:—TARAK NATH MAJUMDAR, D.P.H. (Cal.), L.M.S., F.C.S., WALTER HUME KERR, M.A., B.Sc., and GEORGE FRANCIS SCOTT ELLIOT, M.A. (Cantab.), B.Sc., etc.

SEVENTH ORDINARY MEETING.

Monday, 3rd February 1913.

Professor T. Hudson Beare, B.Sc., M.Inst.C.E., Vice-President, in the Chair.

The following Communications were read:—

1. A New Method of Starting Mercury Vapour Apparatus. By JOHN S. ANDERSON, M.A., B.Sc., and GEORGE B. BURNSIDE. Communicated by Professor A. GRAY, F.R.S. *Proc.*, vol. xxxiii. pp. 117–123.

2. On the Electron Theory of Thermo-electricity. By JOHN M'WHAN, M.A., B.Sc. Communicated by Professor A. GRAY, F.R.S. *Proc.*, vol. xxxiii. pp. 169–176.

3. On the Application of Manley's Differential Densimeter to the Study of Sea Waters on Board Ship. By NORMAN PHILLIPS CAMPBELL, B.A. (Oxon.). Communicated by Sir JOHN MURRAY, K.C.B. *Proc.*, vol. xxxiii. pp. 124–136.

EIGHTH ORDINARY MEETING.

Monday, 17th February 1913.

Dr Horne, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. Polychæta of the Families Serpulidae and Sabellidae, collected by the Scottish National Antarctic Expedition. By HELEN L. M. PIXELL, B.Sc., F.Z.S. Communicated by Dr J. H. ASHWORTH. *Trans.*, vol. xlix. pp. 347–358.

2. The Error caused by "Lag" in a Recording Instrument: An Experimental Study. By Dr J. R. MILNE and H. LEVY, M.A., B.Sc. *Proc.*, vol. xxxiii. pp. 103–116.

NINTH ORDINARY MEETING.

Monday, 3rd March 1913.

Professor F. O. Bower, F.R.S., Vice-President, in the Chair.

The following communications were read:—

1. Obituary Notice of R. H. Traquair, M.D., LL.D., F.R.S. By Dr HORNE, F.R.S. *Proc.*, vol. xxxiii. pp. 336–341.

2. The Absorption of Light by Inorganic Salts. No. IX.: Solutions of Copper, Nickel, and Cobalt Salts in Alcohol and in Acetone. By Dr R. A. HOUSTOUN and A. H. GRAY, M.A., B.Sc. *Proc.*, vol. xxxiii. pp. 137–146.

3. The Absorption of Light by Inorganic Salts. No. X. By Dr R. A. HOUSTOUN and Mr CHARLES COCHRANE. *Proc.*, vol. xxxiii. pp. 147–155.

4. The Absorption of Light by Inorganic Salts. No. XI.: Conclusion. By Dr R. A. HOUSTOUN. *Proc.*, vol. xxxiii. pp. 156–165.

5. Note on a Comparison of Records of Atmospheric Electric Potential at two Stations in Edinburgh. By Dr G. A. CARSE and G. SHEARER, M.A., B.Sc., and H. JAMESON, B.Sc. *Proc.*, vol. xxxiii. pp. 194–199.

TENTH ORDINARY MEETING.

Monday, 17th March 1913.

Dr B. N. PEACH, F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. Measurements and Weights of Antarctic Seals taken by the Scottish National Antarctic Expedition. By Dr W. S. BRUCE. *Trans.*, vol. xlix. pp. 567–577.

2. The Pterobranchia of the Scottish National Antarctic Expedition. By Dr S. F. HARMER, F.R.S., and Dr W. G. RIDWOOD. Communicated by Dr J. H. ASHWORTH. *Trans.*, vol. xlix. pp. 531-565.

3. On Intestinal Respiration in Annelids; with Considerations on the Origin and Evolution of the Vascular System in that Group. By Professor J. STEPHENSON, M.B., D.Sc. *Trans.*, vol. xlix. pp. 735-829.

4. On the Occurrence of Functional Teeth in the Upper Jaw of the Sperm Whale. By Dr JAMES RITCHIE and Mr A. J. H. EDWARDS. Communicated by Sir WILLIAM TURNER, K.C.B. *Proc.*, vol. xxxiii. pp. 166-168.

The following gentlemen signed the Roll, and were duly admitted Fellows of the Society:—Dr THEOPHILUS BULKELEY HYSLOP, Mr WALTER HUME KERR, Mr GEORGE FRANCIS SCOTT ELLIOT, and The Most Hon. THE MARQUIS OF LINLITHGOW.

The following Candidate for Fellowship was balloted for, and declared duly elected:—HARRY AVERY REID, F.R.C.V.S., etc.

ELEVENTH ORDINARY MEETING.

Monday, 5th May 1913.

Principal Sir William Turner, K.C.B., President, in the Chair.

The following Communications were read:—

1. Scottish National Antarctic Expedition: The Skulls of Antarctic Seals. By Dr W. S. BRUCE. *Trans.*, vol. xlix. pp. 345-346.

2. Report on the Bryozoa of the Scottish National Antarctic Expedition. By LAURA R. THORNLEY. Communicated by Dr W. S. BRUCE.

3. The Compressibilities of Dilute Solutions of Certain Inorganic Salts. By WILLIAM WATSON, M.A., B.Sc. Communicated by the GENERAL SECRETARY. *Proc.*, vol. xxxiii. pp. 282-291.

Dr J. H. ASHWORTH signed the Roll, and was duly admitted a Fellow of the Society.

TWELFTH ORDINARY MEETING.

Monday, 26th May 1913.

Professor T. Hudson Beare, B.Sc., M.Inst.C.E., Vice-President, in the Chair.

The following Communications were read:—

1. Scottish National Antarctic Expedition, 1902-1904: Deep-Sea Deposits. By Dr J. H. HARVEY PIRIE. *Trans.*, vol. xlix. pp. 645-686.

2. The Foraminifera of the Scottish National Antarctic Expedition. By Mr F. GORDON PEARCEY. Communicated by Dr J. H. HARVEY PIRIE. *Trans.*, vol. xlix., Pt. IV.

3. The Effect of Thermal Treatment, and the Effect of Longitudinal Strain in Inducing a Sensitive State in Certain Magnetic Materials. By Miss MARGARET MOIR, M.A., B.Sc. Communicated by Professor A. GRAY, F.R.S. *Proc.*, vol. xxxiii. pp. 243-256.

THIRTEENTH ORDINARY MEETING.

Monday, 2nd June 1913.

Principal Sir William Turner, K.C.B., F.R.S., President, in the Chair.

The following Communications were read:—

1. Contributions to the Craniology of the People of the Empire of India. Part IV.—Bhils, Frontier Tribes of Burma and Pakokku, South Shan Tribes, Tibetans. By Principal Sir Wm. TURNER, K.C.B., F.R.S., President of the Society. *Trans.*, vol. xlix. pp. 705-734.

2. Scottish National Antarctic Expedition: Glaciation of the South Orkneys. By Dr J. H. HARVEY PIRIE. *Trans.*, vol. xlix. pp. 831-863.

The following Candidates for Fellowship were balloted for, and declared duly elected:—ALEXANDER PHILIP, M.A., LL.B., ALEXANDER RUSSELL BROWN, M.A., B.Sc., DUGALD M'LELLAN, M.Inst.C.E., and ALEXANDER CLEGHORN, M.Inst.C.E.

FOURTEENTH ORDINARY MEETING.

Monday, 16th June 1913.

Dr B. N. PEACH, F.R.S., Vice-President, in the Chair.

The following Communication was read :—

The Properties of the Gyrostat, illustrated by Experiments with New Gyrostats and with New Gyrostatic Models, including Pole Balancing Tops, Gyrostatic Bicycle Riders, and Gyrostatic Motor Cars. By Dr J. G. GRAY, Lecturer in Physics in the University of Glasgow.

Mr ALEXANDER PHILIP signed the Roll, and was duly admitted a Fellow of the Society.

FIFTEENTH ORDINARY MEETING.

Monday, 7th July 1913.

Dr Horne, F.R.S., Vice-President, in the Chair.

The CHAIRMAN read the names of the following gentlemen, proposed by the Council as Honorary Fellows of the Society; they will be balloted for at the Ordinary Meeting on 3rd November 1913 :—

AS BRITISH HONORARY FELLOWS :—

1. HORACE LAMB, M.A., Sc.D., D.Sc., LL.D., F.R.S., Professor of Mathematics in the University of Manchester.
2. Sir WILLIAM TURNER THISELTON-DYER, K.C.M.G., C.I.E., M.A., LL.D., F.R.S., formerly Director of the Royal Botanic Gardens, Kew.

AS FOREIGN HONORARY FELLOWS :—

1. SANTIAGO RAMÓN Y CAJAL, F.M.R.S., Professor of Histology and Pathological Anatomy in the University of Madrid.
2. GEORGE ELLERY HALE, F.M.R.S., Director of the Mount Wilson Solar Observatory (Carnegie Institution of Washington), Pasadena, California.
3. ÉMIL CLEMENT JUNGFLEISCH, Mem.Inst.Fr., Professor of Organic Chemistry in the College of France, Paris.
4. VITO VOLTERRA, F.M.R.S., Sc.D., Ph.D., Professor of Mathematics and Physics in the University of Rome.
5. CHARLES RENÉ ZEILLER, Mem.Inst.Fr., Professor of Plant Palaeontology in the National Superior School of Mines, Paris.

The following communications were read :—

1. Plane Strain in a Wedge, with Application to Masonry Dams. By S. D. CAROTHERS, A.R.C.Sc.I. Communicated by the GENERAL SECRETARY. *Proc.*, vol. xxxiii. pp. 292–306.
2. The Corals of the Scottish National Antarctic Expedition. By J. STANLEY GARDINER, M.A., F.R.S. Communicated by Dr J. H. ASHWORTH. *Trans.*, vol. xl ix. pp. 687–689.
3. The Schizopoda, Stomatopoda, and Non-Antarctic Isopoda of the Scottish National Antarctic Expedition. By Dr WALTER M. TATTERSALL. Communicated by Dr J. H. ASHWORTH. *Trans.*, vol. xl ix. pp. 865–894.
4. On Some Pseudo-hermaphrodite Examples of *Daphnia pulex*. By Dr J. H. ASHWORTH. *Proc.*, vol. xxxiii. pp. 307–316.

SIXTEENTH AND LAST ORDINARY MEETING.

Monday, 21st July 1913.

Dr Horne, F.R.S., Vice-President, in the Chair.

The following Prizes were presented :—

1. The GUNNING VICTORIA JUBILEE PRIZE for the quadrennial period 1908–12 to Professor J. NORMAN COLLIE, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the *Transactions* of the Society.
2. The MAKDOUGALL-BRISBANE PRIZE for the biennial period 1910–12 to Dr JOHN BROWNLEE, for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the *Proceedings* of the Society within and prior to the prescribed period.

On making the presentations the CHAIRMAN read the following notes on the work of the recipients of the prizes :—

GUNNING VICTORIA JUBILEE PRIZE.

Some of Dr Collie's earliest papers were published conjointly with Professor Letts in our *Transactions*, namely those of 1881 and 1882—on the Salts of Tetrabenzyl-Phosphonium, and on the action of Sodium Phosphide on Haloid Ethers, and on the Salts of Tetrabenzyl-Phosphonium.

In 1885 he began a long series of very important papers on Keten compounds and the establishment of their connection with most of the well-known types of compounds naturally occurring. He also established the quadrivalence of oxygen in organic compounds, and we further owe to him an extremely ingenious space-formula for benzene, papers on Phosphines and Phosphonium compounds, and the demonstration of Synthesis by means of the silent electric discharges.

He has been among the most active in investigating the rare gases, and his writings cover the Occurrence of Helium in Minerals, the Phosphorescence of Neon when shaken with Mercury, and the startling observation of the Occurrence of Helium and Neon in Hydrogen after sparking, which has proved a matter for division of opinion among chemists. This list does not exhaust Dr Collie's activities ; he has written on Mineralogy, on a new form of Barometer, and on other subjects ; and as no account is now complete without mentioning a man's recreations, we may mention that Dr Collie has done much mountain-climbing, including first ascents in the Alps, Himalayas, Canadian Rockies, and the Lofoten Islands. His accounts of these and other explorations are embodied in the journals of the Alpine Club, the Royal Geographical Society, and in his books.

MAKDOUGALL-BRISBANE PRIZE.

In the series of papers on the mathematics of Mendelian heredity Dr Brownlee discusses in the first place the effect of different percentages in the parental stock on the correlation coefficient obtained between parent and offspring. He next shows that, if the correlation is calculated by applying Professor Pearson's method of fourfold division to the figures got by Mendelian experiment, values comparable to those calculated by the product-moment method result. He further finds that, with exactly the same type of heredity, higher values of the correlation coefficient are obtained when more than two races mix. As Professor Pearson's data for determining the law of ancestral heredity are very largely of this nature, and as they are claimed to be in harmony with the Mendelian law, grave doubt is thrown upon the legitimacy of the method by which his law has been investigated. Further, in two papers, Dr Brownlee shows how different types of Mendelian inheritance lead to a variety of curves. He gives methods of fitting statistics to these curves, adding a theorem regarding the moments of compound distributions. He has also analysed the whole of Dr Beddoe's original survey of Hair and Eye-Colour in Britain and the West of Europe, and has shown that all Dr Beddoe's observations can be explained on the basis of Mendelian inheritance.

Other papers presented during the period have dealt with the application of statistical methods to problems of epidemiology and of the variation of immunity from age to age.

At the request of the Council an Address was delivered :—

On the Sun-Spot Period and its Terrestrial Relations. By Professor R. A. SAMPSON, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland.

The following paper was read :—

The Chaetognatha of the Scottish National Antarctic Expedition. By A. PRINGLE JAMESON, B.Sc. Communicated by Dr W. S. BRUCE. *Trans.*, vol. xlix., Pt. IV.

The following Candidate for Fellowship was balloted for, and declared duly elected :—Rev. JAMES MARCHANT.

LAWS OF THE SOCIETY,

As revised 26th October 1908.

[By the Charter of the Society (printed in the *Transactions*, vol. vi. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

II.

The fees of
Ordinary
Fellows residing
in Scotland.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.* Fellows may compound for these contributions on such terms as the Council may from time to time fix.

III.

Payment to
cease after
25 years.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from further payment.

IV.

Fees of Non-
Resident
Ordinary
Fellows.

Case of Fellows
becoming Non-
Resident.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any further payment. In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

* A modification of this rule, in certain cases, was agreed to at a Meeting of the Society held on the 3rd January 1831.

At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

V.

Members failing to pay their contributions for three successive years (due Defaulters, application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

None but Ordinary Fellows shall bear any office in the Society, or vote in the Privileges of choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Ordinary Fellows. Society.

VII.

The number of Ordinary Fellows shall be unlimited.

Numbers
unlimited.

VIII.

All Ordinary Fellows of the Society who are not in arrear of their Annual Contributions shall be entitled to receive, gratis, copies of the parts of the Transactions of the Society which shall be published subsequent to their admission, upon application, either personally or by an authorised agent, to the Librarian, provided they apply for them within five years of the date of publication of such parts.

Copies of the parts of the Proceedings shall be distributed to all Fellows of the Society, by post or otherwise, as soon as may be convenient after publication.

IX.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the Ordinary Fellows. Mode of Recommending purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall be exhibited publicly in the Society's rooms for one month, after which it shall be considered by the Council. If the Candidate be approved by the Council, notice of the day fixed for the election shall be given in the circulars of at least two Ordinary Meetings of the Society.

X.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

* "A. B., a gentleman well versed in science (*or Polite Literature, as the case may be*), being to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

XI.

Royal Personages.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X.

XII.

Recommendation of Honorary Fellows.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one Meeting, and printed in the circulars for Two Ordinary Meetings of the Society, previous to the day of election.

Mode of election.

Election of Ordinary Fellows.

The election of Ordinary Fellows shall take place only at one Afternoon Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote.

Ordinary Meetings.

XIII.

The Ordinary Meetings shall be held on the first and third Mondays of each month from November to March, and from May to July, inclusive; with the exception that when there are five Mondays in January, the Meetings for that month shall be held on its second and fourth Mondays. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make.

The Transactions.

XIV.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall deem it expedient to publish in the Transactions of the Society, and shall superintend the printing of the same.

How Published.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

* We hereby recommend _____ for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

XVII.

That there shall be formed a Council, consisting—First, of such gentlemen as The Council may have filled the office of President ; and Secondly, of the following to be annually elected, viz.:—a President, Six Vice-Presidents (two at least of whom shall be Resident), Twelve Ordinary Fellows as Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote.

XVIII

Four Councillors shall go out annually, to be taken according to the order in Retiring Councillors. which they stand on the list of the Council.

XIX.

An Extraordinary Meeting for the election of Office-Bearers shall be held annually Election of Office-Bearers. on the fourth Monday of October, or on such other lawful day in October as the Council may fix, and each Session of the Society shall be held to begin at the date of the said Extraordinary Meeting.

XX.

Special Meetings of the Society may be called by the Secretary, by direction of Special Meetings; how the Council ; or on a requisition signed by six or more Ordinary Fellows. Notice called. of not less than two days must be given of such Meetings.

XXI.

The Treasurer shall receive and disburse the money belonging to the Society, Treasurer's Duties. granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually ; and at the Extraordinary Meeting in October, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

At the Extraordinary Meeting in October, a professional accountant shall be Auditor. chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the General Secretary's Society, and of the Meetings of the Council, in two distinct books. He shall, under Duties. the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

Secretaries to
Ordinary
Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXV.

Curator of
Museum and
Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the hall, for the inspection of the Fellows.

XXVI.

Use of Museum
and Library.

All articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations as the Council from time to time shall appoint.

XXVII.

Register Book.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

XXVIII.

Power of
Expulsion.

If, in the opinion of the Council of the Society, the conduct of any Fellow is unbecoming the position of a Member of a learned Society, or is injurious to the character and interests of this Society, the Council may request such Fellow to resign; and, if he fail to do so within one month of such request being addressed to him, the Council shall call a General Meeting of the Fellows of the Society to consider the matter; and, if a majority of the Fellows present at such Meeting agree to the expulsion of such Member, he shall be then and there expelled by the declaration of the Chairman of the said Meeting to that effect; and he shall thereafter cease to be a Fellow of the Society, and his name shall be erased from the Roll of Fellows, and he shall forfeit all right or claim in or to the property of the Society.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner :—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1915–1916 for the “best communication on a scientific subject, communicated,* in the first instance, to the Royal Society during the Sessions 1913–1914 and 1914–1915.” Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1914–1915, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.
2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1914.
3. The Competition is open to all men of science.
4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.
5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society.

* For the purposes of this award the word “communicated” shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented * to the Society during the Sessions 1912-13, 1913-14, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate :

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1915-1916.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the fourth Monday in October 1915,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. GUNNING, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

**AWARDS OF THE KEITH, MAKDOUGALL - BRISBANE,
NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.**

I. KEITH PRIZE.

- 1ST BIENNIAL PERIOD, 1827-29.—Dr BREWSTER, for his papers “on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals,” published in the Transactions of the Society.
- 2ND BIENNIAL PERIOD, 1829-31.—Dr BREWSTER, for his paper “on a New Analysis of Solar Light,” published in the Transactions of the Society.
- 3RD BIENNIAL PERIOD, 1831-33.—THOMAS GRAHAM, Esq., for his paper “on the Law of the Diffusion of Gases,” published in the Transactions of the Society.
- 4TH BIENNIAL PERIOD, 1833-35.—Professor J. D. FORBES, for his paper “on the Refraction and Polarization of Heat,” published in the Transactions of the Society.
- 5TH BIENNIAL PERIOD, 1835-37.—JOHN SCOTT RUSSELL, Esq., for his researches “on Hydrodynamics,” published in the Transactions of the Society.
- 6TH BIENNIAL PERIOD, 1837-39.—Mr JOHN SHAW, for his experiments “on the Development and Growth of the Salmon,” published in the Transactions of the Society.
- 7TH BIENNIAL PERIOD, 1839-41.—Not awarded.
- 8TH BIENNIAL PERIOD, 1841-1843.—Professor JAMES DAVID FORBES, for his papers “on Glaciers,” published in the Proceedings of the Society.
- 9TH BIENNIAL PERIOD, 1843-45.—Not awarded.
- 10TH BIENNIAL PERIOD, 1845-47.—General Sir THOMAS BRISBANE, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.
- 11TH BIENNIAL PERIOD, 1847-49.—Not awarded.
- 12TH BIENNIAL PERIOD, 1849-51.—Professor KELLAND, for his papers “on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations,” published in the Transactions of the Society.
- 13TH BIENNIAL PERIOD, 1851-53.—W. J. MACQUORN RANKINE, Esq., for his series of papers “on the Mechanical Action of Heat,” published in the Transactions of the Society.
- 14TH BIENNIAL PERIOD, 1853-55.—Dr THOMAS ANDERSON, for his papers “on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances,” published in the Transactions of the Society.
- 15TH BIENNIAL PERIOD, 1855-57.—Professor BOOLE, for his Memoir “on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments,” published in the Transactions of the Society.
- 16TH BIENNIAL PERIOD, 1857-59.—Not awarded.
- 17TH BIENNIAL PERIOD, 1859-61.—JOHN ALLAN BROUN, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers “on the Horizontal Force of the Earth’s Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally,” published in the Transactions of the Society.
- 18TH BIENNIAL PERIOD, 1861-63.—Professor WILLIAM THOMSON, of the University of Glasgow, for his Communication “on some Kinematical and Dynamical Theorems.”
- 19TH BIENNIAL PERIOD, 1863-65.—Principal FORBES, St Andrews, for his “Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars,” published in the Transactions of the Society.
- 20TH BIENNIAL PERIOD, 1865-67.—Professor C. PIAZZI SMYTH, for his paper “on Recent Measures at the Great Pyramid,” published in the Transactions of the Society.
- 21ST BIENNIAL PERIOD, 1867-69.—Professor P. G. TAIT, for his paper “on the Rotation of a Rigid Body about a Fixed Point” published in the Transactions of the Society.
- 22ND BIENNIAL PERIOD, 1869-71.—Professor CLERK MAXWELL, for his paper “on Figures, Frames, and Diagrams of Forces,” published in the Transactions of the Society.

- 23RD BIENNIAL PERIOD, 1871-73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.
- 24TH BIENNIAL PERIOD, 1873-75.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."
- 25TH BIENNIAL PERIOD, 1875-77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.
- 26TH BIENNIAL PERIOD, 1877-79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II. having appeared in the volume for 1877-78.
- 27TH BIENNIAL PERIOD, 1879-81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.
- 28TH BIENNIAL PERIOD, 1881-83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.
- 29TH BIENNIAL PERIOD, 1883-85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.
- 30TH BIENNIAL PERIOD, 1885-87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.
- 31ST BIENNIAL PERIOD, 1887-89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.
- 32ND BIENNIAL PERIOD, 1889-91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv. of the Society's Transactions.
- 33RD BIENNIAL PERIOD, 1891-93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv., xxxvi., and xxxvii. of the Society's Transactions.
- 34TH BIENNIAL PERIOD, 1893-95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.
- 35TH BIENNIAL PERIOD, 1895-97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.
- 36TH BIENNIAL PERIOD, 1897-99.—Dr JAMES BURGESS, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} dt$, with extended Tables of Values," printed in vol. xxxix. of the Transactions of the Society.
- 37TH BIENNIAL PERIOD, 1899-1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.
- 38TH BIENNIAL PERIOD, 1901-03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., &c., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I., II., likewise published in the Transactions of the Society.
- 39TH BIENNIAL PERIOD, 1903-05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxus*," published in the Transactions of the Society within the period.
- 40TH BIENNIAL PERIOD, 1905-07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.
- 41ST BIENNIAL PERIOD, 1907-09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."
- 42ND BIENNIAL PERIOD, 1909-11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

II. MAKDOUGALL-BRISBANE PRIZE.

- 1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.
- 2ND BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his “Memoir of the Life and Writings of Dr Robert Whytt,” published in the Transactions of the Society.
- 3RD BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. “Icarus,” for his paper “on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology,” published in the Transactions of the Society.
- 4TH BIENNIAL PERIOD, 1864–66.—Not awarded.
- 5TH BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper “on the Connection between Chemical Constitution and Physiological Action,” published in the Transactions of the Society.
- 6TH BIENNIAL PERIOD, 1868–70.—Not awarded.
- 7TH BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper “on the Homological Relations of the Ccelenterata,” published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.
- 8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper “on the Germ Theory of Putrefaction and the Fermentive Changes,” communicated to the Society, 7th April 1873.
- 9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper “on the Diurnal Oscillation of the Barometer,” published in the Transactions of the Society.
- 10TH BIENNIAL PERIOD, 1876–78.—Professor ARCHIBALD GEIKIE, for his paper “on the Old Red Sandstone of Western Europe,” published in the Transactions of the Society.
- 11TH BIENNIAL PERIOD, 1878–80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper “on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination,” published in the Transactions of the Society.
- 12TH BIENNIAL PERIOD, 1880–82.—Professor JAMES GEIKIE, for his “Contributions to the Geology of the North-West of Europe,” including his paper “on the Geology of the Faroes,” published in the Transactions of the Society.
- 13TH BIENNIAL PERIOD, 1882–84.—EDWARD SANG, Esq., LL.D., for his paper “on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor,” and generally for his Recalculation of Logarithms both of Numbers and Trigonometrical Ratios, —the former communication being published in the Proceedings of the Society.
- 14TH BIENNIAL PERIOD, 1884–86.—JOHN MURRAY, Esq., LL.D., for his papers “On the Drainage Areas of Continents, and Ocean Deposits,” “The Rainfall of the Globe, and Discharge of Rivers,” “The Height of the Land and Depth of the Ocean,” and “The Distribution of Temperature in the Scottish Lochs as affected by the Wind.”
- 15TH BIENNIAL PERIOD, 1886–88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled “History of Volcanic Action during the Tertiary Period in the British Isles,” published in the Transactions of the Society.
- 16TH BIENNIAL PERIOD, 1889–90.—Dr LUDWIG BECKER, for his paper on “The Solar Spectrum at Medium and Low Altitudes,” printed in vol. xxxvi. Part I. of the Society’s Transactions.
- 17TH BIENNIAL PERIOD, 1890–92.—HUGH ROBERT MILL, Esq., D.Sc., for his papers on “The Physical Conditions of the Clyde Sea Area,” Part I. being already published in vol. xxxvi. of the Society’s Transactions.
- 18TH BIENNIAL PERIOD, 1892–94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx. pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.
- 19TH BIENNIAL PERIOD, 1894–96.—Professor JOHN G. M’KENDRICK, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society’s publications.
- 20TH BIENNIAL PERIOD, 1896–98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.
- 21ST BIENNIAL PERIOD, 1898–1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled “Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland,” printed in vol. xxxix. of the Transactions of the Society.

- 22ND BIENNIAL PERIOD, 1900-02.—Dr ARTHUR T. MASTERMAN, for his paper entitled “The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development,” printed in vol. xl. of the Transactions of the Society.
- 23RD BIENNIAL PERIOD, 1902-04.—Mr JOHN DOUGALL, M.A., for his paper on “An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate,” published in vol. xli. of the Transactions of the Society.
- 24TH BIENNIAL PERIOD, 1904-06.—JACOB E. HALM, Ph.D., for his two papers entitled “Spectroscopic Observations of the Rotation of the Sun,” and “Some Further Results obtained with the Spectroheliometer,” and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.
- 25TH BIENNIAL PERIOD, 1906-08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, “On the Fossil Osmundaceæ,” and 2nd, “On the Origin of the Adaxially-curved Leaf-trace in the Filicales,” communicated by him conjointly with Dr R. Kidston.
- 26TH BIENNIAL PERIOD, 1908-10.—ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., for his series of papers bearing upon “The Temperature Distribution in Fresh-water Lochs,” and especially upon “The Temperature Seiche.”
- 27TH BIENNIAL PERIOD, 1910-12.—JOHN BROWNLEE, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.

III. THE NEILL PRIZE.

- 1ST TRIENNIAL PERIOD, 1856-59.—Dr W. LAUDER LINDSAY, for his paper “on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliate Lichens,” published in the Transactions of the Society.
- 2ND TRIENNIAL PERIOD, 1859-61.—ROBERT KAYE GREVILLE, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.
- 3RD TRIENNIAL PERIOD, 1862-65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.
- 4TH TRIENNIAL PERIOD, 1865-68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper “on the Structure of the British Nemerteans, and on some New British Annelids,” published in the Transactions of the Society.
- 5TH TRIENNIAL PERIOD, 1868-71.—Professor WILLIAM TURNER, for his papers “on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Foetal Membranes in the Cetacea,” published in the Transactions of the Society.
- 6TH TRIENNIAL PERIOD, 1871-74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.
- 7TH TRIENNIAL PERIOD, 1874-77.—Dr RAMSAY H. TRAQUAIR, for his paper “on the Structure and Affinities of *Tristichopterus alatus* (Egerton),” published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.
- 8TH TRIENNIAL PERIOD, 1877-80.—JOHN MURRAY, Esq., for his paper “on the Structure and Origin of Coral Reefs and Islands,” published (in abstract) in the Proceedings of the Society.
- 9TH TRIENNIAL PERIOD, 1880-83.—Professor HERDMAN, for his papers “on the Tunicata,” published in the Proceedings and Transactions of the Society.
- 10TH TRIENNIAL PERIOD, 1883-86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palaeontology of Scotland, published in the Transactions of the Society.
- 11TH TRIENNIAL PERIOD, 1886-89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.
- 12TH TRIENNIAL PERIOD, 1889-92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.
- 13TH TRIENNIAL PERIOD, 1892-95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

- 14TH TRIENNIAL PERIOD, 1895-98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.
- 15TH TRIENNIAL PERIOD, 1898-1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix. of the Transactions of the Society.
- 16TH TRIENNIAL PERIOD, 1901-04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxus*, published in the Philosophical Transactions of the Royal Society, London.
- 17TH TRIENNIAL PERIOD, 1904-07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoïd Fishes, based on a study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.
- 1ST BIENNIAL PERIOD, 1907-09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."
- 2ND BIENNIAL PERIOD, 1909-11.—JAMES MURRAY, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award : see Neill Prize Regulations).

IV. GUNNING VICTORIA JUBILEE PRIZE.

- 1ST TRIENNIAL PERIOD, 1884-87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.
- 2ND TRIENNIAL PERIOD, 1887-90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.
- 3RD TRIENNIAL PERIOD, 1890-93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's Publications.
- 4TH TRIENNIAL PERIOD, 1893-96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.
- 1ST QUADRENNIAL PERIOD, 1896-1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.
- 2ND QUADRENNIAL PERIOD, 1900-04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures : his earliest papers being published in the Transactions and Proceedings of the Society.
- 3RD QUADRENNIAL PERIOD, 1904-08.—Professor GEORGE CHRYSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."
- 4TH QUADRENNIAL PERIOD, 1908-12.—Professor J. NORMAN COLLIE, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

THE COUNCIL OF THE SOCIETY,

October 1913.

PRESIDENT.

JAMES GEIKIE, LL.D., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh.

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BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S., formerly District Superintendent and Acting Paleontologist of the Geological Survey of Scotland.

SIR EDWARD ALBERT SCHÄFER, M.R.C.S., LL.D., F.R.S., Professor of Physiology in the University of Edinburgh.

GENERAL SECRETARY.

CARGILL G. KNOTT, D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh.

SECRETARIES TO ORDINARY MEETINGS.

ROBERT KIDSTON, LL.D., F.R.S., F.G.S.

ARTHUR ROBINSON, M.D., M.R.C.S., Professor of Anatomy in the University of Edinburgh.

TREASURER.

JAMES CURRIE, M.A.

CURATOR OF LIBRARY AND MUSEUM.

JOHN SUTHERLAND BLACK, M.A., LL.D.

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JAMES H. ASHWORTH, D.Sc., Lecturer on Invertebrate Zoology in the University of Edinburgh.

JAMES GORDON GRAY, D.Sc., Lecturer on Physics in the University of Glasgow.

RALPH A. SAMPSON, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland, and Professor of Astronomy in the University of Edinburgh.

D'ARCY W. THOMPSON, C.B., B.A., F.L.S., Professor of Natural History in the University College, Dundee.

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A. P. LAURIE, M.A., D.Sc., Principal of the Heriot-Watt College, Edinburgh.

JOHN GRAHAM KERR, M.A., F.R.S., Professor of Zoology in the University of Glasgow.

LEONARD DOBBIN, Ph.D., Lecturer on Chemistry in the University of Edinburgh.

ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., W.S., D.Sc.

SOCIETY'S REPRESENTATIVE ON GEORGE HERIOT'S TRUST.

WILLIAM ALLAN CARTER, M.Inst.C.E.

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**ALPHABETICAL LIST OF THE ORDINARY FELLOWS
OF THE SOCIETY,**

Corrected to 1st January 1914.

N.B.—*Those marked * are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.

K.	"	"	"	Keith Medal.
N.	"	"	"	Neill Medal.
V. J.	"	"	"	the Gunning Victoria Jubilee Prize.
C.	"	"	"	contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Election.				Service on Council, etc.
1898	C.	* Abercromby, the Hon. John, LL.D., 62 Palmerston Place, Edinburgh		
1898		Adami, Prof. J. G., M.A., M.D. Cantab., F.R.S., Professor of Pathology in M'Gill University, Montreal		
1911		* Adams, Archibald Campbell, A.M.Inst.Mech.E., A.M.Inst.E.E., Consulting Engineer, 1 Old Smithills, Paisley		
1896		* Affleck, Sir Jas. Ormiston, M.D., LL.D., F.R.C.P.E., 38 Heriot Row, Edinburgh		
1871		Agnew, Sir Stair, K.C.B., M.A., formerly Registrar - General for Scotland, 22 Buckingham Terrace, Edinburgh	5	1882-85, 1886-89, 1891-93, 1895-98.
1875	C. K.	Aitken, John, LL.D., F.R.S., Ardenlea, Falkirk		
	V. J.			
1895		* Alford, Robert Gervase, M.Inst.C.E., 1 Windmill Hill, Hampstead, London, N.W.		
1889		Alison, John, M.A., Head Master, George Watson's College, Edinburgh		
1894		Allan, Francis John, M.D., C.M. Edin., M.O.H. City of Westminster, West- minster City Hall, Charing Cross Road, London		
1888	C.	Allardice, R. E., M.A., Professor of Mathematics in Stanford University, Palo Alto, Santa Clara Co., California	10	
1906		Anderson, Daniel E., M.D., B.A., B.Sc., Green Bank, Merton Lane, Highgate, London, N.		
1893		Anderson, J. Macvicar, Architect, 6 Stratton Street, London		
1883		Anderson, Sir Robert Rowand, LL.D., 16 Rutland Square, Edinburgh		
1905		Anderson, William, F.G.S., P.O. Box 635, Sydney, New South Wales, Australia		
1905		* Anderson, William, M.A., Head Science Master, George Watson's College, Edin- burgh, 29 Lutton Place, Edinburgh	15	
1903		Anderson-Berry, David, M.D., LL.D., F.R.S.L., M.R.A.S., F.S.A. (Scot.), Versailles, Highgate, London, N.		
1905		* Andrew, George, M.A., B.A., H.M.I.S., Balwherie, Strathearn Road, Broughty Ferry		
1881	C.	Anglin, A. H., M.A., LL.D., M.R.I.A., Professor of Mathematics, Queen's College, Cork		
1906		Appleton, Colonel Arthur Frederick, F.R.C.V.S., 22 Avenue Mansions, Finchley Road, London		
1899		Appleyard, James R., Royal Technical Institute, Salford, Manchester	20	
1893		* Archer, Walter E., 17 Sloane Court, London, S.W.		
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, Syracuse University, Syracuse, N.Y., U.S.A.		
1907		* Archibald, James, M.A., Head Master, St Bernard's School, 1 Leamington Terrace, Edinburgh		
1911	C.	* Ashworth, James Hartley, D.Sc., Lecturer on Invertebrate Zoology, University of Edinburgh, 4 Cluny Terrace, Edinburgh		1912-

Date of Election.			Service on Council, etc.
1907			
1896	C.	* Badre, Muhammad, Ph.D., Almuneerah, Cairo, Egypt	25
	C.	* Baily, Francis Gibson, M.A., M.Inst.E.E., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh, Newbury, Colinton, Midlothian	1909-12.
1877	C.	Balfour, I. Bayley, M.A., Sc.D., M.D., LL.D., F.R.S., F.L.S., King's Botanist in Scotland, Professor of Botany in the University of Edinburgh and Keeper of the Royal Botanic Garden, Inverleith House, Edinburgh	1888-91.
1905	C.	Balfour-Browne, William Alexander Francis, M.A., Barrister-at-Law, 26 Barton Road, Cambridge	
1892		* Ballantyne, J. W., M.D., F.R.C.P.E., 19 Rothesay Terrace, Edinburgh	
1902	C.	Bannerman, W. B., C.S.I., I.M.S., M.D., D.Sc., Surgeon General, Indian Medical Service, Madras, India	30
1889		Barbour, A. H. F., M.A., LL.D., F.R.C.P.E., 4 Charlotte Square, Edinburgh	
1886		Barclay, A. J. Gunion, M.A., 729 Great Western Road, Glasgow	
1883	C.	Barclay, G. W. W., M.A., Raeden House, Aberdeen	
1910		* Barclay, Lewis Bennett, C.E., 13 Cargill Terrace, Edinburgh	
1903		Bardswell, Noël Dean, M.D., M.R.C.P. Ed. and Lond., King Edward VII. Sanatorium, Midhurst	35
1882	C.	Barnes, Henry, M.D., LL.D., 6 Portland Square, Carlisle	
1904		Barr, Sir James, M.D., LL.D., F.R.C.P. Lond., 72 Rodney Street, Liverpool	
1874		Barrett, Sir William F., F.R.S., M.R.I.A., formerly Professor of Physics, Royal College of Science, Dublin, 6 De Vesci Terrace, Kingstown, County Dublin	
1887		Bartholomew, J. G., LL.D., F.R.G.S., The Geographical Institute, Duncan Street, Edinburgh	1909-12.
1895	C.	Barton, Edwin H., D.Sc., A.M.Inst.E.E., Fellow Physical Society of London, Professor of Experimental Physics, University College, Nottingham	40
1904		* Baxter, William Muirhead, Glenalmond, Sciennes Gardens, Edinburgh	
1913		Beard, Joseph, F.R.C.S. (Edin.), M.R.C.S. (Eng.), L.R.C.P. (Lond.), D.P.H. (Camb.), Medical Officer of Health and School Medical Officer, City of Carlisle, 15 Brunswick Street, Carlisle	
1888		Beare, Thomas Hudson, B.Sc., M.Inst.C.E., Professor of Engineering in the University of Edinburgh (VICE-PRESIDENT)	{ 1907-1909. V.P 1909-
1897	C.	* Beattie, John Carruthers, D.Sc., Professor of Physics, South African College, Cape Town	
1892		Beck, Sir J. H. Meiting, Kt., M.D., M.R.C.P.E., Drostdy, Tulbagh, Cape Province, South Africa	45
1893	B. C.	* Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow	
1882	C.	Beddard, Frank E., M.A. Oxon., F.R.S., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London	
1887		Begg, Ferdinand Faithfull, Bartholomew House, London	
1906		Bell, John Patrick Fair, F.Z.S., Fulforth, Witton Gilbert, Durham	
1900		* Bennett, James Bower, C.E., 5 Hill Street, Edinburgh	50
1887		Bernard, J. Mackay, of Dunsinian, B.Sc., Dunsinian, Perth	
1893	C.	* Berry, George A., M.D., C.M., F.R.C.S., 31 Drumsheugh Gardens, Edinburgh	
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Professor of Anatomy in the University of Melbourne, Victoria, Australia	
1904		* Beveridge, Erskine, LL.D., St Leonards Hill, Dunfermline	
1880	C.	Birch, De Burgh, C.B., M.D., Professor of Physiology in the University of Leeds, 8 Osborne Terrace, Leeds	55
1907		* Black, Frederick Alexander, Solicitor, 59 Academy Street, Inverness	
1884	C.	Black, John S., M.A., LL.D. (CURATOR OF LIBRARY AND MUSEUM), 6 Oxford Terrace, Edinburgh	{ 1891-94. Cur. 1906-
1897		* Blaikie, Walter Biggar, LL.D., The Loan, Colinton	
1904	C.	* Bles, Edward J., M.A., D.Sc., Elterholm, Cambridge	
1898	C.	* Blyth, Benjamin Hall, M.A., V.P.Inst.C.E., 17 Palmerston Place, Edinburgh	60
1894		* Bolton, Herbert, M.Sc., F.G.S., F.Z.S., Director of the Bristol Museum and Art Gallery, Bristol	
1872	C.	Bottomley, J. Thomson, M.A., D.Sc., LL.D., F.R.S., F.C.S., 13 University Gardens, Glasgow	
1886		Bower, Frederick O., M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow, 1 St John's Terrace, Hillhead, Glasgow (VICE-PRESIDENT)	{ 1887-90, 1893-96, 1907-09. V.P 1910-

Alphabetical List of the Ordinary Fellows of the Society.

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Date of Election.			Service on Council, etc.
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., A.Inst.C.E., A.Inst.M.E., M.Inst.E.E., etc., 4 Albert Square, Manchester	
1901		Bradbury, J. B., M.D., Downing Professor of Medicine, University of Cambridge	65
1903	C.	* Bradley, O. Charnock, M.D., D.Sc., Principal, Royal Veterinary College, Edinburgh	1907-10.
1886		Bramwell, Byrom, M.D., F.R.C.P.E., 23 Drumsheugh Gardens, Edinburgh	1890-93.
1907		* Bramwell, Edwin, M.B., F.R.C.P.E., F.R.C.P. Lond., 24 Walker Street, Edinburgh	
1912		Brider, Adolphus Edward, M.D. (Edin.), F.R.C.P. (Edu.), B.Sc. (Paris), B.L. (Paris), Foley Lodge, Langham Street, London, W.	
1895		Bright, Charles, M.Inst.C.E., M.Inst.E.E., F.R.A.S., F.G.S., Consulting Engineer to the Commonwealth of Australia, The Grange, Leigh, Kent, and Members' Mansions, Victoria Street, London, S.W.	70
1893		Brock, G. Sandison, M.D., 6 Corso d'Italia, Rome, Italy	
1901	C.	* Brodie, W. Brodie, M.B., Thaxted, Dunmow, Essex	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, South African College, Cape Town	
1864	C.	Brown, Alex. Crum, M.A., M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent, Edinburgh	1865-68, 1869-72, 1873-75, 1876-78, 1911-13. Sec. 1879-1905. V-P 1905-11.
1913	C.	* Brown, Alexander Russell, M.A., B.Sc., Science Master, Buckhaven Junior Student Centre, Norfield, Buckhaven	75
1898		* Brown, David, F.C.S., F.I.C., Willowbrae House, Willowbrae Road, Edinburgh	
1911		* Brown, David Rainy, Chemical Manufacturer (J. F. Macfarlan & Co.), 93 Abbeyhill, Edinburgh	
1883	C.	Brown, J. J. Graham, M.D., F.R.C.P.E., 3 Chester Street, Edinburgh	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 64 Upper Berkeley Street, Portman Square, London, W.	
1909	B. C.	* Brownlee, John, M.A., M.D., D.Sc., Ruchill Hospital, Bilsland Drive, Glasgow	80
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh	
1906	C.	* Bruce, William Speirs, LL.D., Director of the Scottish Oceanographical Laboratory, Edinburgh, Antarctica, Joppa, Midlothian	1909-12.
1898	K. C.	* Bryce, T. H., M.A., M.D. (Edin.), Professor of Anatomy in the University of Glasgow, 2 The University, Glasgow	1911-
1870	C. K.	Buchanan, John Young, M.A., F.R.S., 26 Norfolk Street, Park Lane, London, W.	1878-81, 1884-86.
1887	C.	Buist, J. B., M.D., F.R.C.P.E., 1 Clifton Terrace, Edinburgh	85
1905		Bunting, Thomas Lowe, M.D., 27 Denton Road, Scotswood, Newcastle-on-Tyne	
1902		* Burgess, A. G., M.A., Mathematical Master, Edinburgh Ladies' College, 64 Strathearn Road, Edinburgh	
1894	C. K.	* Burgess, James, C.I.E., LL.D., Hon. A.R.I.B.A., F.R.G.S., Hon. M. Imp. Russ. Archael. Soc., and Amer. Or. Soc., M. Soc. Asiat. de Paris, M.R.A.S., H. Corr. M. Batavian Soc. of Arts and Sciences, and Berlin Soc. Anthrop., H. Assoc. Finno-Ugrian Soc. (VICE-PRESIDENT), 22 Seton Place, Edinburgh	1895-98, 1899-1902. V-P 1908-
1902		* Burn, Rev. John Henry, B.D., The Parsonage, Ballater	
1887		Burnet, John James, Architect, 18 University Avenue, Hillhead, Glasgow	90
1888		Burns, Rev. T., D.D., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent, Edinburgh	
1896		* Butters, J. W., M.A., B.Sc., Rector of Ardrossan Academy	
1887	C.	Cadell, Henry Moubray, of Grange, B.Sc., Linlithgow	
1897		* Caird, Robert, LL.D., Shipbuilder, Greenock	
1910		* Calderwood, Rev. Robert Sibbald, Minister of Cambuslang, The Manse, Cambuslang, Lanarkshire	95
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, South Bank, Canaan Lane, Edinburgh	
1894		* Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S. Eng., Anatomy Department, Middlesex Hospital Medical School, London, W.	

Date of Election.		Service on Council, etc.
1904	* Campbell, Charles Duff, 21 Montague Terrace, Inverleith Row, Edinburgh	
1899	C. * Carlier, Edmund W. W., M.D., M.Sc., F.E.S., Professor of Physiology, University, Birmingham	100
1910	Carnegie, David, M.Inst.C.E., M.Inst.Mech.E., M.I.S.Inst., 33-35 Charterhouse Square, London, E.C.	
1905	C. * Carse, George Alexander, M.A., D.Sc., Lecturer on Natural Philosophy, University of Edinburgh, 3 Middleby Street, Edinburgh	
1901	Carlaw, H. S., M.A., D.Sc., Professor of Mathematics in the University of Sydney, New South Wales	
1905	Carter, Joseph Henry, F.R.C.V.S., Rowley Hall, Burnley, Lancashire	
1898	* Carter, Wm. Allan, M.Inst.C.E., 32 Great King Street, Edinburgh (Society's Representative on George Heriot's Trust)	105
1898	Carus-Wilson, Cecil, F.R.G.S., F.G.S., Waldegrave Park, Strawberry Hill, Middlesex, and Sandacres Lodge, Parkstone-on-Sea, Dorset	
1908	Cavanagh, Thomas Francis, M.D., The Hospital, Bella Coola, B.C., Canada	
1882	Cay, W. Dyce, M.Inst.C.E., 39 Victoria Street, Westminster, London	
1899	Chatham, James, Actuary, 7 Belgrave Crescent, Edinburgh	
1912	Chaudhuri, Banawari Lal, B.A.(Cal.), B.Sc. (Edin.), Assistant Superintendent, Natural History Section, Indian Museum, 120 Lower Circular Road, Calcutta, India	110
1874	Chiene, John, C.B., M.D., LL.D., F.R.C.S.E., Emeritus Professor of Surgery in the University of Edinburgh, Barnton Avenue, Davidson's Mains	
1891	* Clark, John B., M.A., Head Master of Heriot's Hospital School, Lauriston, Garleffin, Craigie Drive, Edinburgh	
1911	* Clark, William Inglis, D.Sc., 29 Lauder Road, Edinburgh	
1903	* Clarke, William Eagle, F.L.S., Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh, 35 Braid Road, Edinburgh	
1909	Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Gateshead, 13 The Crescent, Gateshead-on-Tyne	115
1913	* Cleghorn, Alexander, M.Inst.C.E., Marine Engineer, 14 Hatfield Drive, Kelvinside, Glasgow	
1875	Clouston, Sir T. S., M.D., LL.D., F.R.C.P.E., 26 Heriot Row, Edinburgh	
1904	C. Coker, Ernest George, M.A., D.Sc., Professor of Mechanical Engineering and Applied Mechanics, City and Guilds Technical College, Finsbury, Leonard Street, City Road, London, E.C.	
1904	Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1888	V. J. Collie, John Norman, Ph.D., D.Sc., LL.D., F.R.S., F.C.S., F.I.C., F.R.G.S., Professor of Organic Chemistry in the University College, Gower Street, London	120
1904	* Colquhoun, Walter, M.A., M.B., 18 Walmer Crescent, Ibrox, Glasgow	
1909	C. Comrie, Peter, M.A., B.Sc., Head Mathematical Master, Boroughmuir Junior Student Centre, 19 Craighouse Terrace, Edinburgh	
1886	Connan, Daniel M., M.A.	
1872	Constable, Archibald, LL.D., 11 Thistle Street, Edinburgh	
1894	Cook, John, M.A., formerly Principal, Central College, Bangalore, Director of Meteorology in Mysore, and Fellow, University of Madras, India, 10 Elm Place, Aberdeen	125
1891	* Cooper, Charles A., LL.D., 41 Drumsheugh Gardens, Edinburgh	
1905	* Corrie, David, F.C.S., Nobel's Explosives Company, Polmont, Stirlingshire	
1914	* Coutts, William Barron, M.A., B.Sc., 33 Dalhousie Terrace, Edinburgh	
1911	* Cowan, Alexander C., Papermaker, Valleyfield House, Penicuik, Midlothian	
1908	Craig, James Ireland, M.A., B.A., Controller of the Department of General Statistics, 14 Abdin Street, Cairo : The Koubbeh Gardens, near Cairo, Egypt	130
1875	Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place, Edinburgh	
1907	* Cramer, William, Ph.D., Lecturer in Physiological Chemistry in the University of Edinburgh, Physiological Department, The University, Edinburgh	
1903	Crawford, Lawrence, M.A., D.Sc., Professor of Mathematics in the South African College, Cape Town	
1887	Crawford, William Caldwell, 1 Lockharton Gardens, Colinton Road, Edinburgh	
1870	Crichton-Browne, Sir Jas., M.D., LL.D., D.Sc., F.R.S., Lord Chancellor's Visitor and Vice-President and Treasurer of the Royal Institution of Great Britain, 45 Hans Place, S.W., and Royal Courts of Justice, Strand, London	135
1886	Croom, Sir John Halliday, M.D., F.R.C.P.E., Professor of Midwifery in the University of Edinburgh, late President, Royal College of Surgeons, Edinburgh, 25 Charlotte Square, Edinburgh	

Alphabetical List of the Ordinary Fellows of the Society.

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Date of Election.	Service on Council, etc.
1898	* Currie, James, M.A. Cantab. (TREASURER), Larkfield, Goldenacre, Edinburgh
1904	* Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock
1885	Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London
1912	* Darbshire, Arthur Dukinfield, M.A., Lecturer in Genetics at the University of Edinburgh, 63 Frederick Street, Edinburgh
1884	Davy, R., F.R.C.S. Eng., Consulting Surgeon to Westminster Hospital, Burstone House, Bow, North Devon
1894	* Denny, Sir Archibald, Bart., LL.D., Cardross Park, Cardross, Dumbartonshire
1869	Dewar, Sir James, Kt., M.A., LL.D., D.C.L., D.Sc., F.R.S., V.P.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London
V. J.	1872-74.
1905	* Dewar, James Campbell, C.A., 27 Douglas Crescent, Edinburgh
1906	* Dewar, Thomas William, M.D., F.R.C.P., Kincairn, Dunblane
1904	Dickinson, Walter George Burnett, F.R.C.V.S., Boston, Lincolnshire
1884	Dickson, the Right Hon. Charles Scott, K.C., LL.D., 22 Moray Place, Edinburgh
1888	C. Dickson, Henry Newton, M.A., D.Sc., 160 Castle Hill, Reading
1876	C. Dickson, J. D. Hamilton, M.A., Senior Fellow and formerly Tutor, St Peter's College, Cambridge
1885	C. Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California, Wesley Avenue, Los Angeles, California, U.S.A.
	150
1897	* Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue, Edinburgh
1904	* Dobbie, James Johnston, M.A., D.Sc., LL.D., F.R.S., Principal of the Government Laboratories, London, 4 Vicarage Gate, Kensington, London, W.
1881	C. Dobbin, Leonard, Ph.D., Lecturer on Chemistry in the University of Edinburgh, 6 Wilton Road, Edinburgh
1867	C. Donaldson, Sir James, M.A., LL.D., Principal of the University of St Andrews
1905	* Donaldson, Rev. Wm. Galloway, F.R.G.S., F.E.I.S., The Manse, Forfar
1882	Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh
1901	* Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow, 2 Royal Crescent, Glasgow
1866	Douglas, David, 22 Drummond Place, Edinburgh
1910	* Douglas, Loudon MacQueen, Author and Lecturer, 3 Lauder Road, Edinburgh
1908	C. Drinkwater, Harry, M.D., M.R.C.S. (Eng.), F.L.S., Lister House, Wrexham, North Wales
1901	* Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh
1904	* Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh
1903	Dunstan, John, M.R.C.V.S., Inversnaid, Liskeard, Cornwall
1892	C. Dunstan, M. J. R., M.A., F.I.C., F.C.S., Principal, South-Eastern Agricultural College, Wye, Kent
1899	* Duthie, George, M.A., Inspector-General of Education, Salisbury, Rhodesia
1906	C. * Dyson, Frank Watson, M.A., LL.D., F.R.S., Astronomer Royal, Royal Observatory, Greenwich
1893	Edington, Alexander, M.D., Howick, Natal
1904	* Edwards, John, 4 Great Western Terrace, Kelvinside, Glasgow
1904	* Elder, William, M.D., F.R.C.P.E., 4 John's Place, Leith
1875	Elliot, Daniel G., American Museum of Natural History, Central Park West, New York, N.Y., U.S.A.
1913	* Elliot, George Francis Scott, M.A. (Cantab.), B.Sc., F.R.G.S., F.L.S., Drumwhill, Mossdale
1906	C. * Ellis, David, D.Sc., Ph.D., Lecturer in Botany and Bacteriology, Glasgow and West of Scotland Technical College, Glasgow
1897	C. * Erskine-Murray, James Robert, D.Sc., 77 Kingsfield Road, Watford, Herts
1884	Evans, William, F.F.A., 38 Morningside Park, Edinburgh
1879	C. N. Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.Z.S., Regius Professor of Natural History, University of Edinburgh, Craigybield, Penicuik, Midlothian
1902	* Ewen, John Taylor, B.Sc., M.I.Mech.E., H.M. Inspector of Schools, 104 King's Gate, Aberdeen
	1882-85, 1904-07. V-P 1907-12.

Date of Election.			Service on Council, etc.
1878	C.	Ewing, Sir James Alfred, K.C.B., M.A., B.Sc., LL.D., M.Inst.C.E., F.R.S., Director of Naval Education, Admiralty, Frogmore, Edenbridge, Kent	1888-91.
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Guy's Hospital (Bacteriological Department), London	
1910	*	Fairgrieve, Mungo M'Callum, M.A. (Glasg.), M.A. (Cambridge), Master at the Edinburgh Academy, 37 Queen's Crescent, Edinburgh	
1875		Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds	180
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., Lecturer on Geography, The University, Glasgow.	
1888	C.	Fawsitt, Charles A., 9 Foremount Terrace, Dowanhill, Glasgow	
1883	C.	Felkin, Robert W., M.D., F.R.G.S., Fellow of the Anthropological Society of Berlin, 47 Bassett Road, North Kensington, London, W.	
1899	*	Fergus, Andrew Freeland, M.D., 22 Blythswood Square, Glasgow	
1907	*	Fergus, Edward Oswald, 12 Clainment Gardens, Glasgow	185
1904	*	Ferguson, James Haig, M.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent, Edinburgh	
1888	*	Ferguson, John, M.A., LL.D., Professor of Chemistry in the University of Glasgow	
1898	*	Findlay, John R., M.A. Oxon., 27 Drumsheugh Gardens, Edinburgh	
1899	*	Finlay, David W., B.A., M.D., LL.D., F.R.C.P., D.P.H., Emeritus Professor of Medicine in the University of Aberdeen, Honorary Physician to His Majesty in Scotland, 23 Dundonald Road, Glasgow, W.	
1911		Fleming, John Arnold, F.C.S., etc., Pottery Manufacturer, Woodburn, Rutherglen, Glasgow	190
1906	*	Fleming, Robert Alexander, M.A., M.D., F.R.C.P.E., Assistant Physician, Royal Infirmary, 10 Chester Street, Edinburgh	
1900	C. N.	* Flett, John S., M.A., D.Sc., LL.D., F.R.S., Director of the Geological Survey of Scotland, 33 George Square, Edinburgh	
1872	C.	Forbes, Professor George, M.A., M.Inst.C.E., M.Inst.E.E., F.R.S., F.R.A.S., 11 Little College Street, Westminster, S.W.	
1904		Forbes, Norman Hay, F.R.C.S.E., L.R.C.P. Lond., M.R.C.S. Eng., Corres. Memb. Soc. d'Hydrologie médicale de Paris, Drumminor, Church Stretton, Salop	
1892	*	Ford, John Simpson, F.C.S., 4 Nile Grove, Edinburgh	195
1910	*	Fraser, Alexander, Actuary, 17 Eildon Street, Edinburgh	
1858		Fraser, A. Campbell, Fellow of the British Academy, Hon. D.C.L. Oxford, LL.D., Litt.D., Emeritus Professor of Logic and Metaphysics in the University of Edinburgh, 34 Melville Street, Edinburgh	1879-82.
1896	*	Fraser, John, M.B., F.R.C.P.E., formerly one of H.M. Commissioners in Lunacy for Scotland, 54 Great King Street, Edinburgh	
1867	C.	Fraser, Sir Thomas R., Kt., M.D., LL.D., Sc.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh, Honorary Physician to the King in Scotland, 13 Drumsheugh Gardens, Edinburgh. (VICE-PRESIDENT)	1870-73, 1877-79, 1883-86, 1894-97.
1914	K. B.	*	V.P 1911-
1891	*	Fraser, William, Managing Director, Neill & Co., Ltd., Printers, 17 Eildon Street, Edinburgh	200
1891	*	Fullarton, J. H., M.A., D.Sc., 23 Porchester Gardens, London, W.	
1891	*	Fulton, T. Wemyss, M.D., Scientific Superintendent, Scottish Fishery Board, 41 Queen's Road, Aberdeen	
1907	*	Galbraith, Alexander, Superintendent Engineer, Cunard Line, Liverpool, 93 Trinity Road, Bootle, Liverpool	
1888	C.	Galt, Alexander, D.Sc., Keeper of the Technological Department, Royal Scottish Museum, Edinburgh	
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	205
1899		Gatehouse, T. E., A.M. Inst.C.E., M.Inst.M.E., M.Inst.E.E., Fairfield, 128 Tulse Hill, London, S.W.	
1867	C.	Gayner, Charles, M.D., F.L.S.	
1909	*	Geddes, Auckland C., M.D., Professor of Anatomy, Royal College of Surgeons in Ireland, Dublin	
1880	C.	Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh	
1861	C. B.	Geikie, Sir Archibald, O.M., K.C.B., D.C.L. Oxf., D.Sc., LL.D., Ph.D., Late Pres. R.S., Foreign Member of the Reale Accad. Lincei, Rome, of the National Acad. of the United States, of the Academies of Stockholm, Christiania, Göttingen, Corresponding Member of the Institute of France and of the Academies of Berlin, Vienna, Munich, Turin, Belgium, Philadelphia, New York, etc., Shepherd's Down, Haslemere, Surrey	1869-72, 1874-76, 1879-82.
			210

Date of Election.			Service on Council, etc.
1871	C. B.	Geikie, James, LL.D., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh (PRESIDENT), Kilmorie, Colinton Road, Edinburgh	1882-85, 1888-91, 1897-99. V-P 1892-97. 1900-05. P. 1913-
1914		Gemmell, John Edward, M.B., C.M., Hon. Surgeon Hospital for Women and Maternity Hospital; Hon. Gynaecologist, Victoria Central Hospital, Liscard, 28 Rodney Street, Liverpool.	
1909		* Gentle, William, B.Sc., 12 Mayfield Road, Edinburgh	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University, 15 South Lauder Road, Edinburgh	
1912	C.	* Gibson, Arnold Hartley, D.Sc., Professor of Engineering, University College, Dundee	215
1910		* Gibson, Charles Robert, Lynton, Mansewood, by Pollokshaws	
1890		* Gibson, George A., M.A., LL.D., Professor of Mathematics in the University of Glasgow, 10 The University, Glasgow	
1911		Gidney, Henry A. J., L.M. and S. Socts. Ap. (Lond.), F.R.C.S. (Edin.), D.P.H. (Camb.), D.O. (Oxford), Army Specialist Public Health, c/o Thomas Cook & Sons, Ludgate Circus, London	1905-08. 1912-13.
1900		Gilchrist, Douglas A., B.Sc., Professor of Agriculture and Rural Economy, Armstrong College, Newcastle-upon-Tyne	
1880		Gilruth, George Ritchie, Surgeon, 53 Northumberland Street, Edinburgh	220
1907		Gilruth, John Anderson, M.R.C.V.S., D.V.Sc. (Melb.), Administrator, Government House, Darwin Northern Territory, Australia	
1909		* Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., Capenoch, Thornhill, Dumfriesshire	
1911		Gladstone, Reginald John, M.D., F.R.C.S. (Eng.), Lecturer on Embryology and Senior Demonstrator of Anatomy, Middlesex Hospital, London, 22 Regent's Park Terrace, London, N.W.	
1898		* Glaister, John, M.D., F.R.F.P.S. Glasgow, D.P.H. Camb., Professor of Forensic Medicine in the University of Glasgow, 3 Newton Place, Glasgow	
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.S.A. (Eng.), Lecturer on Physiology, Middlesex Hospital, London, Annandale Lodge, Vanbrugh Park, Blackheath, London, S.E.	225
1901		Goodwillie, James, M.A., B.Sc., Liberton, Edinburgh	
1899		* Goodwin, Thomas S., M.B., C.M., F.C.S., 25 Worple Road, Isleworth, and Derwent Lodge, London Road, Spring-grove, Isleworth, Middlesex	
1913	C.	* Gordon, William Thomas, M.A., D.Sc. (Edin.), B.A. (Cantab.), Lecturer in Palaeontology, Edinburgh University, Goldie Lea, Eskbank, Midlothian	
1897		Gordon-Munn, John Gordon, M.D., Heigham Hall, Norwich	
1891		* Graham, Richard D., 11 Strathearn Road, Edinburgh	230
1898	C.	* Gray, Albert A., M.D., 4 Clairmont Gardens, Glasgow	
1883	C.	Gray, Andrew, M.A., LL.D., F.R.S., Professor of Natural Philosophy in the University of Glasgow	1903-06. V-P 1906-09.
1910		Gray, Bruce M'Gregor, C.E., A.M.Inst C.E., Westbourne Grove, Selby, Yorkshire	
1909	C.	* Gray, James Gordon, D.Sc., Lecturer in Physics in the University of Glasgow, 11 The University, Glasgow	1913-
1910		* Green, Charles Edward, Publisher, Gracemount House, Liberton	235
1886		Greenfield, W. S., M.D., F.R.C.P.E., LL.D., Emeritus Professor of General Pathology in the University of Edinburgh, Kirkbrae, Elie, Fife	
1897		Greenlees, Thomas Duncan, M.D. Edin., Rostrevor, Kirtleton Avenue, Weymouth, Dorset	
1905	C.	* Gregory, John Walter, D.Sc., F.R.S., Professor of Geology in the University of Glasgow, 4 Park Quadrant, Glasgow	1908-11.
1906		Greig, Edward David Wilson, M.D., B.Sc., Captain, H.M. Indian Medical Service, Byculla Club, Bombay, India	
1905		Greig, Robert Blyth, F.Z.S., Board of Agriculture for Scotland, 29 St. Andrew Square, Edinburgh	240
1910		* Grimshaw, Percy Hall, Assistant Keeper, Natural History Department, The Royal Scottish Museum, 49 Comiston Drive, Edinburgh	
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Newbattle Terrace, Edinburgh	
1907	C.	* Gulliver, Gilbert Henry, B.Sc., A.M.I.Mech.E., Lecturer in Experimental Engineering in the University of Edinburgh, 10 Stanley Street, Portobello	

Date of Election.			Service on Council, etc.
1911	C.	* Gunn, James Andrew, M.A., M.D., D.Sc., Department of Pharmacology, University Museum, Oxford	
1888	C.	Guppy, Henry Brougham, M.B., Rosario, Salcombe, Devon	245
1911	C.	* Guy, William, F.R.C.S., L.R.C.P., L.D.S. Ed., Consulting Dental Surgeon, Edinburgh Royal Infirmary ; Dean, Edinburgh Dental Hospital and School ; Lecturer on Human and Comparative Dental Anatomy and Physiology, 11 Wemyss Place, Edinburgh	
1910	B. C.	Gwynne-Vaughan, D. T., F.L.S., Professor of Botany, Queen's University, Belfast, The Cottage, Balmoral, Belfast	
1911		Hall-Edwards, John Francis, L.R.C.P. (Edin.), Hon. F.R.P.S., Senior Medical Officer in charge of X-ray Department, General Hospital, Birmingham, 141A and 141B Great Charles Street (Newhall Street), Birmingham	
1905	B. C.	* Halm, Jacob E., Ph.D., Chief Assistant Astronomer, Royal Observatory, Cape Town, Cape of Good Hope	
1899		Hamilton, Allan M'Lane, M.D., LL.D., 36 East 40th Street, New York, U.S.A.	250
1876	C.	Hannay, J. Ballantyne, Cove Castle, Loch Long	
1896	C.	* Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., F.S.A. Scot., Professor of Physiology in the Dalhousie University, Halifax, Nova Scotia	
1914		Harrison, Edward Philip, Ph.D., Professor of Physics, Presidency College, University of Calcutta, The Observatory, Alipore, Calcutta	
1888	C.	Hart, D. Berry, M.D., F.R.C.P.E., 5 Randolph Cliff, Edinburgh	
1869		Hartley, Sir Charles A., K.C.M.G., M.Inst.C.E., 26 Pall Mall, London	255
1881		Harvie-Brown, J. A., of Quarter, LL.D., F.Z.S., Dunipace House, Larbert, Stirlingshire	
1880	C.	Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff	
1892	C.	* Heath, Thomas, B.A., formerly Assistant Astronomer, Royal Observatory, Edinburgh, 11 Cluny Drive, Edinburgh	
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India	
1890	C.	Helme, T. Arthur, M.D., M.R.C.P., M.R.C.S., 3 St Peter's Square, Manchester	260
1900		Henderson, John, D.Sc., A.Inst.E.E., Kinnoul, Warwick's Bench Road, Guildford, Surrey	
1908		* Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Bristol	
1890	C.	Hepburn, David, M.D., Professor of Anatomy in the University College of South Wales and Monmouthshire, Cardiff	
1881	C. N.	Herdman, W. A., D.Sc., F.R.S., Past Pres.L.S., Professor of Natural History in the University of Liverpool, Croxteth Lodge, Ullet Road, Liverpool	
1908		* Hewat, Archibald, F.F.A., F.I.A., 13 Eton Terrace, Edinburgh	265
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Valentine Mount, Freshwater Bay, Isle of Wight	
1902		* Hinxman, Lionel W., B.A., Geological Survey Office, 33 George Square, Edinburgh	
1904		Hobday, Frederick T. G., F.R.C.V.S., 6 Berkely Gardens, Kensington, London, W.	
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Professor of Chemistry and Physics at the Ordnance College, Woolwich, 89 Shooter's Hill Road, Blackheath, Kent	
1911		Holland, William Jacob, LL.D. St Andrews, etc., Director Carnegie Institute, Pittsburg, Pa., 5545 Forbes Street, Pittsburg, Pa., U.S.A.	270
1881	C. N.	Horne, John, LL.D., F.R.S., F.G.S., formerly Director of the Geological Survey of Scotland, 12 Keith Crescent, Blackhall, Midlothian	
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley	
1904	C.	* Horsburgh, Ellice Martin, M.A., B.Sc., Lecturer in Technical Mathematics, University of Edinburgh, 11 Granville Terrace, Edinburgh	
1897		Houston, Alex. Cruikshanks, M.B., C.M., D.Sc., 19 Fairhazel Gardens, South Hampstead, London, N.W.	
1912	C.	* Houston, Robert Alexander, M.A., Ph.D., D.Sc., Lecturer in Physical Optics, University, Glasgow, 11 Cambridge Drive, Glasgow	275
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., Professor of Anatomy in the University of Durham, 14 Burdon Terrace, Newcastle-on-Tyne	
1899		Howie, W. Lamond, F.C.S., 26 Neville Court, Abbey Road, Regent's Park, London, N.W.	
1883	C.	Hoyle, William Evans, M.A., D.Sc., M.R.C.S., Director of the Welsh National Museum ; Crowland, Llandaff, Wales	

1902-05,
1906-07.
V-P
1907-1913.

Date of Election.		Service on Council, etc.
1910	Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt	
1886	Hunt, Rev. H. G. Bonavia, Mus.D. Dub., Mus.B. Oxon., The Vicarage, Burgess Hill, Sussex	280
1911	Hunter, Gilbert Macintyre, M.Inst.C.E., M.Inst.E.S., Resident Engineer Nitrate Railways, Iquique, Chile, and Maybole, Ayrshire	
1887	C. Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian	
1887	C. Hunter, William, M.D., M.R.C.P.L. and E., M.R.C.S., 54 Harley Street, London	
1908	Hyslop, Theophilus Bulkeley, M.D., M.R.C.P.E., 5 Portland Place, London, W.	
1882	C. Inglis, J. W., M.Inst.C.E., 26 Pitt Street, Edinburgh	285
1912	* Inglis, Robert John Mathieson, A.M.Inst.C.E., Engineer, Northern Division, North British Railway, Tantah, Peebles	
1904	C. Innes, R. T. A., Director, Government Observatory, Johannesburg, Transvaal	
1904	* Ireland, Alexander Scott, S.S.C., 2 Buckingham Terrace, Edinburgh	
1875	Jack, William, M.A., LL.D., Emeritus Professor of Mathematics in the University of Glasgow	
1894	Jackson, Sir John, C.V.O., LL.D., 48 Belgrave Square, London	290
1889	James, Alexander, M.D., F.R.C.P.E., 14 Randolph Crescent, Edinburgh	
1901	* Jardine, Robert, M.D., M.R.C.S. F.R.F.P.S. Glas., 20 Royal Crescent, Glasgow	
1912	C. * Jeffrey, George Rutherford, M.D. (Glasg.), F.R.C.P. (Edin.), etc., Bootham Park Private Mental Hospital, York	
1906	C. * Jehu, Thomas James, M.A., M.D., F.G.S., Lecturer in Geology, University of St Andrews, St Ronan's, St Andrews	
1900	* Jerdan, David Smiles, M.A., D.Sc., Ph.D., Temora, Colinton, Midlothian	295
1895	Johnston, Col. Henry Halcyro, C.B., R.A.M.S., D.Sc., M.D., F.L.S., Orphir House, Kirkwall, Orkney	
1903	C. * Johnston, Thomas Nicol, M.B., C.M., Pogbie, Humbie, East Lothian	
1902	Johnstone, George, Lieut. R.N.R., formerly Marine Superintendent, British India Steam Navigation Co., 26 Comiston Drive, Edinburgh	
1874	Jones, Francis, M.Sc., Lecturer on Chemistry, 17 Whalley Range, Whalley Range, Manchester	
1905	Jones, George William, M.A., B.Sc., LL.B., Scottish Tutorial Institute, Edinburgh and Glasgow, 25 North Bridge: Coraldene, Kirk Brae, Liberton, Edinburgh	300
1888	Jones, John Alfred, M.Inst.C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras, c/o Messrs Parry & Co., 70 Gracechurch Street, London	
1907	* Kemp, John, M.A., Sea Bank School, North Berwick	
1912	Kennedy, Robert Foster, M.D. (Queen's Univ., Belfast), M.B., B.Ch. (R.U.I.), Instructor in Neurology, and Chief of the Neurological Clinic, Cornell University, New York, 52 West 53rd Street, New York, U.S.A.	
1909	Kenwood, Henry Richard, M.B., Chadwick Professor of Hygiene in the University of London, 126 Queen's Road, Finsbury Park, London, N.	
1908	* Kerr, Andrew William, F.S.A. Scot., Royal Bank House, St Andrew Square, Edinburgh	305
1903	C. N. * Kerr, John Graham, M.A., F.R.S., Professor of Zoology in the University of Glasgow	
1891	Kerr, Joshua Law, M.D., The Chequers, Mittagong, Sydney, Australia	
1913	* Kerr, Walter Hume, M.A.; B.Sc., Lecturer on Engineering Drawing and Structural Design in the University of Edinburgh	
1908	Kidd, Walter Aubrey, M.D., 12 Montpelier Row, Blackheath, London	
1886	C. N. Kidston, Robert, LL.D., F.R.S., F.G.S. (SECRETARY), 12 Clarendon Place, Stirling	310
1907	* King, Archibald, M.A., B.Sc., formerly Rector of the Academy, Castle Douglas; Junior Inspector of Schools, La Maisonneuve, Clarkston, Glasgow	
1880	King, W. F., Lonend, Russell Place, Trinity, Leith	
1883	Kinnear, the Right Hon. Lord, P.C., one of the Senators of the College of Justice, 2 Moray Place, Edinburgh	
1878	Kintore, the Right Hon. the Earl of, P.C., G.C.M.G., M.A. Cantab., LL.D. Cambridge, Aberdeen and Adelaide, Keith Hall, Inverurie, Aberdeenshire	
1901	* Knight, Rev. G. A. Frank, M.A., St Leonard's United Free Church, Perth	315
1907	* Knight, James, M.A., D.Sc., F.C.S., F.G.S., Head Master, St James' School, Glasgow, The Shieling, Uddingston, by Glasgow	

1904-07,
1913-1891-94,
1903-06.Sec.
1909-

Date of Election.			Service on Council, etc.
1880	C. K.	Knott, C. G., D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh (formerly Professor of Physics, Imperial University, Japan) (GEN. SECRETARY), 42 Upper Gray Street, Edinburgh	1894-97, 1898-01, 1902-05. Sec. 1905-1912. Gen. Sec. 1912-
1886	C.	Laing, Rev. George P., 17 Buckingham Terrace, Edinburgh	
1878	C.	Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics, University of St Andrews	
1910		* Lauder, Alexander, D.Sc., F.I.C., Lecturer in Agricultural Chemistry, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 320	
1885	C.	Laurie, A. P., M.A., D.Sc., Principal of the Heriot-Watt College, Edinburgh	1908-11, 1913-
1894	C.	* Laurie, Malcoim, B.A., D.Sc., F.L.S., 19 Merchiston Park, Edinburgh	
1910	C.	* Lawson, A. Anstruther, B.Sc., Ph.D., D.Sc., F.L.S., Professor of Botany, University of Sydney, New South Wales, Australia	
1905		* Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	
1910	C.	* Lee, Gabriel W., D.Sc., Palaeontologist, Geological Survey of Scotland, 33 George Square, Edinburgh 325	
1903		* Leighton, Gerald Rowley, M.D., Local Government Board, 125 George Street, Edinburgh	
1874	C. K.	Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast	
1910		Levie, Alexander, F.R.C.V.S., D.V.S.M., Veterinary Surgeon, Lecturer on Veterinary Science, Veterinary Infirmary, 12 Derwent Street, Derby	
1905		* Lightbody, Forrest Hay, 56 Queen Street, Edinburgh	
1889		Lindsay, Rev. James, M.A., D.D., B.Sc., F.R.S.L., F.G.S., M.R.A.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Associate of the Philosophical Society of Louvain, Annick Lodge, Irvine 330	
1912		* Lindsay, John George, M.A., B.Sc. (Edin.), Science Master, Royal High School, 33 Lauriston Gardens, Edinburgh	
1912		* Linlithgow, The Most Honourable the Marquis of, Hopetoun House, South Queensferry	
1903		Liston, William Glen, M.D., Captain, Indian Medical Service, c/o Grindlay, Groom & Co., Bombay, India	
1903		* Littlejohn, Henry Harvey, M.A., M.B., B.Sc., F.R.C.S.E., Professor of Forensic Medicine, Dean of the Faculty of Medicine in the University of Edinburgh, 11 Rutland Street, Edinburgh	
1898		* Lothian, Alexander Veitch, M.A., B.Sc., Training College, Cowcaddens, Glasgow 335	
1884		Low, George M., Actuary, 11 Moray Place, Edinburgh	
1888		Lowe, D. F., M.A., LL.D., formerly Head Master of Heriot's Hospital School, Lauriston, 19 George Square, Edinburgh	
1900		Lusk, Graham, Ph.D., M.A., Professor of Physiology, Cornell University Medical College, New York, N.Y., U.S.A.	
1894		* Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire	
1887		M'Aldowie, Alexander M., M.D., Glengarriff, Leckhampton, Cheltenham 340	
1907		MacAlister, Donald Alexander, A.R.S.M., F.G.S., 26 Thurloe Square, South Kensington, London, S.W.	
1891		Macallan, John, F.I.C., 3 Rutland Terrace, Clontarf, Dublin	
1883		M'Bride, P., M.D., F.R.C.P.E., 10 Park Avenue, Harrogate, and Hill House, Withypool, Dunster, Somerset	
1903		* M'Cormick, Sir W. S., M.A., LL.D., Secretary to the Carnegie Trust for the Universities of Scotland, 13 Douglas Crescent, Edinburgh 1910-13.	
1905		* Macdonald, Hector Munro, M.A., F.R.S., Professor of Mathematics, University of Aberdeen, 52 College Bounds, Aberdeen 345	1908-11.
1897	C.	* Macdonald, James A., M.A., B.Sc., H.M. Inspector of Schools, Stewarton, Kilmacolm	
1904		* Macdonald, John A., M.A., B.Sc., King Edward VII. School, Johannesburg, Transvaal	
1886		Macdonald, the Right Hon. Sir J. H. A. (Lord Kingsburgh) P.C., K.C., K.C.B., LL.D., F.R.S., M.Inst.E.E., Lord Justice-Clerk, and Lord President of the Second Division of the Court of Session, 15 Abercromby Place, Edinburgh 1889-92.	
1904		Macdonald, William, B.Sc., M.Sc., Agriculturist, Editor <i>Transvaal Agricultural Journal</i> , Department of Agriculture, Pretoria Club, Pretoria, Transvaal	

Date of Election.			Service on Council, etc.
1886	C.	Macdonald, William J., M.A., 15 Comiston Drive, Edinburgh	350
1901	C.	* MacDougall, R. Stewart, M.A., D.Sc., Professor of Biology, Royal Veterinary College, Edinburgh, 9 Dryden Place, Edinburgh	
1910		Macewen, Hugh Allan, M.B., Ch.B., D.P.H. (Lond. and Camb.), Local Government Board, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, M.B., B.Sc., LL.D., Principal, and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London	
1885	C.	Macfarlane, J. M., D.Sc., Professor of Botany and Director of the Botanic Garden, University of Pennsylvania, Philadelphia, Pennsylvania, U.S.A.	
1897		* MacGillivray, Angus, C.M., M.D., D.Sc., 23 South Tay Street, Dundee	355
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, Middlesex	
1903		* M'Intosh, Donald C., M.A., D.Sc., 3 Glenisla Gardens, Edinburgh	
1911		M'Intosh, John William, A.R.C.V.S., 14 Templar Street, Myatts Park, London, S.E.	
1869	C. N.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, Pres. Ray. Society, 2 Abbotsford Crescent, St Andrews	1885-88.
1895	C.	* Macintyre, John, M.D., 179 Bath Street, Glasgow	360
1882		Mackay, John Sturgeon, M.A., LL.D., formerly Mathematical Master in the Edinburgh Academy, 69 Northumberland Street, Edinburgh	{ 1895-98, 1900-03.
1873	C. B.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Physiology in the University of Glasgow, Maxieburn, Stonehaven	{ 1875-78, 1885-88, 1893-94, 1900-02. V-P 1894-1900.
1912	C.	M'Kendrick, Anderson Gray, M.B., Major, Indian Medical Service, Officiating Statistical Officer to the Government of India, Sanitary and Medical Departments, Chillingham, Simla, Punjab, India; Dunmoir, Stonehaven (till Jan. 1915)	
1900	C.	* M'Kendrick, John Souttar, M.D., F.R.F.P.S.G., 2 Buckingham Terrace, Glasgow	
1910	C.	* Mackenzie, Alister, M.A., M.D., D.P.H., Principal, College of Hygiene and Physical Training, Dunfermline	365
1911		* M'Kenzie, Kenneth John, M.A., Master of Method to Leith School Board, 30 Craighall Crescent, Leith	
1894		* Mackenzie, Robert, M.D., Napier, Nairn	
1904		* Mackenzie, W. Leslie, M.A., M.D., D.P.H., LL.D., Medical Member of the Local Government Board for Scotland, 1 Stirling Road, Trinity	
1910		* MacKinnon, James, M.A., Ph.D., Professor of Ecclesiastical History, Edinburgh University, 12 Lygon Road, Edinburgh	
1904		* Mackintosh, Donald James, M.V.O., M.B., C.M., LL.D., Supt. Western Infirmary, Glasgow	370
1869	C.	MacLagan, R. C., M.D., F.R.C.P.E., 5 Coates Crescent, Edinburgh	
1899		Maclean, Ewan John, M.D., M.R.C.P. Lond., 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A., D.Sc., M.Inst.E.E., Professor of Electrical Engineering in the Royal Technical College, 51 Kersland Terrace, Hillhead, Glasgow	
1913		* M'Lellan, Dugald, M.Inst.C.E., District Engineer, Caledonian Railway, 42 Ormendale Terrace, Murrayfield, Edinburgh	
1907		* Macnair, Peter, Curator of the Natural History Collections in the Glasgow Museums, Kelvingrove Museum, Glasgow	375
1898	C.	Mahalanobis, S. C., B.Sc., Professor of Physiology, Presidency College, Calcutta, India	
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F.C.S., Health Officer, Calcutta, IV., 37 Lower Chitpore Road, Calcutta, India	
1908		Mallik, Devendranath, B.A., B.Sc., Professor of Physics and Mathematics, Patna College, Bankipur, Bengal, India	
1912		Maloney, William Joseph, M.D.(Edin.), Professor of Neurology at Fordham University, New York City, N.Y., U.S.A.	
1913		Marchant, Rev. James, F.R.A.S., Director, National Council for Promotion of Race-Regeneration; Literary Adviser to House of Cassell; 42 Great Russell Street, London, W.C.	380
1880	C.	Marsden, R. Sydney, M.D., C.M., D.Sc., D.P.H., Hon. L.A.H. Dub., M.R.I.A., F.I.C., M.O.H., Rowallan House, Cearns Road, and Town Hall, Birkenhead	
1909	C.	* Marshall, C. R., M.D., M.A., Professor of Materia Medica and Therapeutics, Medical School, Dundee, Arnsheen, Westfield Terrace, West Newport, Fife	
1882	C.	Marshall, D. H., M.A., Professor, Union and Alwington Avenue, Kingston, Ontario, Canada	

Date of Election.			Service on Council, etc.
1901	C.	* Marshall, F. H. A., Sc.D., Lecturer on Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge	
1903		Martin, Nicholas Henry, F.L.S., F.C.S., Ravenswood, Low Fell, Gateshead	385
1912		* Martin, Sir Thomas Carlaw, LL.D., J.P., Director, Royal Scottish Museum, 4 Gordon Terrace, Edinburgh	
1913		Masson, George Henry, M.D., D.Sc., M.R.C.P.E., Port of Spain, Trinidad, British West Indies	
1885	C.	Masson, Orme, D.Sc., F.R.S., Professor of Chemistry in the University of Melbourne	
1898	C. B.	* Masterman, Arthur Thomas, M.A., D.Sc., Inspector of Fisheries, Board of Agriculture, Whitehall, London	1902-04.
1911		Mathews, Gregory Macalister, F.L.S., F.Z.S., Langley Mount, Watford, Herts	390
1906		* Mathieson, Robert, F.C.S., Rillbank, Innerleithen	
1902		Matthews, Ernest Romney, A.M.Inst.C.E., F.G.S., Bessemer Prizeman, Soc. Engineers, Bridlington, Yorkshire	
1901	C.	* Menzies, Alan W. C., M.A., B.Sc., Ph.D., F.C.S., Professor of Chemistry, Oberlin College, Oberlin, Ohio, U.S.A.	
1888		Methven, Cathcart W., M.Inst.C.E., F.R.I.B.A., Durban, Natal, S. Africa	
1902	C.	Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Professor of Mathematics, Syracuse University, Syracuse, N.Y., U.S.A.	395
1885	C. B.	Mill, Hugh Robert, D.Sc., LL.D., 62 Camden Square, London	
1908		* Miller, Alexander Cameron, M.D., F.S.A. Scot., Craig Linnhe, Fort-William, Inverness-shire	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow	
1905		* Miller-Milne, C. H., M.A., Head Master, Daniel Stewart's College, 4 Campbell Road, Murrayfield, Edinburgh	
1909		Mills, Bernard Langley, M.D., F.R.C.S.E., M.R.C.S., D.P.H., Lt.-Col. R.A.M.C., formerly Army Specialist in Hygiene, 84 Grange Crescent, Sharrow, Sheffield	400
1905		* Milne, Archibald, M.A., B.Sc., Lecturer on Mathematics and Science, Edinburgh Provincial Training College, 108 Comiston Drive, Edinburgh	
1904	C.	* Milne, James Robert, D.Sc., Lecturer on Natural Philosophy, 11 Melville Crescent, Edinburgh	
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1899		* Milroy, T. H., M.D., B.Sc., Professor of Physiology in Queen's College, Belfast, Thornlee, Malone Park, Belfast	
1889	C.	Mitchell, A. Crichton, D.Sc., Hon. Doc. Sc. (Genève), formerly Director of Public Instruction in Travancore, India, 103 Trinity Road, Edinburgh	405
1897		Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		* Mitchell, James, M.A., B.Sc., Cruiach, Lochgilphead	
1899		* Mitchell-Thomson, Sir Mitchell, Bart., 6 Charlotte Square, Edinburgh	
1911		Modi, Edalji Manejki, D.Sc., LL.D., Litt.D., F.C.S., etc., Proprietor and Director of Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	
1906	C.	Moffat, Rev. Alexander, M.A., B.Sc., Professor of Physical Science, Christian College, Madras, India	410
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., Combe Bank, near Sevenoaks, Kent	
1887	C.	Moos, N. A. F., L.C.E., B.Sc., Professor of Physics, Elphinstone College, and Director of the Government Observatory, Colaba, Bombay, India	
1896		* Morgan, Alexander, M.A., D.Sc., Principal, Edinburgh Provincial Training College, 1 Midmar Gardens, Edinburgh	
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony	
1901		Moses, O. St John, I.M.S., M.D., D.Sc., F.R.C.S., Captain, Professor of Medical Jurisprudence, 26 Park Street, Wellesley, Calcutta, India	415
1892	C.	Mossman, Robert C., Acting Editor, British Rainfall Organization's Publications, 62 Camden Square, London, N.W.	
1874	C. K.	Muir, Thomas, C.M.G., M.A., LL.D., F.R.S., Superintendent-General of Education for Cape Colony, Education Office, Cape Town, and Mowbray Hall, Rosebank, Cape Colony	1885-88. V-P 1888-91.
1888	C.	Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers	
1907		Muirhead, James M. P., J.P., F.R.S.L., F.S.S., Royal Automobile Club, Pall Mall, London, S.W.	
1887		Mukhopâdhyay, Asûtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta, India	420

Alphabetical List of the Ordinary Fellows of the Society. 381

Date of Election.			Service on Council, etc.
1891	C.	* Munro, Robert, M.A., M.D., LL.D., Hon. Memb. R.I.A., Hon. Memb. Royal Society of Antiquaries of Ireland, Elmbank, Largs, Ayrshire	{ 1894-97, 1900-03. V-P 1903-08.
1896		* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent, Edinburgh	{ 1881-84, 1889-91, 1901-03.
1907	C. N.	* Murray, James, Hill Farm Bungalow, Froxfield, Hants, England	{ Sec. 1891-1900. V-P 1884-89, 1903-06.
1877	C.	Murray, Sir John, K.C.B., LL.D., Ph.D., D.Sc., F.R.S., Knight of the Royal Prussian Order <i>Pour le Mérite</i> , Norwegian Order of St Olav, Director of the "Challenger" Expedition Publications. Office, Villa Medusa, Boswell Road, House, Challenger Lodge, Wardie, and United Service Club	{ 1881-84, 1889-91, 1901-03. Sec. 1891-1900. V-P 1884-89, 1903-06.
1907		Musgrove, James, M.D., F.R.C.S. Edin. and Eng., Bute Professor of Anatomy, University of St Andrews, The Swallowgate, St Andrews	425
1902		Mylne, Rev. R. S., M.A., B.C.L. Oxford, F.S.A. Lond., Great Amwell, Herts	
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P., 28 Angas Street, Adelaide, S. Australia	
1897		Nash, Alfred George, B.Sc., F.R.G.S., C.E., Belretiro, Mandeville, Jamaica, W.I.	
1906		* Newington, Frank A., M.Inst.C.E., M.Inst.E.E., 7 Wester Coates Road, Edinburgh	
1898		Newman, Sir George, M.D., D.P.H. Cambridge, Lecturer on Preventive Medicine, St Bartholomew's Hospital, University of London : Grim's Wood, Harrow Weald, Middlesex	430
1884		Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park, Edinburgh	{ 1885-87, 1892-95, 1897-1900.
1880	C.	Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place, Edinburgh	
1878		Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham	
1906		* O'Connor, Henry, A.M.Inst.C.E., 1 Drummond Place, Edinburgh	
1888		Ogilvie, F. Grant, C.B., M.A., B.Sc., LL.D., Director of the Science Museum and the Geological Survey, 15 Evelyn Gardens, London, S.W.	435
1888		Oliphant, James, M.A., 11 Heathfield Park, Willesden Green, London	
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 123 Harley Street, London, W.	
1895	C.	Oliver, Sir Thomas, M.D., LL.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	
1884	C. K.	Omond, R. Traill, LL.D., 3 Church Hill, Edinburgh	
1914		* Oswald, Alfred, Lecturer in German, Glasgow Provincial Training College, Nordheim, Bearsden, Glasgow	440
1908		Page, William Davidge, F.C.S., F.G.S., M. Inst. M.E., 10 Clifton Dale, York	
1905		Pallin, William Alfred, F.R.C.V.S., Major in the Army Veterinary Corps, c/o Messrs Holt & Co., 3 Whitehall Place, London	
1914		Pare, John William, M.B., C.M., M.D., L.D.S., Lecturer in Dental Anatomy, National Dental Hospital, 64 Brook Street, Grosvenor Square, London, W.	
1892		Parker, Thomas, M. Inst.C.E., Severn House, Iron Bridge, Salop	
1901		* Paterson, David, F.C.S., Lea Bank, Rosslyn, Midlothian	445
1886	C.	Paton, D. Noël, M.D., B.Sc., F.R.C.P.E., Professor of Physiology in the University of Glasgow, University, Glasgow	{ 1894-97, 1904-06, 1909-12.
1889		Patrick, David, M.A., LL.D., c/o W. & R. Chambers, 339 High Street, Edinburgh	
1892		* Paulin, Sir David, Actuary, 6 Forres Street, Edinburgh	
1881	C. N.	Peach, Benjamin N., LL.D., F.R.S., F.G.S. (VICE-PRESIDENT), formerly District Superintendent and Acting Palaeontologist of the Geological Survey of Scotland, 72 Grange Loan, Edinburgh	{ 1905-08, 1911-1912. V-P 1912-
1907		* Pearce, John Thomson, B.A., B.Sc., School House, Tranent	450
1904		* Peck, James Wallace, M.A., Chief Inspector, National Health Insurance, Scotland, 83 Princes Street, Edinburgh	
1889		Peck, William, F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh	
1887	C. B.	Peddie, Wm., D.Sc., Professor of Natural Philosophy in University College, Dundee, Rosemount, Forthill Road, Broughty Ferry	1904-07, 1908-11.
1893		Perkin, Arthur George, F.R.S., 8 Montpellier Terrace, Hyde Park, Leeds	

Date of Election.		Service on Council, etc.
1913	* Philip, Alexander, M.A., LL.B., Writer, The Mary Acre, Brechin	455
1889	Philip, Sir R. W., M.A., M.D., F.R.C.P.E., 45 Charlotte Square, Edinburgh	
1907	C. Phillips, Charles E. S., Castle House, Shooter's Hill, Kent	
1905	* Pinkerton, Peter, M.A., D.Sc., Rector, High School, Glasgow	
1908	C. * Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., European Hospital, Mombasa, British East Africa	
1911	* Pirie, James Simpson, Civil Engineer, 28 Scotland Street, Edinburgh	460
1906	Pitchford, Herbert Watkins, F.R.C.V.S., Bacteriologist and Analyst, Natal Government, The Laboratory, Pietermaritzburg, Natal	
1886	Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow	
1888	Prain, Sir David, Lt.-Col., Indian Medical Service (Retired), C.M.G., C.I.E., M.A., M.B., LL.D., F.L.S., F.R.S., For. Memb. K. Svensk. Vetensk. Akad.; Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acreale; Pharm. Soc. Gt. Britain; Corr. Memb. K. Bayer Akad. Wiss., etc.; Director, Royal Botanic Gardens, Kew, Surrey	
1902	* Preller, Charles Du Riche, M.A., Ph.D., A.M.Inst.C.E., 61 Melville Street, Edinburgh	
1892	* Pressland, Arthur J., M.A. Camb., Edinburgh Academy	465
1875	C. Prevost, E. W., Ph.D., Weston, Ross, Herefordshire	
1908	* Pringle, George Cossar, M.A., Rector of Peebles Burgh and County High School, Bloomfield, Peebles	
1903	* Pullar, Laurence, The Lea, Bridge of Allan	
1911	Purdy, John Smith, M.D., C.M. (Aberd.), D.P.H. (Camb.), F.R.G.S., Chief Health Officer for Tasmania, Islington, Hobart, Tasmania	
1898	* Purves, John Archibald, D.Sc., 13 Albany Street, Edinburgh	470
1897	* Rainy, Harry, M.A., M.B., C.M., F.R.C.P. Ed., 16 Great Stuart Street, Edinburgh	
1899	* Ramage, Alexander G., 8 Western Terrace, Murrayfield, Edinburgh	
1884	Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, Curator of Australian Museum, Sydney, N.S.W.	
1911	* Rankin, Adam A., Vice-President, British Astronomical Association, West of Scotland Branch, 324 Cow Road, Broomhill, Glasgow, W.	
1891	* Rankine, John, K.C., M.A., LL.D., Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place, Edinburgh	475
1904	Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham	
1900	Raw, Nathan, M.D., M.R.C.P. (London), B.S., F.R.C.S., D.P.H., 66 Rodney Street, Liverpool	
1883	C. Readman, J. B., D.Sc., F.C.S., Belmont, Hereford	
1889	Redwood, Sir Boverton, Bt., D.Sc. (Hon.), F.I.C., F.C.S., A.Inst.C.E., The Cloisters, 18 Avenue Road, Regent's Park, London, N.W.	
1902	Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., Barrister-at-Law, National Liberal Club, Whitehall Place, London	480
1902	Reid, George Archdall O'Brien, M.B., C.M., 9 Victoria Road South, Southsea, Hants	
1913	Reid, Harry Avery, F.R.C.V.S., D.V.H., Bacteriologist and Pathologist, Department of Agriculture, Wellington, New Zealand	
1908	C. * Rennie, John, D.Sc., Lecturer on Parasitology, and Assistant to the Professor of Natural History, University of Aberdeen, 60 Desswood Place, Aberdeen	
1913	* Richardson, Harry, M.Inst.E.E., M.Inst.M.E., General Manager and Chief Engineer, Electricity Supply, Dundee and District, The Cottage, Craigie, Broughty Ferry	
1908	Richardson, Linsdall, F.L.S., F.G.S., Organising Inspector of Technical Education for the Gloucestershire Education Committee, 10 Oxford Parade, Cheltenham	485
1875	Richardson, Ralph, W.S., 10 Magdalae Place, Edinburgh	
1906	C. * Ritchie, William Thomas, M.D., F.R.C.P.E., 9 Atholl Place, Edinburgh	
1898	C. Roberts, Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa	
1880	Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John Street, Manchester	
1900	* Robertson, Joseph M'Gregor, M.B., C.M., 26 Buckingham Terrace, Glasgow	490
1896	* Robertson, Robert, M.A., 25 Mansionhouse Road, Edinburgh	
1902	C. * Robertson, Robert A., M.A., B.Sc., Lecturer on Botany in the University of St Andrews	
1896	C. * Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., 2 Mayfield Gardens, Edinburgh	

Alphabetical List of the Ordinary Fellows of the Society.

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Date of Election.		Service on Council, etc.
1910	* Robinson, Arthur, M.D., M.R.C.S., Professor of Anatomy, University of Edinburgh, 35 Coates Gardens, Edinburgh (SECRETARY)	1910-1912.
1881	Rosebery, the Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh	Sec. 1912-
1909	C. * Ross, Alex. David, M.A., D.Sc., F.R.A.S., Professor of Mathematics and Physics, University of Western Australia, Perth, Western Australia	495
1906	* Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School, Dunaura, Heugh Street, Falkirk	
1902	C. * Russell, James, 22 Glenorchy Terrace, Edinburgh	
1880	Russell, Sir James A., M.A., B.Sc., M.B., F.R.C.P.E., LL.D., Woodville, Canaan Lane, Edinburgh	
1904	Sachs, Edwin O., Architect, Chairman of the British Fire Prevention Committee, Vice-President of the International Fire Service Council, 8 Waterloo Place, Pall Mall, London, S.W.	500
1906	Saleeby, Caleb William, M.D., 13 Greville Place, London	
1912	* Sampson, Ralph Allen, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland, Professor of Astronomy, University, Edinburgh, Royal Observatory, Edinburgh	1912-
1903	* Samuel, John S., 8 Park Avenue, Glasgow	
1903	* Sarolea, Charles, Ph.D., D.Litt., Lecturer on French Language, Literature, and Romance Philology, University of Edinburgh, 21 Royal Terrace, Edinburgh	
1891	Sawyer, Sir James, Kt., M.D., F.R.C.P., F.S.A., J.P., Consulting Physician to the Queen's Hospital, 31 Temple Row, Birmingham	505
1900	C. * Schäfer, Sir Edward Albert, M.R.C.S., LL.D., F.R.S. (VICE-PRESIDENT), Professor of Physiology in the University of Edinburgh	1900-03, 1906-09. V.P. 1913-
1885	C. Scott, Alexander, M.A., D.Sc., F.R.S., 34 Upper Hamilton Terrace, London, N.W.	
1880	Scott, J. H., M.B., C.M., M.R.C.S., Professor of Anatomy in the University of Otago, New Zealand	
1889	Scougal, A. E., M.A., LL.D., formerly H.M. Senior Chief Inspector of Schools and Inspector of Training Colleges, 1 Wester Coates Avenue, Edinburgh	
1902	Senn, Nicholas, M.D., LL.D., Professor of Surgery, Rush Medical College, Chicago, U.S.A.	510
1871	Simpson, Sir A. R., M.D., Emeritus Professor of Midwifery in the University of Edinburgh, 52 Queen Street, Edinburgh	
1908	* Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., 43 Manor Place, Edinburgh	
1900	C. * Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh, 25 Chester Street, Edinburgh	
1911	C. Simpson, Sutherland, M.D., D.Sc.(Edin.), Professor of Physiology, Medical College, Cornell University, Ithaca, N.Y., U.S.A., 118 Eddy Street, Ithaca, N.Y., U.S.A.	
1900	Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. Edin., H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay, India	515
1903	* Skinner, Robert Taylor, M.A., Governor and Head Master, Donaldson's Hospital, Edinburgh	
1901	* Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1891	C. K. * Smith, Alexander, B.Sc., Ph.D., Department of Chemistry, Columbia University, New York, N.Y., U.S.A.	
1882	C. Smith, C. Michie, C.I.E., B.Sc., F.R.A.S., formerly Director of the Kodaikānal and Madras Observatories, Winsford, Kodaikānal, South India	
1885	Smith, George, F.C.S., 5 Rosehall Terrace, Falkirk	520
1911	* Smith, Stephen, B.Sc., Goldsmith, 12 Murrayfield Avenue, Edinburgh	
1907	C. Smith, William Ramsay, D.Sc., M.D., C.M., Permanent Head of the Health Department, South Australia, Belair, South Australia	
1880	Smith, William Robert, M.D., D.Sc., LL.D., Professor of Forensic Medicine and Toxicology in King's College, University of London, and Principal of the Royal Institute of Public Health, 36 Russell Square, London, W.C.	
1899	Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Medical Officer of Health, Coventry	
1880	Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., Fellow of University College, Oxford, and Professor of Geology and Palaeontology in the University of Oxford	525
1910	* Somerville, Robert, B.Sc., Science Master, High School, Dunfermline, 38 Cameron Street, Dunfermline	

Date of Election.		Service on Council, etc.
1889	C. Somerville, Wm., M.A., D.Sc., D.Oec., Sibthorpius Professor of Rural Economy and Fellow of St John's College in the University of Oxford, 121 Banbury Road, Oxford	
1911	C. * Sommerville, Duncan M'Laren Young, M.A., D.Sc., Lecturer in Mathematics and in Applied Mathematics, University of St Andrews	
1882	Sorley, James, 82 Onslow Gardens, London	
1896	* Spence, Frank, M.A., B.Sc., 25 Craigie Drive, Edinburgh	530
1874	C. Sprague, T. B., M.A., LL.D., Actuary, 29 Buckingham Terrace, Edinburgh	
1906	Squance, Thomas Coke, M.D., F.R.M.S., F.S.A.Scot., Physician and Pathologist in the Sunderland Infirmary, President Sunderland Antiquarian Society, Sunderland Naturalists' Association, 15 Grange Crescent, Sunderland	1885-87.
1891	* Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College, Edinburgh	
1912	C. Stephenson, John, M.B., D.Sc. (Lond.), Indian Medical Service, Professor of Biology, Government College, Lahore, India.	
1910	* Stephenson, Thomas, F.C.S., Editor of the <i>Prescriber</i> , Examiner to the Pharmaceutical Society, 9 Woodburn Terrace, Edinburgh	535
1886	C. Stevenson, Charles A., B.Sc., M.Inst.C.E., 28 Douglas Crescent, Edinburgh	
1884	Stevenson, David Alan, B.Sc., M.Inst.C.E., 84 George Street, Edinburgh	
1888	C. Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, Usher Institute of Public Health, Warrender Park Road, Edinburgh	
1902	* Stockdale, Herbert Fitton, Director of the Royal Technical College, Glasgow, Clairinch, Upper Helensburgh, Dumbartonshire	
1889	Stockman, Ralph, M.D., F.R.C.P.E., Professor of Materia Medica and Therapeutics in the University of Glasgow	1903-05.
1906	Story, Fraser, Professor of Forestry, University College, Bangor, North Wales	
1907	* Strong, John, M.A., Rector of Montrose Academy, Peel Place, Montrose	
1903	Sutherland, David W., M.D., M.R.C.P., Captain, Indian Medical Service, Professor of Pathology and Materia Medica, Medical College, Lahore, India	
1905	Swithinbank, Harold William, Denham Court, Denham, Bucks	
1912	* Syme, William Smith, M.D. (Edin.), 10 India Street, Glasgow	545
1885	C. Symington, Johnson, M.D., F.R.C.S.E., F.R.S., Professor of Anatomy in Queen's College, Belfast	1892-
1904	* Tait, John W., B.Sc., Rector of Leith Academy, 18 Netherby Road, Leith	
1898	C. Tait, William Archer, D.Sc., M.Inst.C.E., 38 George Square, Edinburgh	
1895	Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., Professor of Geology, University of Utah, Salt Lake City, Utah, U.S.A.	
1890	C. Tanakadate, Akitu, Professor of Natural Philosophy in the Imperial University of Japan, Tokyo, Japan	550
1870	Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899	* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy	
1892	Thackwell, J. B., M.B., C.M., 423A Battersea Park Road, London, S.W.	
1885	C. Thompson, D'Arcy W., C.B., B.A., F.L.S., Professor of Natural History in University College, Dundee	{ 1892-95, 1896-99, 1907-10, 1912-}
1907	* Thompson, John Hannay, M.Sc. (Durh.), M.Inst.C.E., M.Inst.Mech.E., Engineer to the Dundee Harbour Trust, Earlville, Broughty Ferry	555
1905	* Thoms, Alexander, 7 Playfair Terrace, St Andrews	
1887	Thomson, Andrew, M.A., D.Sc., F.I.C., Rector, Perth Academy, Ardenlea, Pitcullen, Perth	
1911	* Thomson, Frank Wyville, M.A., M.B., C.M., D.P.H., D.T.M., Lt.-Col. I.M.S. (Retired), Bonyside, Linlithgow	
1896	* Thomson, George Ritchie, M.B., C.M., General Hospital, Johannesburg, Transvaal	
1903	Thomson, George S., F.C.S., Ferma Albion, Marculesci, Roumania	560
1906	* Thomson, Gilbert, M.Inst.C.E., 164 Bath Street, Glasgow	
1887	C. Thomson, J. Arthur, M.A., LL.D., Regius Professor of Natural History in the University of Aberdeen	1906-08.
1906	C. Thomson, James Stuart, F.L.S., Zoological Department, University, Manchester	
1880	Thomson, John Millar, LL.D., F.R.S., Professor of Chemistry in King's College, London, 18 Lansdowne Road, London, W.	
1899	* Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow	565
1912	C. Thomson, Robert Black, M.B., Edin., Professor of Anatomy, South African College, Cape Town	
1870	Thomison, Spencer C., Actuary, 10 Eglinton Crescent, Edinburgh	

Alphabetical List of the Ordinary Fellows of the Society. 385

Date of Election.			Service on Council, etc.
1882		Thomson, Wm., M.A., B.Sc., LL.D., Registrar, University of the Cape of Good Hope, University Buildings, Cape Town	
1876	C.	Thomson, William, Royal Institution, Manchester	
1911		* Tosh, James Ramsay, M.A., D.Sc. (St Ands.), Thursday Island, Queensland, Australia	570
1888		Turnbull, Andrew H., Actuary, The Elms, Whitehouse Loan, Edinburgh	
1905		* Turner, Arthur Logan, M.D., F.R.C.S.E., 27 Walker Street, Edinburgh	
1906	C.	* Turner, Dawson F. D., B.A., M.D., F.R.C.P.E., M.R.C.P., Lecturer on Medical Physics, Surgeons' Hall, Physician in charge of Radium Treatment, Royal Infirmary, Edinburgh, 37 George Square, Edinburgh	
1861	K. N.		1866-68,
	C.	Turner, Sir William, K.C.B., M.B., F.R.C.S.L. and E., LL.D., D.C.L., D.Sc., F.R.S., Late Pres. R.S.E., Knight of the Royal Prussian Order <i>Pour le Mérite</i> , Principal and Vice-Chancellor of the University of Edinburgh, 6 Eton Terrace, Edinburgh	1895-97. Sec.
1895		Turton, Albert H., M.I.M.M., 171 George Road, Erdington, Birmingham	1869-91.
1898	C.	* Tweedie, Charles, M.A., B.Sc., Lecturer on Mathematics in the University of Edinburgh, Duns, Berwickshire	V-P
1889		Underhill, T. Edgar, M.D., F.R.C.S.E., Dunedin, Barnt Green, Worcestershire	1891-95,
1910		Vincent, Swale, M.D. Lond, D.Sc. Edin., etc., Professor of Physiology, University of Manitoba, Winnipeg, Canada	1897-1903. P.
1911	C.	* Walker, Henry, M.A., D.Sc., Head Physics Master, Kilmarnock Academy and Technical School, 30 M'Lelland Drive, Kilmarnock	1908-1913.
1891	C. B.	* Walker, James, D.Sc., Ph.D., LL.D., F.R.S., Professor of Chemistry in the University of Edinburgh, 5 Wester Coates Road, Edinburgh	1903-05, 1910-13.
1873	C.	Walker, Robert, M.A., LL.D., University, Aberdeen	580
1902		* Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen	
1886	C.	Wallace, R., F.L.S., Professor of Agriculture and Rural Economy in the University of Edinburgh	
1898		Wallace, Wm., M.A., Belvedere, Alberta, Canada	
1891		* Walmsley, R. Mullineux, D.Sc., Principal of the Northampton Institute, Clerkenwell, London	585
1907		Waters, E. Wynston, Medical Officer, H.B.M. Administration, E. Africa, Malindi, British East Africa Protectorate, <i>via</i> Mombasa	
1901	C.	* Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, King's College, London	
1904		* Watson, Charles B. Boog, Huntly Lodge, 1 Napier Road, Edinburgh	
1911		* Watson, James A. S., B.Sc., etc., Assistant in Agriculture, University of Edinburgh, Downieken, Dundee	
1900		* Watson, Thomas P., M.A., B.Sc., Principal, Keighley Institute, Keighley	590
1910		* Watson, William John, M.A., LL.D. Aberdeen, B.A. Oxon., Rector of the Royal High School, Edinburgh, 17 Merchiston Avenue, Edinburgh	
1907		* Watt, Andrew, M.A., Secretary to the Scottish Meteorological Society, 6 Woodburn Terrace, Edinburgh	1912-
1911		Watt, James, W.S., F.F.A., 24 Rothesay Terrace, Edinburgh	
1911		* Watt, Rev. Lauchlan Maclean, B.D., Minister of St Stephen's Parish, 7 Royal Circus, Edinburgh	
1896		Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynaecology, Rush Medical College, 1748 Harrison Street, Chicago, Ill., U.S.A.	595
1907	B. C.	* Wedderburn, Ernest Maclagan, M.A., LL.B., W.S., D.Sc., 7 Dean Park Crescent, Edinburgh	1913-
1903	C.	* Wedderburn, J. H. Maclagan, M.A., D.Sc., 95 Mercer Street, Princeton, N.J., U.S.A.	
1904		Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma, The Education Office, Rangoon, Burma	
1896		Wenley, Robert Mark, M.A., D.Sc., D.Phil., Litt.D., LL.D., Professor of Philosophy in the University of Michigan, Ann Arbor, Michigan, U.S.A.	
1909	C.	* Westergaard, Reginald Ludovic Andreas Emil, Ph.D., Professor of Technical Mycology, Heriot-Watt College, Hafnia, Liberton, Edinburgh	600
1896	C.	White, Philip J., M.B., Professor of Zoology in University College, Bangor, North Wales	
1911		* Whittaker, Charles Richard, F.R.C.S. (Edin.), F.S.A. (Scot.), Lynwood, Hatton Place, Edinburgh	

Date of Election.		Service on Council, etc.
1912	* Whittaker, Edmund Taylor, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh, 35 George Square, Edinburgh	1912-
1879	Will, John Charles Ogilvie, of Newton of Pitfodels, M.D., 17 Bon-Accord Square, Aberdeen	
1908	* Williamson, Henry Charles, M.A., D.Sc., Naturalist to the Fishery Board for Scotland, Marine Laboratory, Aberdeen	605
1910 C.	* Williamson, William, 9 Plewlands Terrace, Edinburgh	
1900	Wilson, Alfred C., F.C.S., Voewood Croft, Stockton-on-Tees	
1911	* Wilson, Andrew, M.Inst.C.E., 51 Queen Street, Edinburgh	
1902	* Wilson, Charles T. R., M.A., F.R.S., 21 Grange Road, Cambridge, Sidney Sussex College, Cambridge	
1895	Wilson-Barker, David, R.N.R., F.R.G.S., Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," off Greenhithe, Kent	610
1882	Wilson, George, M.A., M.D., LL.D.	
1891	* Wilson, John Hardie, D.Sc., University of St Andrews, 39 South Street, St Andrews	
1902	Wilson, William Wright, F.R.C.S.E., M.R.C.S., Cottesbrook House, Acock's Green, Birmingham	
1908 C.	* Wood, Thomas, M.D., Eastwood, 182 Ferry Road, Bonnington, Leith	
1886	Woodhead, German Sims, M.D., F.R.C.P.E., Professor of Pathology in the University of Cambridge	615
1884	Woods, G. A., M.R.C.S., 1 Hammerton Road, Bromley, Kent	
1911	* Wrigley, Kuriel Whitelhead, B.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh	
1890	* Wright, Johnstone Christie, Conservative Club, Edinburgh	
1896	* Wright, Sir Robert Patrick, Chairman of the Board of Agriculture for Scotland, Corsewall, Colinton, Midlothian	
1882	Young, Frank W., F.C.S., H.M. Inspector of Science and Art Schools, 32 Buckingham Terrace, Botanic Gardens, Glasgow	620
1892	Young, George, Ph.D., "Bradda," Church Crescent, Church End, Finchley, London, N.	
1896 C.	* Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton	
1904	Young, R. B., M.A., D.Sc., F.G.S., Professor of Geology and Mineralogy in the South African School of Mines and Technology, Johannesburg, Transvaal	

LIST OF HONORARY FELLOWS OF THE SOCIETY

At 1st January 1914.

HIS MOST GRACIOUS MAJESTY THE KING.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X.).

Elected,	
1897	Emile Hilaire Amagat, Membre de l'Institut, St Satur, Cher, France.
1900	Arthur Auwers, Bellevue-Strasse 55, Berlin-Lichterfelde, Germany.
1900	Adolf Ritter von Baeyer, Universität, München, Germany.
1905	Waldemar Christofer Brögger, K. Frederiks Universitet, Christiania, Norway.
1905	Moritz Cantor, Gaisbergstrasse 15, Heidelberg, Germany.
1902	Jean Gaston Darboux, Secrétaire de l'Institut, Paris, France.
1910	Hugo de Vries, Universiteit, Amsterdam, Holland.
1905	Paul Ehrlich, K. Institut für Experimentelle Therapie, Sandhofstrasse 44, Frankfurt-a.-M., Germany.
1908	Emil Fischer, Universität, Berlin, Germany.
1910	Karl F. von Goebel, Universität, München, Germany.
1905	Paul Heinrich von Groth, Universität, München, Germany.
1888	Ernst Haeckel, Universität, Jena, Germany.
1913	George Ellery Hale, Mount Wilson Solar Observatory (Carnegie Institution of Washington), Pasadena, California, U.S.A.
1883	Julius Hann, Universität, Wien, Austria.
1908	George William Hill, West Nyack, New York, U.S.A.
1913	Emil Clement Jungfleisch, Collège de France, Paris, France.
1910	Jacobus Cornelius Kapteyn, Universiteit, Groningen, Holland.
1897	Gabriel Lippmann, Université, Paris, France.

Elected

- 1895 Carl Menger, Wien ix., Fuchstallerg, 2, Austria.
 1910 Elie Metchnikoff, Institut Pasteur, Paris, France.
 1910 Albert Abraham Michelson, University, Chicago, U.S.A.
 1897 Fridtjof Nansen, K. Frederiks Universitet, Christiania, Norway.
 1908 Henry Fairfield Osborn, Columbia University and American Museum of Natural History, New York, N.Y., U.S.A.
 1910 Wilhelm Ostwald, Gross-Bothen, bei Leipzig, Germany.
 1908 Ivan Petrovitch Pavlov, Wedenskaja Strasse 4, St Petersburg, Russia.
 1910 Frederick Ward Putnam, Peabody Museum of Harvard University, Cambridge, Mass., U.S.A.
 1889 Georg Hermann Quincke, Bergstrasse 41, Heidelberg, Germany.
 1913 Santiago Ramón y Cajal, Universidad, Madrid, Spain.
 1908 Magnus Gustaf Retzius, Högskolan, Stockholm, Sweden.
 1908 Augusto Righi, Regia Università, Bologna, Italy.
 1905 Eduard Suess, Afrikanergasse 9, Wien 11/2, Austria.
 1913 Vito Volterra, Regia Università, Rome, Italy.
 1905 Wilhelm Waldeyer, Universität, Berlin, Germany.
 1910 August F. L. Weismann, Universität, Freiburg-im-Breisgau, Germany.
 1905 Wilhelm Wundt, Universität, Leipzig, Germany.
 1913 Charles René Zeiller, Ecole Nationale Supérieure des Mines, Paris, France.

Total, 36.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X.).

- 1892 Colonel Alexander Ross Clarke, C.B., R.E., F.R.S., Strathmore, Reigate, Surrey.
 1900 Sir David Ferrier, Kt., M.A., M.D., LL.D., F.R.S., Emer. Professor of Neuro-Pathology, King's College, London, 34 Cavendish Square, London, W.
 1900 Andrew Russell Forsyth, M.A., Sc.D., LL.D., Math.D., F.R.S., Chief Professor of Mathematics in the Imperial College of Science and Technology, London, formerly Sadlerian Professor of Pure Mathematics in the University of Cambridge, Imperial College of Science, London, S.W.
 1910 James George Frazer, D.C.L., LL.D., Litt.D., F.B.A., Fellow of Trinity College, Cambridge, Professor of Social Anthropology in the University of Liverpool, Trinity College, Cambridge.
 1892 Sir David Gill, K.C.B., LL.D., F.R.S., formerly His Majesty's Astronomer at the Cape of Good Hope, 34 De Vere Gardens, Kensington, London, W.
 1895 Albert C. L. G. Günther, M.A., M.D., Ph.D., F.R.S., 2 Lichfield Road, Kew Gardens, Surrey.
 1908 Sir Alexander B. W. Kennedy, Kt., LL.D., F.R.S., Past Pres. Inst. C.E., 1 Queen Anne Street, Cavendish Square, London, W.
 1913 Horace Lamb, M.A., Sc.D., D.Sc., LL.D., F.R.S., Professor of Mathematics in the University of Manchester.
 1908 Sir Edwin Ray Lankester, K.C.B., LL.D., F.R.S., 29 Thurloe Place, S. Kensington, London, S.W.
 1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., M.P. University of Cambridge since 1911, Lucasian Professor of Mathematics in the University of Cambridge, St John's College, Cambridge.
 1900 Archibald Liversidge, M.A., LL.D., F.R.S., Emer. Professor of Chemistry in the University of Sydney, Fieldhead, Combe Warren, Kingston, Surrey.
 1908 Sir James A. H. Murray, LL.D., D.C.L., D.Litt., Ph.D., Litt.D., F.S.A., Corresp. Member of the Institute of France, etc., Editor of the Oxford English Dictionary, Oxford.
 1905 Sir William Ramsay, K.C.B., LL.D., F.R.S., formerly Professor of Chemistry in the University College, London, 19 Chester Terrace, Regent's Park, London, N.W.
 1886 The Rt. Hon. Lord Rayleigh, O.M., P.C., J.P., D.C.L., LL.D., D.Sc. Dub., F.R.S., Corresp. Mem. Inst. of France, Terling Place, Witham, Essex.
 1908 Charles Scott Sherrington, M.A., M.D., LL.D., F.R.S., Holt Professor of Physiology in the University of Liverpool, 16 Grove Park, Liverpool.
 1913 Sir William Turner Thiselton-Dyer, K.C.M.G., C.I.E., M.A., LL.D., F.R.S., formerly Director of the Royal Botanic Gardens, Kew; The Ferns, Witcombe, Gloucester.
 1905 Sir Joseph John Thomson, D.Sc., LL.D., F.R.S., Cavendish Professor of Experimental Physics, University of Cambridge, Trinity College, Cambridge.
 1900 Sir Thomas Edward Thorpe, Kt., C.B., D.Sc., LL.D., F.R.S., formerly Principal of the Government Laboratories, Imperial College of Science and Technology, South Kensington, London, S.W., Whinfield Salcombe, South Devon.

Total, 18.

ORDINARY FELLOWS OF THE SOCIETY ELECTED

During Session 1912-13.

(Arranged according to their date of election.)

18th November 1912.

WILLIAM THOMAS GORDON, M.A., D.Sc., B.A. GEORGE HENRY MASSON, M.D., D.Sc., M.R.C.P.E.

2nd December 1912.

JOSEPH BEARD, F.R.C.S.E., M.R.C.S., L.R.C.P., etc.
HARRY RICHARDSON, M.Inst.E.E., M.I.M.E.

20th January 1913.

TARAK NATH MAJUMDAR, D.P.H. (Cal.), L.M.S., F.C.S.
WALTER HUME KERR, M.A., B.Sc.,
GEORGE FRANCIS SCOTT ELLIOT, M.A., B.Sc., F.R.G.S., F.L.S.

17th March 1913.

HARRY AVERY REID, F.R.C.V.S., etc.

2nd June 1913.

ALEXANDER PHILIP, M.A., LL.B. DUGALD M'LELLAN, M.Inst.C.E.
ALEXANDER RUSSELL BROWN, M.A., B.Sc. ALEXANDER CLEGHORN, M.Inst.C.E.

21st July 1913.

Rev. JAMES MARCHANT.

ORDINARY FELLOWS DECEASED AND RESIGNED

During Session 1912-13.

DECEASED.

RAMSAY HEATLEY TRAQUAIR, M.D., LL.D., F.R.S., F.G.S.	WALTER WHITEHEAD, F.R.C.S.E.
ANDREW JAMIESON, M.Inst.C.E.	JAMES GORDON MACGREGOR, D.Sc., LL.D., F.R.S.
JOHN WILLIAM SHEPHERD.	WILLIAM COLIN MACKENZIE, M.D., F.R.C.S.
Lieut.-Col. FREDERICK BAILEY.	JOHN PENNY, M.B., C.M., D.Sc.
JOHN M'ARTHUR, F.C.S.	JAMES M'CUBBIN, B.A.
ROBERT M. FERGUSON, Ph.D., LL.D.	WILLIAM GAYTON, M.D., M.R.C.P.E.
A. BEATSON BELL.	HUGH MARSHALL, D.Sc., F.R.S.
GEORGE ALEXANDER GIBSON, D.Sc., M.D., LL.D., F.R.C.P.E.	ALEXANDER MACFARLANE, M.A., D.Sc., LL.D.
Sir WILLIAM WHITE, K.C.B., F.R.S.	Sir WALTER NOEL HARTLEY, D.Sc., F.R.S.
J. J. KIRK DUNCANSON, M.D.	JOHN MACMILLAN, M.A., D.Sc., M.B., C.M.
	Sir JOHN BATTY TUKE, M.D., D.Sc., LL.D.
	WILLIAM DONALDSON, M.A.

RESIGNED.

Lieut.-Col. JOHN CAMPBELL.

FOREIGN HONORARY FELLOW DECEASED.

LOUIS JOSEPH TROOST.

BRITISH HONORARY FELLOW DECEASED.

Sir GEORGE HOWARD DARWIN, K.C.B., M.A., LL.D., F.R.S.

A B S T R A C T
OF
THE ACCOUNTS OF JAMES CURRIE, ESQ.
As Treasurer of the Royal Society of Edinburgh.
SESSION 1912-1913.

I. ACCOUNT OF THE GENERAL FUND.

CHARGE.

1. Arrears of Contributions at 1st October 1912	£102 18 0
2. Contributions for present Session :—	
1. 135 Fellows at £2, 2s. each	£283 10 0
141 Fellows at £3, 3s. each	444 3 0
	—————
	£727 13 0
2. Fees of Admission and Contributions of eight new Resident Fellows at £5, 5s. each	42 0 0
3. Fees of Admission of five new Non-Resident Fellows at £26, 5s. each	131 5 0
4. Commutation Fees in lieu of future Contributions of two Fellows	24 17 0
	—————
3. Contribution for 1913-1914 paid in advance	925 15 0
4. Interest received—	
Interest, less Tax £22, 11s. 8d.	£364 10 3
Annuity from Edinburgh and District Water Trust, less Tax £3, 1s. 2d.	49 8 10
	—————
5. Transactions and Proceedings sold	413 19 1
6. Annual Grant from Government	107 1 2
7. Income Tax repaid for year to 5th April 1913	600 0 0
	—————
	25 12 11
Amount of the Charge	£2178 9 2

DISCHARGE.

1. TAXES, INSURANCE, COAL AND LIGHTING :—	
Inhabited House Duty	£0 6 3
Insurance	10 10 0
Coal to 15th July 1913	21 13 7
Gas to 10th February 1913	0 2 8
Electric Light to 22nd September 1913	11 6 2
Water 1912-13	4 4 0
	—————
	£48 2 8
2. SALARIES :—	
Interim General Secretary for period from 3rd November 1911 to 1st October 1912	£90 19 2
Librarian	118 6 8
Assistant Librarian	42 10 0
Office Keeper	86 14 0
Treasurer's Clerk	25 0 0
	—————
	363 9 10
Carry forward	£411 12 6

	Brought forward	. . . £411 12 6
3. EXPENSES OF TRANSACTIONS :—		
Neill & Co., Ltd., Printers	£479 4 3	
Do. for illustrations	10 8 0	
M'Farlane & Erskine, Lithographers	82 10 0	
Hislop & Day, Engravers	5 3 9	
Orrick & Son, Bookbinders	£178 11 6	
<i>Less</i> —Received from Mr J. Y. Buchanan for binding Vol. 49, Part 1	37 15 0	
	140 16 6	
H. G. Herring, Photographs	£11 8 9	
London Stereoscopic Co., Ltd.	25 6 0	
Dr F. A. Bather, for blocks	5 0 0	
	£41 14 9	
<i>Less</i> —Received from Royal Society, London, towards illustrations for Memoir	35 0 0	
	6 14 9	
Chas. Butterworth, for blocks	2 14 6	
G. M. Woodward, Lithographer	5 0 0	
West Newman & Co., Engravers	3 13 2	
J. Bartholomew & Co., Engravers	20 0 0	
	756 4 11	
4. EXPENSES OF PROCEEDINGS :—		
Neill & Co., Ltd., Printers	£430 9 4	
Do. (for illustrations)	8 0 6	
Hislop & Day, Engravers	20 10 1	
John Fowler & Co., Engravers	6 4 0	
	465 3 11	
5. BOOKS, PERIODICALS, NEWSPAPERS, ETC. :—		
Otto Schulze & Co., Booksellers	£130 8 7	
James Thin, do.	71 17 3	
R. Grant & Son, do.	7 7 8	
Wm. Green & Sons, do.	0 15 6	
International Catalogue of Scientific Literature	17 0 0	
Robertson & Scott, News Agents	1 8 0	
Egypt Exploration Funds Subscription	3 3 0	
Ray Society do.	1 1 0	
Paleontographical Society do.	1 1 0	
Journal de Conchyliologie	1 15 1	
Orrick & Son, Bookbinders	30 19 3	
Arthur Giles, Publisher	0 15 0	
The Times to 6th February 1914	2 12 0	
Arthur F. Bird, Publisher	2 5 6	
T. & A. Constable, Printers	0 18 0	
	273 6 10	
6. OTHER PAYMENTS :—		
Neill & Co., Ltd., Printers	£65 19 6	
E. Sawers, Purveyor	30 16 8	
S. Duncan, Tailor (uniforms)	4 14 0	
Lantern Exhibitions, etc., at Lectures	9 17 6	
Lindsay, Jamieson & Haldane, C.A., Auditors	6 6 0	
Post Office Telephone Rent	10 0 0	
A. Cowan & Sons, Ltd.	4 16 6	
G. Waterston & Sons	3 7 0	
Gillies & Wright, Joiners	6 6 6	
R. Graham, Slater	6 4 0	
Dr Theodore Rettie, Grant in aid of investigation into causes of louping ill in sheep	£50 0 0	
<i>Less</i> —Grant from Treasury out of Development Fund	50 0 0	
	0 0 0	
Petty Expenses, Postages, Carriage, etc.	117 8 10	
	265 16 6	
7. INTEREST PAID ON BORROWED MONEY :—		
Makerstoun Magnetic Meteorological Observation Fund	7 0 2	
Carry forward	£2179 4 10	

	Brought forward	£2179 4 10
8. IRRECOVERABLE ARREARS of Contributions written off		5 5 0
9. ARREARS of CONTRIBUTIONS outstanding at 1st October 1913 :—		
Present Session	£67 4 0	
Previous Sessions	38 17 0	
	106 1 0	
	Amount of the Discharge	£2290 10 10
Amount of the Charge		£2178 9 2
Amount of the Discharge		2290 10 10
Excess of Payments over Receipts for 1912-1913		£112 1 8
FLOATING BALANCE DUE BY THE SOCIETY at 1st October 1912		£ 11 14 6
Add Excess of Payments as above		112 1 8
Floating Balance due by the Society at 1st October 1913		£123 16 2
<i>Being—</i>		
Loan from the Makerstoun Magnetic Meteorological Observation Fund		£220 13 6
Less Balance due by Union Bank of Scotland, Ltd., on		
Account Current	£93 13 8	
Balance in hands of Librarian	3 3 8	
	96 17 4	
	£123 16 2	

II. ACCOUNT OF THE KEITH FUND*To 1st October 1913.*

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1912	£29 16 4
2. INTEREST RECEIVED :—	
On £896, 19s. id. North British Railway Company 3 per cent. Debenture Stock for year to Whitsunday 1913, less Tax £1, 11s. 4d.	£25 6 10
On £211, 4s. North British Railway Company 3 per cent. Lien Stock from 31st July 1912 to 30th June 1913, less Tax 6s. 9d.	5 9 4
	30 16 2
3. INCOME TAX repaid for year to 5th April 1913	1 18 1
	£62 10 7

DISCHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1913	£62 10 7
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III. ACCOUNT OF THE NEILL FUND*To 1st October 1913.*

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1912	£33 3 6
2. INTEREST RECEIVED :—	
On £355 London, Chatham and Dover Railway 4½ per cent. Arbitration Debenture Stock for year to 30th June 1913, less Tax 18s. 8d.	15 0 10
3. INCOME TAX repaid for year to 5th April 1913	0 18 8

£49 3 0

DISCHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1913	£49 3 0
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IV. ACCOUNT OF THE MAKDOUGALL-BRISBANE FUND*To 1st October 1913.*

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1912	£185 13 3
2. INTEREST received :— On £365 Caledonian Railway Company 4 per cent. Consolidated Preference Stock No. 2 from 31st July 1912 to 30th June 1913, less Tax 15s. 7d.	12 12 1
3. INCOME TAX repaid for year to 5th April 1913	0 15 7
	<u>£199 0 11</u>

DISCHARGE.

1. Dr John Brownlee—Money Portion of Prize 1910-12	£17 1 10
2. Alexander Kirkwood & Son, Engravers, for Gold Medal	16 0 0
3. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1913	165 19 1
	<u>£199 0 11</u>

**V. ACCOUNT OF THE MAKERSTOUN MAGNETIC METEOROLOGICAL
OBSERVATION FUND***To 1st October 1913.*

CHARGE.

1. BALANCE due by General Fund at 1st October 1912	£218 13 4
2. INTEREST received on Balances due by General Fund at Deposit Receipt Rates to 1st October 1913	7 0 2
	<u>£225 13 6</u>

DISCHARGE.

1. W. C. M. Lewis, Grant in aid of the publication of the Annual Tables of Constants and Numerical Data, Chemical, Physical and Technological	£5 0 0
2. BALANCE due by General Fund at 1st October 1913	220 13 6
	<u>£225 13 6</u>

VI. ACCOUNT OF THE GUNNING VICTORIA JUBILEE PRIZE FUND*To 1st October 1913.*

(Instituted by Dr R. H. GUNNING of Edinburgh and Rio de Janeiro.)

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1912	£119 5 4
2. INTEREST received on £1000 North British Railway Company Consolidated Lien Stock from 31st July 1912 to 30th June 1913, less Tax £1, 12s. 1d.	25 17 11
3. INCOME TAX repaid for year to 5th April 1913	1 12 1
	<u>£146 15 4</u>

DISCHARGE.

1. Professor J. N. Collie, Prize for 1908-12	£105 0 0
2. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1913	41 15 4
	<u>£146 15 4</u>

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 1st October 1913.

1. GENERAL FUND—

1. £2090, 9s. 4d. three per cent. Lien Stock of the North British Railway Company at 75½ per cent., the selling price at 1st October 1913	£1,578 6 0
2. £8519, 14s. 3d. three per cent. Debenture Stock of do. at 76⅔ per cent., do.	6,506 18 7
3. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at 156 per cent., do	1,365 0 0
4. £1811 four per cent. Debenture Stock of the Caledonian Railway Company at 102½ per cent., do	1,856 5 6
5. £35 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 108½ per cent., do	37 19 6
6. Arrears of Contributions, as per preceding Abstract of Accounts	106 1 0
	<u>£11,450 10 7</u>
Deduct Floating Balance due by the Society, as per preceding Abstract of Accounts	123 16 2
	<u>AMOUNT £11,326 14 5</u>

Exclusive of Library, Museum, Pictures, etc., Furniture of the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—

1. £896, 19s. 1d. three per cent. Debenture Stock of the North British Railway Company at 76⅔ per cent., the selling price at 1st October 1913	£685 1 0
2. £211, 4s. three per cent. Lien Stock of do. at 75½ per cent., do	159 9 1
3. Balance due by Union Bank of Scotland, Ltd., on Account Current	62 10 7
	<u>AMOUNT £907 0 8</u>

3. NEILL FUND—

1. £355 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 108½ per cent., the selling price at 1st October 1913	£385 3 6
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	49 3 0
	<u>AMOUNT £434 6 6</u>

4. MAKDOUGALL-BRISBANE FUND—

1. £365 four per cent. Consolidated Preference Stock No. 2 of the Caledonian Railway Company at 94⅔ per cent., the selling price at 1st October 1913	£346 5 11
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	165 19 1
	<u>AMOUNT £512 5 0</u>

5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—

Balance due by General Fund at 1st October 1913	£220 13 6
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6. GUNNING VICTORIA JUBILEE PRIZE FUND—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £1000 three per cent. Consolidated Lien Stock of the North British Railway Company at 75½ per cent., the selling price at 1st October 1913 . . .	£755 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Account Current . . .	41 15 4
AMOUNT . . .	<u>£796 15 4</u>

EDINBURGH, 16th October 1913.—We have examined the six preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1912-1913, and have found them to be correct. The securities of the various Investments at 1st October 1913, as noted in the above Statement of Funds, have been exhibited to us.

LINDSAY, JAMIESON & HALDANE,
Auditors.

**List of Library Exchanges, Presentations, etc., at
1st January 1914.**

1. TRANSACTIONS AND PROCEEDINGS OF LEARNED SOCIETIES, ACADEMIES, ETC., RECEIVED BY EXCHANGE OF PUBLICATIONS, AND LIST OF PUBLIC INSTITUTIONS ENTITLED TO RECEIVE COPIES OF THE TRANSACTIONS AND PROCEEDINGS OF THE ROYAL SOCIETY OF EDINBURGH. (*For convenience certain Presentations are included in this List.*)

T.P. prefixed to a name indicates that the Institution is entitled to receive *Transactions* and *Proceedings*. P. indicates *Proceedings*.

AFRICA (BRITISH CENTRAL).

ZOMBA.—*Scientific Department.* Meteorological Observations, Fol. (*Presented by H.M. Acting Commissioner and Consul-General.*)

America (North). (See CANADA, UNITED STATES, AND MEXICO.)

AMERICA (SOUTH).

- T.P. BUENOS AIRES (ARGENTINE REPUBLIC).—*Museo Nacional.* Anales.
- P. *Sociedad Physis.* Boletin.
- Oficina Meteorologica Argentina.* Anales. (*Presented.*)
- CORDOBA—
- T.P. *Academia Nacional de Ciencias de la Republica Argentina.* Boletin.
- T.P. *National Observatory.* Annals.—Maps.
- T.P. LA PLATA (ARGENTINE REPUBLIC).—*Museo de La Plata.*
- LIMA (PERU). *Cuerpo de Ingenieros de Minas del Peru.* Boletin. (*Presented.*)
- P. MONTEVIDEO (URUGUAY).—*Museo Nacional.* Anales (Flora Uruguay).
- T.P. PARÀ (BRAZIL).—*Museu Paraense de Historia Natural e Ethnographia.* Boletin.
- P. QUITO (ECUADOR).—*Observatorio Astronomico y Meteorologico.*
- RIO DE JANEIRO (BRAZIL)—
- T.P. *Observatorio.* Annuario.—Boletin Mensal.
- P. *Museu Nacional.* Revista (Archivos).
- SANTIAGO (CHILI)—
- T.P. *Société Scientifique du Chili.* Actes.
- P. *Deutscher Wissenschaftlicher Verein.*
- P. SAN SALVADOR.—*Observatorio Astronómico y Meteorológico.*
- VALPARAISO (CHILI).—*Servicio Meteorologico.* Annuario. (*Presented.*)

AUSTRALIA.

Australasian Association for the Advancement of Science.—Reports. (*Presented.*)

ADELAIDE—

- P. *University Library.*
 P. *Royal Society of South Australia.* Transactions and Proceedings.—Memoirs.
 P. *Royal Geographical Society of Australasia (South Australian Branch).* Proceedings.
Observatory. Meteorological Observations. 4to. (*Presented.*)

BRISBANE—

- T.P. *University of Queensland.*
 P. *Royal Society of Queensland.* Transactions.
 P. *Royal Geographical Society (Queensland Branch).* Queensland Geographical Journal.
 P. *Government Meteorological Office.*
 P. *Water Supply Department.*
 P. *GEELONG (VICTORIA).—Gordon Technical College.*
 T.P. *HOBART.—Royal Society of Tasmania.* Proceedings.

MELBOURNE—

- Commonwealth Bureau of Census and Statistics.* Official Year Book.
 By G. H. Knibbs. (*Presented.*)
National Museum. Memoirs. (*Presented.*)
 T.P. *University Library.*
 P. *Royal Society of Victoria.* Proceedings.—Transactions.

PERTH, W.A.—

- P. *Geological Survey.* Annual Progress Reports.—Bulletins.
Government Statistician's Office. Monthly Statistical Abstract. (*Presented.*)

SYDNEY—

- T.P. *University Library.* Calendar.—Reprints of Papers from Science Laboratories.
 T.P. *Department of Mines and Agriculture (Geological Survey), N.S.W.* Records.—Annual Reports.—Palaeontology. Mineral Resources.
 T.P. *Linnean Society of New South Wales.* Proceedings.
 T.P. *Royal Society of New South Wales.* Journal and Proceedings.
 P. *Australian Museum.* Records.—Reports.—Memoirs.—Catalogues.
N.S.W. Government. Fisheries Report. (*Presented.*)

AUSTRIA.

CRACOW—

- T.P. *Académie des Sciences.* Rozprawy Wydziału matematyczno-przyrodniczego (Proceedings, Math. and Nat. Sciences Cl.).—Rozprawy Wydziału filologicznego (Proc., Philological Section).—Rozprawy Wydziału historyczno-filozoficznego (Proc., Hist.-Phil. Section).—Sprawozdanie Komisji do badania historii sztuki w Polsce (Proc., Commission on History of Art in Poland).—Sprawozdanie Komisji fizyjograficznej (Proc., Commission on Physiography).—Geological Atlas of Galicia; Text, Maps.—Bulletin International, etc.

GRATZ—

- T.P. *Naturwissenschaftlicher Verein für Steiermark.* Mittheilungen.
 P. *Chemisches Institut der K. K. Universität.*
 P. *LEMBERG.—Société Scientifique de Chevtchenko.*

PRAGUE—

- P. *Deutscher Nat.-Med. Verein für Böhmen "Lotos."*—"Lotos."
- T.P. *K. K. Sternwurte.* Magnetische und Meteorologische Beobachtungen.
Astronomische Beobachtungen.
- T.P. *K. Böhmische Gesellschaft.* Sitzungsberichte : Math.-Naturw. Classe ; Phil.-Hist.-Philol. Classe.—Jahresbericht,—and other publications.
- T.P. *Ceská Akademie Císaře Františka Josefa pro Vědy Slovesnost a Umění.* Almanach.—Vestnik (Proceedings).—Rozpravy (Transactions) : Phil.-Hist. Class ; Math.-Phys. Cl. ; Philol. Cl.—Historický Archiv.—Bulletin International, Résumé des Travaux présentés,—and other publications of the Academy.
- P. SARAJEVO (BOSNIA).—*The Governor-General of Bosnia-Herzegovina.* Ergebnisse der Meteorologischen Beobachtungen.

TRIESTE—

- P. *Società Adriatica di Scienze Naturali.*
- P. *Museo Civico di Storia Naturale.*
- P. *Osservatorio Marittimo.* Rapporto Annuale.

VIENNA—

- T.P. *Kais. Akademie der Wissenschaften.* Denkschriften : Math.-Naturwissenschaftliche Classe ; Philosophisch-Historische Classe — Sitzungsberichte der Math.-Naturwissenschaftlichen Classe ; Abtheil. I., II.a, II.b, III. ; Philosoph.-Historische Classe.—Almanach.—Mittheilungen der Erdbeben Commission.
- T.P. *K. K. Geologische Reichsanstalt.* Abhandlungen.—Jahrbücher.—Verhandlungen.
- T.P. *Oesterreichische Gesellschaft für Meteorologie.* Meteorologische Zeitschrift.
- T.P. *K. K. Zoologisch-Botanische Gesellschaft.* Verhandlungen.—Abhandlungen.
- P. *K. K. Naturhistorisches Hofmuseum.* Annalen.
- K. K. Central-Anstalt für Meteorologie und Erdmagnetismus. Jahrbücher. 4to.—Allgemeiner Bericht und Chronik. 8vo. (*Presented.*)
- K. K. Militär Geographisches Institut. Astronomisch-Geodätischen Arbeiten.—Astronomische Arbeiten. 4to.—Längenbestimmungen. 4to.—Die Ergebnisse der Triangulierungen. 4to. (*Presented.*)
- Zoologisches Institut der Universität und der Zoologischen Station in Triest.* Arbeiten. (*Purchased.*)

BELGIUM.

BRUSSELS—

- T.P. *Académie Royale des Sciences, des Lettres et des Beaux Arts de Belgique.* Mémoires.—Bulletins.—Annuaire.—Biographie Nationale.
- T.P. *Musée Royal d'Histoire Naturelle.* Mémoires.
- T.P. *Musée du Congo.* Annales.—Botanique. Zoologie. Ethnographie et. Anthropologie. Linguistique, etc.
- T.P. *L'Observatoire Royal de Belgique, Uccle.* Annuaire.—Annales Astronomiques —Annales Météorologiques.—Annales.—Physique du Globe.—Bulletin Climatologique.—Observations Météorologiques.

BRUSSELS—*continued*—

- T.P. *Société Scientifique.* Annales.
 P. *Société Belge d'Astronomie.* Ciel et Terre. (*Purchased.*)
 T.P. GHENT.—*University Library.*
 T.P. LOUVAIN.—*University Library.*

BOSNIA-HERZEGOVINA. (*See AUSTRIA.*)

BULGARIA.

- P. SOFIA.—*Station Centrale Météorologique de Bulgarie.* Bulletin Mensuel.—
 Bulletins Annuaires.

CANADA.

- EDMONTON (ALBERTA).—*Department of Agriculture.* Annual Report.—
 (*Presented.*)
 P. HALIFAX (NOVA SCOTIA).—*Nova Scotian Institute of Science.* Proceedings
 and Transactions.
 T.P. KINGSTON.—*Queen's University.*
 MONTREAL—
 P. *Natural History Society.* Proceedings.
 P. *Canadian Society of Civil Engineers.* Transactions.—Annual Reports.
 OTTAWA—
 T.P. *Royal Society of Canada.* Proceedings and Transactions.
 T.P. *Geological Survey of Canada.* Annual Reports.—Palæozoic Fossils.—Maps,
 Memoirs, and other Publications.
 P. *Literary and Scientific Society.* Transactions.
 T.P. QUEBEC.—*Literary and Philosophical Society.* Transactions.
 TORONTO—
 T.P. *University.* University Studies. (History. Psychological Series. Geological
 Series. Economic Series. Physiological Series. Biological Series.
 Physical Science Series. Papers from the Chemical Laboratory.) etc.
 T.P. *Canadian Institute.* Transactions.
 P. *Royal Astronomical Society of Canada.* Journal.—Astronomical Handbook.

CAPE COLONY. (*See UNION OF SOUTH AFRICA.*)

CEYLON.

- COLOMBO—
 T.P. *Museum.* Spolia Zeylanica. Annual Report.

CHINA.

- HONG KONG—
 P. *Royal Observatory.* Monthly Meteorological Bulletin.—Report.

DENMARK.

COPENHAGEN—

- T.P. *Académie Royale de Copenhague.* Mémoires: Classe des Sciences.—*Oversigt.*
 P. *Naturhistorisk Forening.* Videnskabelige Meddelelser.
 P. *Danish Biological Station.* Report.
Conseil Permanent International pour l'Exploration de la Mer. Publications de circonstance.—Rapports et Procès-Verbaux de Réunions.—Bulletin des Résultats acquis pendant les croisières périodiques.—Bulletin Statistique. (*Presented.*)
Kommissionen for Havundersøgelse. Meddelelser: Série Fiskeri. Série Plankton. Série Hydrografi.—Skrifter. (*Presented.*)
University (Zoological Museum). Reports of the Danish Ingolf-Expedition. (*Presented.*)

EGYPT.

- T.P. CAIRO.—*School of Medicine.* Records.
Ministry of Finance (Survey Dept.: Archaeological Survey of Nubia). Bulletin, Reports, Papers. (*Presented.*)

ENGLAND AND WALES.

BIRMINGHAM—

- P. *Philosophical Society.* Proceedings.
University. Calendar. (*Presented.*)

CAMBRIDGE—

- T.P. *Philosophical Society.* Transactions and Proceedings.
 T.P. *University Library—Observatory.* Report.—Observations.
 T.P. CARDIFF.—*University College of South Wales.*
 COVENTRY.—Annual Report of the Health of the City. (*Presented by Dr Snell.*)
 P. ESSEX.—*Essex Field Club.* The Essex Naturalist.
 T.P. GREENWICH.—*Royal Observatory.* Astronomical, Magnetical, and Meteorological Observations.—Photo-heliographic Results and other Publications.
 T.P. HARPENDEN (HERTS.).—*Rothamsted Exp. Station.* (*Lawes Agricultural Trust.*)

LEEDS—

- T.P. *Philosophical and Literary Society.* Reports.
 P. *Yorkshire Geological and Polytechnic Society.* Proceedings.

LIVERPOOL—

- T.P. *University College Library.*
 P. *Biological Society.* Proceedings and Transactions.
 P. *Geological Society.* Proceedings.

LONDON—

- P. *Admiralty.* Nautical Almanac and Astronomical Ephemeris.—Health of the Navy (Annual Report).
 T.P. *Anthropological Institute.* Journal.

LONDON—*continued*—

- T.P. *Athenæum Club.*
British Antarctic Expedition, 1907–09. Reports on Scientific Investigations.
(Presented.)
- T.P. *British Association for the Advancement of Science.* Reports.
- T.P. *British Museum (Copyright Office).* Reproductions from Illuminated Manuscripts.
- T.P. *British Museum. Natural History Department.* Catalogues, Monographs, Lists, etc. *National Antarctic Expedition, 1901–04.* Publications.
- T.P. *Chemical Society.* Journal. Abstract of Proceedings.
- P. *Faraday Society.* Transactions.
- T.P. *Geological Society.* Quarterly Journal.—Geological Literature.—Abstract of Proceedings.
- T.P. *Geological Survey of the United Kingdom.* Summary of Progress. Memoirs.
- P. *Geologists' Association.* Proceedings.
- T.P. *Hydrographic Office.*
- T.P. *Imperial Institute.*
- T.P. *Institution of Civil Engineers.* Minutes of Proceedings, etc.
- T.P. *Institution of Electrical Engineers.* Journal.
- P. *Institution of Mechanical Engineers.* Proceedings.
- T.P. *International Catalogue of Scientific Literature.* (Purchased.)
- T.P. *Linnean Society.* Journal: Zoology; Botany.—Transactions: Zoology; Botany.—Proceedings.
- P. *Mathematical Society.* Proceedings.
- P. *Meteorological Office.* Report of the Meteorological Committee to the Lords Commissioners of H.M. Treasury.—Reports of the International Meteorological Committee.—Hourly Readings.—Weekly Weather Reports.—Monthly and Quarterly Summaries.—Meteorological Observations at Stations of First and Second Order, and other Publications. Geophysical Journal.—Geophysical Memoirs.
- Mineralogical Society of Great Britain and Ireland.* Mineralogical Magazine and Journal. (Presented.)
- National Antarctic Expedition, 1901–04.* (Presented.)
- Optical Society.* Transactions. (Purchased.)
- P. *Pharmaceutical Society.* Journal.—Calendar.
- P. *Physical Society.* Proceedings.
- T.P. *Royal Astronomical Society.* Monthly Notices.—Memoirs.
- T.P. *Royal College of Surgeons.*
- T.P. *Royal Geographical Society.* Geographical Journal.
- T.P. *Royal Horticultural Society.* Journal.
- T.P. *Royal Institution.* Proceedings.
- P. *Royal Meteorological Society.* Quarterly Journal.
- T.P. *Royal Microscopical Society.* Journal.
- P. *Royal Photographic Society.* Photographic Journal.
- T.P. *Royal Society.* Philosophical Transactions.—Proceedings.—Year-Book.—*National Antarctic Expedition, 1901–04,* Publications; and other Publications.
- T.P. *Royal Society of Arts.* Journal.

LONDON—*continued*—

- T.P. *Royal Society of Literature.* Transactions.—Reports.
- T.P. *Royal Society of Medicine.* Proceedings.
- T.P. *Royal Statistical Society.* Journal.
- T.P. *Society of Antiquaries.* Proceedings.—Archæologia ; or Miscellaneous Tracts relating to Antiquity.
- Society of Chemical Industry.* Journal. (*Presented.*)
- Society for Psychical Research.* Journal.—Proceedings. (*Presented by W. C. Crawford, Esq.*)
- Solar Physics Committee.* Annual Report, and other Publications. (*Presented.*)
- T.P. *United Service Institution.*
- T.P. *University College.* Calendar.
- T.P. *University.*
- T.P. *Zoological Society.* Transactions.—Proceedings.
- T.P. The Editor of *Nature*.—*Nature*.
- T.P. The Editor of *The Electrician*.—*Electrician*.
- T.P. The Editor of *Science Abstracts*.—*Science Abstracts*.

MANCHESTER—

- T.P. *Literary and Philosophical Society.* Memoirs and Proceedings.
- T.P. *University.*—Publications—Medical Series. Public Health Series. Anatomical Series. Physical Series. Biological Series. Lectures. *Manchester Museum (University of Manchester)*. Annual Reports—Notes from the Museum.
- P. *Microscopical Society.* Transactions and Annual Report.

NEWCASTLE-ON-TYNE—

- P. *Natural History Society of Northumberland, Durham, etc.* Transactions.
- T.P. *North of England Institute of Mining and Mechanical Engineers.* Transactions.—Annual Reports.
- Cullercoats Dove Marine Laboratory.* Annual Report. (*Presented.*)
- P. *Literary and Philosophical Society.* Proceedings. (*Presented.*)
- University of Durham Philosophical Society.* Proceedings. (*Presented.*)
- P. NORWICH.—*Norfolk and Norwich Naturalists' Society.* Transactions.

OXFORD—

- T.P. *Bodleian Library.*
- P. *Ashmolean Society.* Proceedings and Report.
- P. *Radcliffe Observatory.* Results of Astronomical and Meteorological Observations.
- University Observatory.* Astrographic Catalogue. (*Presented.*)
- P. PENZANCE.—*Royal Geological Society of Cornwall.* Transactions.

- T.P. PLYMOUTH.—*Marine Biological Association.* Journal.
- RICHMOND (SURREY)—

- T.P. *Kew Observatory.*
- P. SCARBOROUGH.—*Philosophical Society.*
- T.P. SHEFFIELD.—*University College.*
- SOUTHPORT.—*Meteorological Observatory.* Results of Observations. Joseph Baxendell, Meteorologist. (*Presented.*)
- T.P. TEDDINGTON (MIDDLESEX).—*National Physical Laboratory.* Collected Researches.—Annual Reports.

P. TRURO.—*Royal Institution of Cornwall. Journal.*

YORK—

T.P. *Yorkshire Philosophical Society. Reports.*

FINLAND.

HELSINGFORS—

Academie Scientiarum Fennicæ. Annales. Sitzungsberichte.—Documenta Historica. (Presented.)

Hydrographisch Biologisch Untersuchungen. (Presented.)

T.P. *Societas Scientiarum Fennica (Société des Sciences de Finlande). Acta Societatis Scientiarum Fennicæ.—Öfversigt.—Meteorologisches Jahrbuch. —Bidrag till Kändedom om Finlands Natur och Folk.*

T.P. *Societas pro Fauna et Flora Fennica. Acta.—Meddelanden.*

P. *Société de Géographie de Finlande. Fennia.—Meddelanden.*

FRANCE.

BESANÇON.—*Université Observatoire National. Bulletin Chronométrique et Bulletin Météorologique. (Presented.)*

BORDEAUX—

T.P. *Société des Sciences Physiques et Naturelles. Mémoires.—Observations Pluviométriques et Thermométriques.—Procès-Verbaux des Séances.*

P. *Société de Géographie Commerciale. Bulletin. L'Observatoire. Catalogue Photographique du Ciel.*

P. CHERBOURG.—*Société Nationale des Sciences Naturelles et Mathématiques. Mémoires.*

P. CONCARNEAU.—*College de France (Laboratoire de Zoologie et de Physiologie Maritime). Travaux Scientifiques.*

P. DIJON.—*Académie des Sciences. Mémoires.*

LILLE—

T.P. *Société des Sciences.*

T.P. *Société Géologique du Nord. Annales.—Mémoires.*

P. *Université de France. Travaux et Mémoires.*

LYONS—

T.P. *Académie des Sciences, Belles Lettres et Arts. Mémoires.*

T.P. *Société d'Agriculture, Histoire Nat. et Arts. Annales.*

T.P. *Université. Annales, Nouv. Série:—I. Sciences, Médecine. II. Droit, Lettres.*

P. *Société Botanique. Annales.—Nouveaux Bulletins.*

P. *Société Linnéenne. Annales.*

MARSEILLES—

T.P. *Faculté des Sciences. Annales.*

P. *Société Scientifique Industrielle. Bulletin.*

T.P. MONTPELLIER.—*Académie des Sciences et Lettres. Mémoires: Section des Sciences; Section des Lettres; Section de Médecine. Bulletin Mensuel.*

- T.P. NANTES.—*Société Scientifique des Sciences Naturelles de l'Ouest de la France.*
Bulletin.
- T.P. NICE.—*L'Observatoire.* Annales.
- PARIS—
- T.P. *Académie des Sciences.* Comptes Rendus. — *Observatoire d'Abbadia:*
Observations, 4to, and other Publications.
- T.P. *Académie des Inscriptions et Belles-Lettres.* Comptes Rendus.
- T.P. *Association Française pour l'Avancement des Sciences.* Comptes Rendus.
- T.P. *Bureau International des Poids et Mesures.* — Procès-Verbaux des Séances.
—Travaux et Mémoires.
- T.P. *Bureau des Longitudes.* Annuaire.
- P. *L'École des Ponts et Chaussées.*
- T.P. *Ministère de la Marine.* (*Service Hydrographique.*) Annales Hydrographiques. Expédition de Charcot, 1903-05. (*See Presentation List.*)
- T.P. *École des Mines.* Annales des Mines.
- T.P. *École Normale Supérieure.* Annales.
- T.P. *École Polytechnique.* Journal.
- P. *École Libre des Sciences Politiques.*
- T.P. *Institut Océanographique.* Annales.
- T.P. *Ministère de l'Instruction Publique.* Expédition de Charcot, 1908-10. (*See Presentation List.*)
- T.P. *Musée Guimet.* Revue de l'Histoire des Religions.—Annales.—Bibliothèque d'Études.
- T.P. *Muséum d'Histoire Naturelle.* Nouvelles Archives.—Bulletin.
- T.P. *L'Observatoire.* Rapport Annuel sur l'État de l'Observatoire.—Annales.—Mémoires.—Carte Photographique du Ciel. Fol.—Catalogue Photographique du Ciel. 4to.
- L'Observatoire d'Astronomie Physique de Meudon.* Annales. (*Presented.*)
- T.P. *Société Nationale d'Agriculture.* Bulletins.—Mémoires.
- P. *Société d'Anthropologie.* Bulletin et Mémoires.
- T.P. *Société Nationale des Antiquaires.* Mémoires.—Bulletin.
- T.P. *Société de Biologie.* Comptes Rendus.
- T.P. *Société d'Encouragement pour l'Industrie Nationale.* Bulletin.
- T.P. *Société Française de Physique.* Journal de Physique.—Annuaire.—Procès-Verbaux.
- T.P. *Société de Géographie.* La Géographie.
- T.P. *Société Géologique de France.* Bulletins.—Mémoires (Paléontologie).
- P. *Sociétés des Jeunes Naturalistes et d'Études Scientifiques.* Feuilles des Jeunes Naturalistes.
- T.P. *Société Mathématique.* Bulletin.
- P. *Société Philomathique.* Bulletin.
- T.P. *Société Zoologique.* Bulletin.—Mémoires.
- P. *Revue Générale des Sciences Pures et Appliquées.*
- T.P. RENNES.—*Société Scientifique et Médicale de l'Ouest.* Bulletin.
- TOULOUSE—
- T.P. *Université.*—*Faculté des Sciences.*—*L'Observatoire.* Annales.
- P. *Académie des Sciences.* Mémoires.

GERMANY.

BERLIN—

- Carte Géologique internationale de l'Europe.* Livres I.-VIII. (complete).
(Presented.)
- T.P. *K. Akademie der Wissenschaften.* Abhandlungen.—Sitzungsberichte.
- T.P. *Physikalische Gesellschaft.* Fortschritte der Physik: 1^{te} Abtheil; Physik der Materie. 2^{te} Abtheil; Physik des Aethers. 3^e Abtheil; Kosmische Physik.—Verhandlungen.
- T.P. *Deutsche Geologische Gesellschaft.* Zeitschrift.—Monatsberichte.
- P. *Deutsche Meteorologische Gesellschaft.* Zeitschrift.
- P. *Königl. Preussisches Meteorologisches Institut.*
- P. *Gesellschaft Naturforschender Freunde.* Sitzungsberichte.—Archiv für Biontologie.
- P. *Kgl. Technische Hochschule.* Programm.
- T.P. *Zoologisches Museum.* Mitteilungen.

BONN—

- P. *Naturhistorischer Verein der Preussischen Rheinlande und Westfalens.* Verhandlungen.
- Niederrheinische Gesellschaft für Natur- und Heilkunde.* Sitzungsberichte.
(Presented.)
- T.P. BREMEN.—*Naturwissenschaftlicher Verein.* Abhandlungen.
- P. BRUNSWICK.—*Verein für Naturwissenschaft.* Jahresberichte.
- P. CARLSRUHE.—*Technische Hochschule.* Dissertations.
- P. CASSEL.—*Verein für Naturkunde.* Berichte.
- T.P. CHARLOTTENBURG.—*Physikalisch-Technische Reichsanstalt.* Abhandlungen.
- P. CHEMNITZ.—*Naturwissenschaftliche Gesellschaft.* Berichte.
- T.P. DANTZIC.—*Naturforschende Gesellschaft.* Schriften.
- Westpreussischer Botanisch-Zoologischer Verein.* Bericht. (Presented.)

ERLANGEN—

- T.P. *University.* Inaugural Dissertations.
- P. *Physikalisch-Medicinische Societät.* Sitzungsberichte.
- T.P. FRANKFURT-AM-MAIN.—*Senckenbergische Naturforschende Gesellschaft.* Abhandlungen.—Berichte.
- P. FRANKFURT-AM-ODER.—*Naturwissenschaftlicher Verein.* Helios.
- P. FREIBURG-I-BR.—*Naturforschende Gesellschaft.* Berichte.

GIESSEN—

- T.P. *University.* Inaugural Dissertations.
- P. *Oberhessische Gesellschaft für Natur- und Heilkunde.* Berichte.
- T.P. GÖTTINGEN.—*K. Gesellschaft der Wissenschaften.* Abhandlungen, Neue Folge: Math.-Phys. Classe; Phil.-Hist. Classe.—Nachrichten: Math.-Phys. Cl.; Phil.-Hist. Cl.; Geschäftliche Mittheilungen.—(Gelehrte Anzeigen. Purchased.)

HALLE—

- T.P. *K. Leopold-Carolinisch-Deutsche Akademie der Naturforscher.* Nova Acta (Verhandlungen).—Leopoldina.
- T.P. *Naturforschende Gesellschaft.* Abhandlungen.
- P. *Verein für Erdkunde.* Mittheilungen.

HALLE—*continued*—

- P. *Naturwissenschaftlicher Verein für Sachsen und Thüringen.*
 P. *Deutsche Mathematiker Vereinigung.* Jahresbericht.

HAMBURG—

- T.P. *Kaiserliche Marine Deutsche Seewarte.* Annalen der Hydrographie, etc.—Jahresbericht.
 T.P. *Naturwissenschaftlicher Verein.* Abhandlungen aus dem Gebiete der Naturwissenschaften.—Verhandlungen.
 T.P. *Naturhistorisches Museum.* Jahrbuch.—Beihefte.—Mitteilungen.
 P. *Verein für Naturwissenschaftliche Unterhaltung.* Verhandlungen.
 T.P. HANNOVER.—*Naturhistorische Gesellschaft.* Jahresbericht.
 T.P. HELGOLAND.—*K. Biologisches Anstalt.* Wissenschaftliche Meeresuntersuchungen (Abtheilung Helgoland).
 T.P. JENA.—*Medizinisch-Naturwissenschaftliche Gesellschaft.* Jenaische Zeitschrift für Naturwissenschaft.—Denkschriften.

KIEL—

- T.P. *Universität.* Dissertations.
 T.P. *Kommission zur Wissenschaftlichen Untersuchung der Deutschen Meere.* Wissenschaftliche Meeresuntersuchungen (Abtheilung Kiel).
 P. *Naturwissenschaftlicher Verein für Schleswig-Holstein.* Schriften.
 T.P. KÖNIGSBERG.—*University.*

LEIPZIG—

- Fürstlich Jablonowskische Gesellschaft. Preisschriften. (*Presented.*)
 T.P. Königl. Sächsische Gesellschaft der Wissenschaften. Berichte: Math.-Phys. Classe; Philologisch-Historische Classe.—Abhandlungen der Math.-Phys. Classe; Phil.-Hist. Classe.
 T.P. Editor of *Annalen der Physik.* Annalen der Physik.
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 T.P. *Yerkes Observatory (University of Chicago).* Publications.
 P. *Academy of Sciences.* Bulletins.—Special Publications.—Bulletins of the Natural History Survey.

CINCINNATI—

- P. *Observatory (University).* Publications.—University Record.
 P. *Society of Natural History.* Journal.
 T.P. CLEVELAND (OHIO).—*Geological Society of America.* Bulletins.
 T.P. CLINTON (IOWA).—*Litchfield Observatory, Hamilton College.*
 COLORADO SPRINGS.—*Colorado College.* Colorado College Studies. (*Presented.*)
 P. CONNECTICUT.—*Connecticut Academy of Arts and Sciences.* Transactions.—Memoirs.
 P. DAVENPORT.—*Academy of Natural Sciences.* Proceedings.
 P. DENVER (COLORADO).—*Scientific Society of Colorado.* Proceedings.
 T.P. DES MOINES (IOWA).—*Iowa Academy of Sciences.* Proceedings.
 P. GARRISON, N.Y.—Editor, *American Naturalist.*
 T.P. GRANVILLE (OHIO).—*Denison University and Scientific Association.* Bulletin of the Scientific Laboratories.

P. INDIANOPOLIS.—*Indiana Academy of Sciences.* Proceedings.

IOWA CITY—

- P. *Geological Survey.* Annual Reports.
 P. *State University. Laboratories of Natural History.* Bulletins.—Contributions from the Physical Laboratories.

IOWA. (*See DES MOINES.*)

ITHACA (N.Y.)—

- P. The Editor, *Physical Review.* (Cornell University.)

ITHACA (N.Y.)—*continued*—

P. The Editors, *Journal of Physical Chemistry*. (Cornell University.)
 T.P. LAWRENCE (KANSAS).—*University of Kansas*. Science Bulletin (University Quarterly).

P. LINCOLN (NEBRASKA).—*University of Nebraska*. Agricultural Experiment Station. Bulletins.

MADISON—

T.P. Wisconsin University. Washburn Observatory. Observations.

P. Wisconsin Academy of Sciences, Arts, and Letters. Transactions.

P. Geological and Natural History Survey. Bulletins.

P. MASSACHUSETTS.—Tufts College Library. Tufts College Studies.

P. MERIDEN (CONN.).—Meriden Scientific Association.

MICHIGAN. (See ANN ARBOR.)

MINNEAPOLIS (MINN.)—

T.P. { University of Minnesota. Studies.—Bulletin of the School of Mines.
 } Geological and Natural History Survey of Minnesota. Reports.

P. Botanical Survey.

MISSOURI. (See ST. LOUIS and ROLLA.)

P. MOUNT HAMILTON (CALIFORNIA).—Lick Observatory. Bulletins.—Publications.

T.P. MOUNT WILSON (CALIFORNIA).—Solar Observatory. Contributions.—Reports.

T.P. NEWHAVEN (CONN.).—Yale College. Astronomical Observatory of Yale University. Transactions.—Reports.

P. NEW ORLEANS.—Academy of Sciences.

NEW YORK—

T.P. American Mathematical Society. Bulletins.—Transactions.

T.P. American Museum of Natural History. Bulletins.—Memoirs.—American Museum Journal.—Annual Reports.—Anthropological Papers.—Guide Leaflets.—Handbook Series.—Monograph Series.

P. American Geographical Society. Bulletin.

P. American Institute of Electrical Engineers. Proceedings.

NEW YORK. (See also ALBANY.)

PHILADELPHIA—

T.P. American Philosophical Society for Promoting Useful Knowledge. Proceedings.—Transactions.

T.P. Academy of Natural Sciences. Proceedings.—Journal.

T.P. University of Pennsylvania. Publications:—Philology, Literature, and Archæology, Mathematics, etc. Contributions from the Zoological and Botanical Laboratories. University Bulletins.—Theses.—Calendar.

T.P. Geological Survey of Pennsylvania.

P. Wagner Free Institute of Science. Transactions.

P. Geographical Society. Bulletin.

P. Commercial Museum.

P. PORTLAND (MAINE).—Society of Natural History. Proceedings.

P. PRINCETON, N. J.—University. Annals of Mathematics.—University Observatory. Contributions.

P. ROCHESTER.—Academy of Science. Proceedings.

T.P. ROLLA (MISS.).—Bureau of Geology and Mines. Biennial Reports, etc.

T.P. SALEM.—*Essex Institute.*

SAINT LOUIS—

T.P. *Academy of Sciences.* Transactions.

P. *Missouri Botanical Garden.* Annual Reports.

P. *Washington University.* University Studies.

T.P. SAN FRANCISCO (CALIFORNIA).—*Academy of Sciences.* Proceedings.—Memoirs.—Occasional Papers.

STANFORD (CALIFORNIA).—*University.* Publications. (*Presented.*)

P. TOPEKA.—*Kansas Academy of Science.* Transactions.

T.P. URBANA.—*University of Illinois.* Bulletins of *State Geological Survey, State Laboratory of Natural History, and Engineering Experiment Station.*

WASHINGTON—

T.P. *U.S. National Academy of Sciences.* Memoirs.

T.P. *Bureau of Ethnology.* Annual Reports.—Bulletins.

T.P. *U.S. Coast and Geodetic Survey.* Annual Reports, etc.

T.P. *U.S. Commission of Fish and Fisheries.* Reports.—Bulletins.

T.P. *U.S. Naval Observatory.* Reports.—Observations.

T.P. *U.S. Geological Survey.* Bulletins.—Annual Reports.—Monographs.—Geologic Atlas of the United States.—Mineral Resources.—Professional Papers.—Water Supply and Irrigation Papers.

Geological Society of America. (*See CLEVELAND.*)

T.P. *Weather Bureau.* (*Department of Agriculture.*) Monthly Weather Review.—Bulletins.—Reports.—Bulletin of the Mount Weather Observatory (now embodied in Monthly Weather Review).

T.P. *Smithsonian Institution.* Miscellaneous Collections.—The same (Quarterly Issue).—Contributions to Knowledge.—Reports.—Annals of the Astrophysical Observatory.—Harriman Alaska Expedition, Vols. I.—V. and VIII.—XIII. 4to.

T.P. *Surgeon-General's Office.* Index Catalogue of the Library. 4to.

T.P. *Carnegie Institution of Washington.* Year-Books.—Publications. Classics of International Law.—*Carriege Foundation for the Advancement of Teaching.* Annual Report.—Bulletin.

T.P. *American Association for the Advancement of Science.* Proceedings.

P. *U.S. National Museum.* Bulletins.—Reports.—Proceedings.—Contributions from the U.S. National Herbarium.

P. *Department of Agriculture.* (*Division of Economic Ornithology and Mammalogy.*) Bulletin.

P. *U.S. Patent Office.*

Washington Academy of Sciences, Journal of the. (*Purchase.*)

Bureau of Standards, Department of Commerce and Labour. Bulletins. (*Presented.*)—Technologic Papers.

WISCONSIN. (*See MADISON.*)

VICTORIA. (*See AUSTRALIA.*)

II. Donations from Authors, etc.

- Agassiz (G. R.). Letters and Recollections of Alexander Agassiz. 8vo. London, 1913.
- Backhouse (T. W.). Catalogue of 9842 Stars for the Epoch of 1900. 4to. Sunderland, 1911.
- Bartholomew (J. G.). The Citizen's Atlas. Fol. Edinburgh, 1912.
- Belgica :—Résultats du Voyage du S.Y., en 1897, 1898, 1899. Rapports Scientifiques. Géologie :—Petrographische Untersuchungen der Gesteinproben, II. Teil, von D. Sistek.
- Zoologie :—Tuniciers. Par E. van Beneden et M. de Selys-Longchamps. 4to. Anvers, 1913.
- Black (J. S.) and Chrystal (George). Life and Lectures of William Robertson Smith. 2 vols. 8vo. London, 1912.
- Bright (Charles). The Story of the Atlantic Cable. 8vo. London.
— Imperial Telegraphic Communication. 8vo. London, 1911.
- Buchanan (J. Y.). Scientific Papers, Vol. I. 8vo. Cambridge, 1913.
- Burgess (Dr James). Books presented by. *See end of list.*
- Cadell (H. M.) of Grange. The Story of the Forth. 4to. Glasgow, 1913.
- Canadian Biology, Contributions to, 1906-1910. 8vo. Ottawa, 1912. (*Presented by Prof. E. E. Prince, Dominion Commissioner of Fisheries.*)
- Carnegie (David) and Gladwyn (Sidney C.). Liquid Steel: its Manufacture and Cost. 8vo. London, 1913.
- Catalogue of Scientific Papers, 1800-1900. Subject-Index. Vol. II. Mechanics. 4to. Cambridge, 1909.
— Vol. III. Physics. Part I. 4to. Cambridge, 1912. (*Purchased.*)
- Catalogue of Serials in the Cape Peninsula. 4to. Cape Town, 1912. (*Presented by Dr Thos. Muir.*)
- Catalogus Codicum Manu Scriptorum Bibliothecæ Regiæ Monacensis. Tomi I. Pars VI. 8vo. München, 1912.
- Census, 12th Decennial, Scotland, 1911. Report, Vol. II. (*Purchased.*) Fol. London, 1913.
- Ceylon Company of Pearl Fishers, Ltd.—Ceylon Marine Biological Reports. Part VI., Jan. 1912. 8vo. Colombo, 1912.
- Charcot (Dr Jean). Expédition Antarctique Française (1903-1905). Hydrographie, Physique. 4to. Paris, 1911. (*From Ministère de la Marine, Paris.*)
— Deuxième Expédition Antarctique Française (1908-1910). Sciences Physiques; Sciences Naturelles. 8 vols. 4to. Paris, 1911-1913. (*From Ministère de l'Instruction Publique, Paris.*)
- Chrystal, Family of late Professor. Books, etc., presented by. *See end of list.*
— George. *See Black, J. S.*
- Congrès International de Botanique, Bruxelles, 1910. Actes, 2 vols. 4to. Bruxelles, 1912.
— Géologique International. Comptes Rendus—Berlin, 1885; London, 1888 (2 vols.); Vienna, 1903. (*Presented by Dr R. Kidston.*)
- Corral (José I. del). Nuevos Métodos para Resolver Ecuaciones Numéricas. 8vo. Madrid, 1912.

- Dyck (Walther von). Georg von Reichenbach. Fol. München, 1912.
- Elliot (Daniel Giraud). A Review of the Primates, Vols. I., II., III. 4to. New York, 1913. (*American Museum of Natural History.*)
- Fabo (Fray P.). Idiomas y Ethnografia de la Region Oriental de Colombia. 8vo. Bogota, 1911.
- Frend (W.). Principles of Algebra. 8vo. London, 1796. (*Presented by W. Forgan, Esq.*)
- Gill (Sir David). History and Description of the Royal Observatory, Cape of Good Hope. Fol. London, 1913. (*Presented by the Astronomer Royal.*)
- Glasgow University. Matriculation Albums, 1728–1858. Transcribed and annotated by the late W. Innes Addison. 4to. Glasgow, 1913.
- Green (Charles E.). The Local Incidence of Cancer. 8vo. Edinburgh, 1912.
- Guppy (H. B.). Studies in Seeds and Fruits. 8vo. London, 1912.
- Harris (Frank). Gravitation. 8vo. London, 1912.
- Hewat (Archibald). Widows' and Pension Funds. 8vo. Edinburgh, 1912.
- Iyer (L. K. A. K.). The Cochin Tribes and Castes. Vols. I. and II. 8vo. Madras, 1909 and 1912.
- Jungfleisch (Émile C.). Notice sur la Vie et les Travaux de Marcellin Berthelot. 8vo. Paris, 1913.
- Kjøbenhavn. Beretning om den anden Skandinaviske Matematikerkongres, 1911. 8vo. Kjøbenhavn, 1912.
- Laplace, Œuvres complètes de. Tomes XIII. and XIV. 4to. Paris, 1904 and 1912. (*Publiées par l'Académie des Sciences.*)
- London, Royal Society of. Catalogue of the Periodical Publications in the Library. 4to. London, 1912.
- Celebration of the 250th Anniversary, July 15–19, 1912. 4to. London, 1913.
- Record of the (3rd edition). 4to. London, 1912.
- Signatures in the first Journal-Book and the Charter-Book, being a facsimile of the Signatures of the Founders, Patrons, and Fellows of the Society from the year 1660 down to the present time. Fol. London, 1912.
- London, University College. Catalogue of Periodicals. 8vo. London, 1912.
- M'Aldowie (A. M.). Prehistoric Time-Measurement in Britain. 8vo. 1912.
- Macdonald (W. R.). The Construction of the Wonderful Canon of Logarithms by John Napier. 4to. Edinburgh, 1889.
- “Mark” Anniversary Volume. (*From Zoological Museum, Harvard University.*) 4to. New York, 1903.
- Mascart (J.). Books presented by. *See end of list.*
- Munro (Robert). Palæolithic Man and Terramara Settlements in Europe: being the Munro Lectures, Edinburgh University, 1912. 4to. Edinburgh, 1912.
- National Antarctic Expedition, 1901–4.
- Natural History, Vol. IV., Zoology and Botany. (*Presented by British Museum.*)
- Meteorology. Part II. 4to. London, 1913. (*Presented by the Royal Society.*)
- Orléans (Duc de). Campagne Arctique de 1907: Annélides Polychètes, par P. Fauvel. Coelenterés du Fond, par Dr H. Broch. Bryozoaires, par O. Norgaard. Crustacés Malacostracés, par Dr Louis Stappers. Fol. Bruxelles, 1912.

- Paton (D. Noël), Edited by. Glasgow University, Physiological Dept., Collected Papers. Vol. II.
- Vol. III. 8vo. Glasgow, 1913.
- Pemberton (H.). A View of Sir Isaac Newton's Philosophy. 4to. London, 1728.
(*From Dr J. Aitken.*)
- Proceedings of the First West Indian Intercolonial Tuberculosis Conference, March 1913. (*Presented by Dr G. H. Masson.*) Fol. Trinidad, 1913.
- Prym (Friedrich) und Rost (Georg). Theorie der Prym'schen Funktionen Erster Ordnung. 4to. Leipzig, 1911.
- Royal Society. See London.
- St Andrews University, 500th Anniversary, Memorial Volume of Scientific Papers. 4to. St Andrews, 1911.
- Scott (William). Clinical Bacteriology and Vaccine Therapy. 8vo. London, 1913.
- Scottish National Antarctic Expedition: Report on the Scientific Results of the Voyage of S.Y. "Scotia." Vol. III.: Botany.
- Vol. VI.: Zoology (Invertebrates, I.-XI.). 4to. Edinburgh, 1912. (*From Dr W. S. Bruce.*)
- Smith (Sigurd). Søren Hjorth, Inventor of the Dynamo-Electric Principle. 8vo. København, 1912.
- Solvay (E.). Sur l'Établissement des Principes fondamentaux de la Gravito-Matérialistique. 8vo. Bruxelles, 1912.
- Sudan, Anglo-Egyptian, Notes on the Mineral Deposits of. By Stanley C. Dunn. (*Presented by the Sudan Government.*) 8vo. Khartoum, 1911.
- Tables Annuelles de Constantes et Données Numériques de Chimie, de Physique, et de Technologie. (*Publiées sous le patronage de l'Association internationale des Académies.*) Vol. I., 1910. 4to. Paris, 1912.
- Vol. II., 1911. 4to. Paris, 1913.
- Thomas (N. W.). Anthropological Report on the Ibo-speaking Peoples of Nigeria. Parts I.-III. 8vo. London, 1913.
- Anthropological Report on the Edo-speaking Peoples. Parts I. and II. 8vo. London, 1910. (*Presented by H.M. Crown Agents for the Colonies.*)
- Turner (Sir Wm.). Marine Mammals in the Anatomical Museum of the University of Edinburgh. 8vo. London, 1912.
- Vialay (Alfred). Essai sur la Genèse et l'Evolution des Roches. 8vo. Paris, 1912. (*Presented by Author.*)
- Contribution a l'Étude des Relations existant entre les Circulations Atmosphériques, l'Electricité Atmosphérique et le Magnétisme Terrestre. 8vo. Paris, 1911.
- Victoria, Geological Survey of. Reports of Progress, Nos. II., III., IV., VII. 8vo. Melbourne, 1874-1884.
- Watson (R. Boog). (1) The Scaphopoda and Gasteropoda of the "Challenger Expedition."
- (2) Original drawings for plates of above. 4to. 1886. (*Presented by Charles B. Boog Watson.*)
- Webb (W. L.). Biography and Unparalleled Discoveries of T. J. J. See. 8vo. Lynn, Mass., 1913.
- Weir (James). The Energy System of Matter. 8vo. London, 1912.
- Zalessky (M. D.). Études Paléobotaniques. 4to. St Pétersbourg, 1911.

The following were presented by James Burgess, C.I.E., LL.D. (December 1912):—

- Euler (L.). *Methodus inveniendi Lineas Curvas Maximi Minimive proprietate gaudentes, etc.* 4to. Lausannae and Genevae, 1744.
- Fuss (P.-H.). *Correspondance Mathématique et Physique de quelques Célèbres Géomètres du XVIII^{ème} Siècle.* Tomes I., II. 8vo. St Pétersbourg, 1843.
- Gill (C.). *Application of the Angular Analysis to the Solution of Indeterminate Problems of the Second Degree.* 8vo. New York and London, 1848.
- Lubbock (J. W.). *On the Theory of the Moon and on the Perturbations of the Planets, Parts I.—VIII.* 8vo. London, 1834–1850.
- *Various Scientific Tracts.* 8vo. London, 1830–40.

The following were presented by the Family of the late Professor George Chrystal (February 1912):—

- Delebecque (André). *Les Lacs Français.* 4to. Paris, 1898.
- Forel (F. A.). *Le Léman: Monographie Limnologique.* Tomes I., II., and III. 8vo. Lausanne, 1892–1904.
- Hildebrandsson (M. le Dr H. H.) et Teisserenc de Bort (M. Léon). *Les Bases de la Météorologie Dynamique; Historique—État de nos Connaissances.* Tomes I. et II. 8vo. Paris, 1907, 1900.
- Lyons (Capt. H. G.). *The Physiography of the River Nile and its Basin.* (Survey Dept., Egypt.) 8vo. Cairo, 1906.
- Also numerous Pamphlets dealing with the subject of Seiches, a box of lantern slides illustrating Professor Chrystal's work on Seiches, and some photographic negatives.

The following were presented by Mons Jean Mascart, Paris (December 1912):—

- Becquerel (Mm. A. E.). *Résumé de l'Histoire de l'Électricité et du Magnétisme.* 8vo. Paris, 1858.
- Berthelot (M.). *Chimie Organique fondée sur la Synthèse.* Tomes I., II. 8vo. Paris, 1860.
- *Leçons sur les Méthodes Générales de Synthèse en Chimie Organique.* 8vo. Paris, 1864.
- *La Synthèse Chimique.* 8vo. Paris, 1876.
- Berzelius (J. J.). *Théorie des Proportions Chimiques.* (2^{ème} édition.) 8vo. Paris, 1835.
- *Traité de Chimie.* (Nouvelle édition.) Tomes I., II., III. 8vo. Paris, 1838–9.
- Beudant (F. S.). *La Minéralogie et la Géologie.* 12mo. Paris, 1851.
- Cahours (A.). *Leçons de Chimie Générale Élémentaire.* Tomes I., II. 12mo. Paris, 1855–6.
- *Traité de Chimie Générale Élémentaire (Chimie Organique).* Tomes I. II., III. 12mo. Paris, 1874–5.
- Caumont (M. de). *Excursions Archéologiques.* 8vo. Paris, 1858.
- Dupuit (J.). *Étudé sur le Mouvement des Eaux Courantes.* 8vo. Paris, 1848.
- Favre (P. A.) et Silbermann (J. T.). *Recherches sur les Quantités de Chaleur Dégagées dans les Actions Chimiques et Moléculaires.* 8vo. Paris, 1853.

- Fontaine (H.). *Électrolyse.* 8vo. Paris, 1892.
- Foussereau (G.). *Polarisation Rotatoire.* 8vo. Paris, 1893.
- Marielle (C. P.). *Répertoire de l'École Impériale Polytechnique. (1794-1853.)*
8vo. Paris, 1855.
- Marqfoy (G.). *Loi des Équivalents et Théorie Nouvelle de la Chimie.* 8vo.
Paris, 1897.
- Mascart (E.). *Électricité Industrielle.* 4to. Paris, 1903.
- Mascart (J.). *Les Progrès et l'Évolution de l'Astronomie.* 8vo. Turin, 1911.
——— *Un Observatoire près d'un Volcan.* 8vo. Turin, 1911.
- Millon (E.). *Éléments de Chimie Organique.* Tomes I., II. 8vo. Paris, 1845-8.
——— *Notice Biographique de, et ses Travaux de Chimie.* 8vo. Paris, 1870.
- Moigno (L'Abbé). *Physique Moléculaire.* 12mo. Paris, 1868.
- Pierre (I.). *Leçons de Chimie appliquée à l'Agriculture.* 8vo. Caen, 1849-50.
——— *Chimie Agricole.* 12mo. Paris, 1852.
- Pouillet (M.). *Notions Générales de Physique et de Météorologie.* 12mo. Paris,
1860.
- Raoult (F. M.). *Tonométrie.* 8vo. Paris, 1900.
- Sorot (J. L.). *Notice Biographique d'Auguste de la Rive.* 8vo. Genève, 1877.
- Sue (P.). *Histoire du Galvanisme.* Tomes I.-IV. 8vo. Paris, 1805.
- Tassilly (E.). *Étude des Propriétés Physiques des Alliages Métalliques.* 8vo.
Paris, 1904.
- Tresca (H.). *Mémoire sur le Poinçonnage des Métaux.* 4to. Paris, 1872.

III. List of Periodicals and Annual Publications added to the Library by Purchase, etc.

Periodicals not found in this List will be found in Exchange List.

Annuals (Works of Reference), see end of List.

- Acta Mathematica.
- American Journal of Science and Arts.
- * —— Naturalist.
- * —— Journal of Mathematics.
- * —— Chemical Journal.
- * —— Journal of Philology.
- Anatomischer Anzeiger.
——— Ergänzungshefte.
- Annalen der Chemie (Liebig's).
- * —— der Physik.
- * —— der Physik. (Beiblätter.)
- Annales de Chimie.
——— d'Hygiène Publique et de Médecine Légale.
——— de Physique.
——— des Sciences Naturelles. Zoologie et Paléontologie.
——— des Sciences Naturelles. Botanique.

* Received by exchange.

Annali dell' Islam. (*Presented.*)

Annals and Magazine of Natural History (Zoology, Botany, and Geology).

____ of Botany.

* ____ of Mathematics. (Princeton, N.J.)

Anthropologie (L').

Arbeiten-Zoologisches Institut der Universität und der Zoologischen Station in Triest.

* Archiv für Mathematik og Naturvidenskab.

* Archiv für Biontologie.

Archives de Biologie.

____ de Zoologie Expérimentale et Générale.

* ____ des Sciences Biologiques.

____ des Sciences Physiques et Naturelles.

____ Italiennes de Biologie.

* Arkiv för Matematik, Astronomi och Fysik. (Stockholm.)

* ____ för Kemi, Mineralogi och Geologi. ,,

* ____ för Botanik. ,,

* ____ för Zoologi. ,,

Astronomie (L').

Astronomische Nachrichten.

Astrophysical Journal.

Athenæum.

Bericht über die Wissenschaftlichen Leistungen in der Naturgeschichte der niederen Thiere. Begründet von R. Leuckart.

Bibliothèque Universelle et Revue Suisse.

____ See Archives des Sciences Physiques et Naturelles.

Biologisches Centralblatt.

Blackwood's Magazine.

Bollettino delle Pubblicazioni Italiane. (*Presented.*)

Bookman.

Botanische Zeitung.

Botanisches Centralblatt.

____ Beiheft.

British Rainfall.

Bulletin Astronomique.

____ des Sciences Mathématiques.

____ Mensuel de la Société Astronomique de Paris. See L'Astronomie.

Cambridge British Flora. By C. E. Moss.

Catalogue of Scientific Papers, 1800–1900. Subject Index.

Centralblatt für Bakteriologie und Parasitenkunde.

____ für Mineralogie, Geologie und Palæontologie.

Ciel et Terre.

Contemporary Review.

Crelle's Journal. See Journal für Reine und Angewandte Mathematik.

Dictionary, New English. Ed. by Sir J. A. H. Murray.

Dingler's Polytechnisches Journal.

Edinburgh Medical Journal.

* Received by exchange.

Edinburgh Review.

Egypt Exploration Fund. Publications.

* Electrician.

Encyklopädie der Mathematischen Wissenschaften.

Engineering.

English Mechanic and World of Science.

* Essex Naturalist.

Fauna und Flora des Golfes von Neapel.

Flora.

Fortnightly Review.

* Gazette Médicale d'Orient.

* Geographical Journal.

* Geographical Magazine (Scottish).

* Geographie (La).

Geological Magazine.

Göttingische Gelehrte Anzeigen.

Indian Antiquary.

——— Engineering. (*Presented.*)

Indian Journal of Medical Research. (*Presented.*)

Intermédiaire (L') des Mathématiciens.

International Catalogue of Scientific Literature.

Internationale Revue der Gesamten Hydrobiologie und Hydrographie.

Jahrbücher für Wissenschaftliche Botanik (Pringsheim).

Jahresbericht über die Fortschritte der Chemie und verwandter Theile anderer
Wissenschaft.

Journal de Conchyliologie.

——— des Débats.

——— de Mathématiques Pures et Appliquées.

——— de Pharmacie et de Chimie.

* ——— de Physique.

——— des Savants.

——— für die Reine und Angewandte Mathematik (Crelle).

——— für Praktische Chemie.

——— of Anatomy and Physiology.

——— of Botany.

——— of Pathology and Bacteriology.

* ——— of Physical Chemistry.

* ——— of the Royal Society of Arts.

——— of the Society of Chemical Industry. (*Presented.*)

——— of the Washington Academy of Sciences.

Knowledge.

Manual of Conchology.

* Mathematische und Naturwissenschaftliche Berichte aus Ungarn.

Mind.

Mineralogical Magazine. (*Presented.*)

Mineralogische und Petrographische Mittheilungen (Tschermak's).

* Received by exchange.

Monist.

* Nature.

——— (La).

Neues Jahrbuch für Mineralogie, Geologie, und Palaeontologie.

——— Beilage.

Nineteenth Century.

Notes and Queries.

Nuova Notarisia (De Toni).

* Nuovo Cimento ; Giornale di Fisica, Chimica e Storia Naturale.

* Nyt Magazin för Naturvidenskaberne.

Observatory.

Optical Society, London, Transactions.

Page's Engineering Weekly. (*Presented.*)

Palæontographical Society's Publications.

Petermann's Mittheilungen.

——— Ergänzungsheft.

* Pharmaceutical Journal.

Philosophical Magazine. (London, Edinburgh, and Dublin.)

* Photographic Journal.

* Physical Review.

Plankton-Expedition Ergebnisse.

Quarterly Journal of Microscopical Science.

——— of Experimental Physiology.

Quarterly Review.

Ray Society's Publications.

Registrar-General's Returns (Births, Deaths, and Marriages). (*Presented.*)

Resultate der Wissenschaftliche Erforschung der Balatonsees.

Review of Neurology and Psychiatry.

* Revue Générale des Sciences Pures et Appliquées.

——— Philosophique de la France et de l'Étranger

——— Politique et Littéraire. (Revue Bleue.)

——— Scientifique. (Revue Rose.)

* ——— Semestrielle des Publications Mathématiques

Saturday Review.

Science.

* Science Abstracts.

——— Progress.

Scotsman.

Scottish Naturalist.

Symons's Meteorological Magazine.

Thesaurus Linguae Latinae.

Times.

Zeitschrift für die Naturwissenschaften.

——— für Krystallographie und Mineralogie.

——— für Wissenschaftliche Zoologie.

Zoological Record.

* Received by exchange.

- Zoologische Jahrbücher. Abteilung für Anatomie und Ontogenie der Tiere.
— Abteilung für Systematik, Geographie und Biologie der Tiere.
— Abteilung für Allgemeine Zoologie und Physiologie der Tiere.
Zoologischer Anzeiger.
— Jahresbericht.

ANNUALS (WORKS OF REFERENCE).

- Annuaire du Bureau des Longitudes.
Slater's Directory. (Scotland.)
County Directory. (Scotland.)
Edinburgh and Leith Directory.
English Catalogue of Books.
Medical Directory.
Minerva (Jahrbuch der Gelehrten Welt).
Minerva (Handbuch der Gelehrten Welt).
* Nautical Almanac.
Oliver & Boyd's Almanac.
University Calendars:—St Andrews, Edinburgh, Aberdeen, Glasgow, London
University College, Birmingham, Belfast, Sydney, N.S.W.; also Calendar of
Royal Technical College, Glasgow.
Wer ist's?
Whitaker's Almanack.
Who's Who.
Who's Who in Science (International).
Willing's Press Guide.
Year-Book of Scientific and Learned Societies of Great Britain and Ireland.
Zoological Record.

* Received by exchange.

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lossing and the loss of the other two children, and the terrible sense
of responsibility that I feel about it. But still I do not understand
why the good people of this country do not see this. We cannot wait
any longer.

— 2 —

Additional to the difficulties of the present situation, we must
face the fact that we have been compelled to leave our home
and our work, and we have no place to go.

It is now time for us to consider what we shall do. We have
no money, and we have no place to go.

We have no money, and we have no place to go.

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and the μ^2 term in the $\langle \bar{q}q \rangle$ potential is given by

$$\mu^2 = -\frac{1}{2} \left(\frac{\partial^2 \langle \bar{q}q \rangle}{\partial T^2} \right)_{T=0} = \frac{1}{2} \left(\frac{\partial^2 \langle \bar{q}q \rangle}{\partial T^2} \right)_{T=0} \left(\frac{m^2}{T^2} \right)$$

where m is the quark mass. The $\langle \bar{q}q \rangle$ potential is given by

$$\langle \bar{q}q \rangle = \frac{1}{2} m^2 T^2 + \frac{1}{2} \mu^2 T^2 + \frac{1}{2} \left(\frac{\partial^2 \langle \bar{q}q \rangle}{\partial T^2} \right)_{T=0} T^4 + \dots$$

Integrating with respect to T , we get the quark number density n_q and the quark chemical potential μ_q as

$$n_q = \frac{1}{2} \left(\frac{\partial \langle \bar{q}q \rangle}{\partial T} \right)_{T=0} = \frac{1}{2} \left(\frac{\partial \langle \bar{q}q \rangle}{\partial T} \right)_{T=0} \left(\frac{m^2}{T^2} \right) = \frac{1}{2} \left(\frac{\partial \langle \bar{q}q \rangle}{\partial T} \right)_{T=0} \left(\frac{m^2}{T^2} \right) \left(\frac{T^2}{T^2} \right) = \frac{1}{2} \left(\frac{\partial \langle \bar{q}q \rangle}{\partial T} \right)_{T=0}$$

and $\mu_q = \frac{1}{2} \left(\frac{\partial \langle \bar{q}q \rangle}{\partial T} \right)_{T=0} T = \frac{1}{2} \left(\frac{\partial \langle \bar{q}q \rangle}{\partial T} \right)_{T=0} \left(\frac{m^2}{T^2} \right) T = \frac{1}{2} \left(\frac{\partial \langle \bar{q}q \rangle}{\partial T} \right)_{T=0} \left(\frac{m^2}{T^2} \right) \left(\frac{T^2}{T^2} \right) = \frac{1}{2} \left(\frac{\partial \langle \bar{q}q \rangle}{\partial T} \right)_{T=0}$

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R. A. HOUSTOUN and A. H. GRAY.

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